Effect of Channel Side Information on the Capacity of MIMO Systems in Correlated Fading Channels

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Abstract — Using antenna arrays at both sides of the wireless communication link (MIMO systems) can result in high channel capacity provided that the channel is ideal, i.e., propagation medium is rich scattering or Rayleigh fading, the antenna arrays at both sides are uncorrelated. Also, the availability of channel side information (CSI) at the transmitter further enhances the capacity of ideal channels. This paper investigates the effect of CSI availability at the transmitter on the capacity of correlated MIMO channels under Rician fading. It is concluded that the availability of CSI at the transmitter is more significant at low SNR, and the improvement in capacity when CSI is available at the transmitter is more pronounced at higher values of the Rician parameter K and correlation parameter r.

Index Terms — Correlation, fading, MIMO, Rician channel, Channel side information

I. INTRODUCTION

There continues to be substantial interest in wireless communication systems that employ multiple transmit and receive antennas, due to their promise for dramatically increasing the performance and capacity to 20-40 bit/s/Hz when the system design is optimal [1]. Multiple-input multiple-output (MIMO) system can provide two types of gains: diversity gain and spatial multiplexing gain or capacity gain. With T transmit and R receives antennas, the system can provide spatial multiplexing gain of $m = min$ (T, R) [2]. However, the correlation of a real-world wireless channel may result in a substantial degradation of the MIMO architecture performance [3]–[5]. Secondly, there is a possibility that a line-of-sight (LOS) component may exist in addition to scattered components. Then, the fading will follow the Rician distribution, degrading the performance of MIMO, compared to Rayleigh fading [6]–[8].

The availability for CSI at the transmitter is also a key parameter in determining the multiplexing gains. It enhances the capacity under ideal channels conditions [9]–[11]. However, the effect of CSI availability at the transmitter on performance of real-world channels needs to be investigated. Recently, several studies have addressed this topic; however, in most of the published work, the joint effect of Rician fading, correlated channels and availability of channel information were not discussed.

In this paper, we investigate the joint effect of Rician fading, correlation, and availability of CSI on the capacity gains of MIMO channels. Capacity gains of slow and frequency nonselective MIMO systems are obtained using hybrid simulations. The effect of AWGN is calculated analytically, whereas the effects of Rician factor, correlated fading, and CSI at the transmitter are obtained using Monte Carlo simulation.

II. OVERVIEW OF THE PAPER

The paper is organized as follows. Section III describes the signal and the channel model. The effect of CSI availability at the transmitter on capacity is presented in sections IV. Section V presents the simulation results and discussion followed by conclusions in section VI.

III. MIMO CHANNEL AND SIGNAL MODEL

Consider a single user MIMO system with T antennas at the transmitter and R antennas at the receiver.

A. MIMO Capacity Signal Model

The system is described by the matrix equation [12]

$$\mathbf{y} = \sqrt{\frac{E_S}{T}}\mathbf{Hs} + \mathbf{n} \tag{1}$$

where E_s is the total energy available at the transmitter, $\mathbf{y}$ is the $R \times 1$ vector of signals received on the R antennas, $\mathbf{s}$ is the $T \times 1$ vector of signals transmitted on the T transmit antennas, $\mathbf{n}$ is the $R \times 1$ noise vector consisting of independent complex Gaussian distributed elements with zero mean and variance σ^2, and $\mathbf{H}$ is the $R \times T$ channel matrix.

B. Correlated Rician fading Channel Model

In Rician fading the elements of $\mathbf{H}$ are non-zero mean complex Gaussians. Hence we can express $\mathbf{H}$ in matrix notation as [13]

$$\mathbf{H} = a\mathbf{H}^{sp} + b\mathbf{H}^{sc} \tag{2}$$

where the specular and scattered components of $\mathbf{H}$ are denoted by superscripts, $a > 0$, $b > 0$ and $a^2 + b^2 = 1$. $\mathbf{H}^{sp}$ is a matrix of unit entries denoted as $\mathbf{H}_1$. If there is no correlation at the transmitter or at the receiver side then the entries of $\mathbf{H}^{sc}$ are independent and identically distributed (i.i.d) complex Gaussian random variables with zero mean and unit magnitude variance, usually denoted by $\mathbf{H}_\omega$. If there is correlated fading then the $\mathbf{H}^{sc}$ matrix can be modeled as [14].

$$\mathbf{H}^{sc} = \mathbf{R_r}^{1/2}\mathbf{H_\omega}\mathbf{R_t}^{1/2} \tag{3}$$

where $\mathbf{R}_t$ and $\mathbf{R}_r$ are the correlation matrix at the transmitter and at the receiver side, respectively. The correlation matrix $\mathbf{R}$ with correlation parameter r is defined as [4].

$$r_{ij} = \begin{cases} r^{j-i}, & i \le j \\ r^{*}{}_{ji}, & i > j \end{cases}, |r| \le 1 \tag{4}$$

where "*" denotes the complex conjugate. The Rician factor, K is defined as a^2/b^2. Thus, the above $\mathbf{H}$ matrix can be written as

$$\mathbf{H} = \sqrt{\frac{K}{K+1}}\mathbf{H}_1 + \sqrt{\frac{1}{K+1}}\mathbf{R}_{\mathrm{r}}{}^{1/2}\mathbf{H}_{\omega}\mathbf{R}_{\mathrm{t}}{}^{1/2} \tag{5}$$

IV. CAPACITY GAIN

In the following, we assume that the channel is perfectly known to the receiver (channel knowledge at the receiver can be maintained via training and tracking) whereas the channel state information at the transmitter may be available or not. Furthermore, we assume an ergodic block fading channel model where the channel remains constant over a block of consecutive symbols, and changes in an independent fashion across blocks. In fading channels there are essentially two notions of capacity: ergodic capacity and outage capacity [1] which relate to the mean and the tail behavior of capacity, respectively.

Ergodic Capacity: This is the time-averaged capacity of a stochastic channel. It is found by taking the mean of the capacity values obtained from a number of independent channel realizations.

Outage Capacity: The q% outage capacity $C_{out,q}$, is defined as the capacity that is guaranteed for $(100 - q)$% of the channel realizations [1], i.e.,

$$P\left(C \le C_{out,q}\right) = q\% \tag{6}$$

A. Channel Unknown at the Transmitter

Acquiring channel knowledge at the transmitter is in general very difficult in practical systems. When the transmitter has no channel state information, it is optimal to evenly distribute the available power ρ among the transmit antennas. The MIMO channel capacity with $\rho = E_s/\sigma^2$ can be written as

$$C = E_{\mathbf{H}}\left\{\log_2 \det\left(\mathbf{I}_R + \frac{\rho}{T}\mathbf{HH}^H\right)\right\} \tag{7}$$

where $E_{\mathbf{H}}\{.\}$ denote the expectation over **H** and the operator $\mathbf{H}^H$ indicates the hermitian of the matrix **H**. Using singular value decomposition (SVD), (7) can be decomposed as

$$C = E_{\mathbf{H}}\left\{\sum_{i=1}^{k} \log_2\left(1 + \frac{\rho}{T}\lambda_i\right)\right\} \qquad (8)$$

where k, ($k \le m$) is the rank of **H**, and λ_i ($i = 1, 2, .., k$) denotes the positive eigenvalues of $\mathbf{HH}^{\mathrm{H}}$.

B. Channel Known at the Transmitter

When the channel parameters are known at the transmitter, i.e., the channel state information (CSI) is available at the transmitter, the capacity given by (7), or (8), can be increased by assigning the transmitted power to various antennas according to the "water-filling" algorithm [12].

$$C = E_{\mathbf{H}}\left\{\sum_{i=1}^{k} \log_2\left(1 + \rho_i\lambda_i\right)\right\} \qquad (9)$$

where ρ_i, is the power assigned to the i^{th} transmitter and μ is chosen to satisfy:

$$\rho_i = (\mu - \lambda_i^{-1})^+ \qquad (10)$$

$$\rho = \sum_{i=1}^{k} \rho_i \qquad (11)$$

and "+" denotes taking only those terms which are positive.

V. SIMULATION RESULTS

The results in this section illustrate the effect of correlated fading and Rician factor on the capacity gains of MIMO fading channels. In view of the fact that **H** matrix represents the channel, the elements of **H** matrix are random and depend upon the type of fading encountered by the channel.

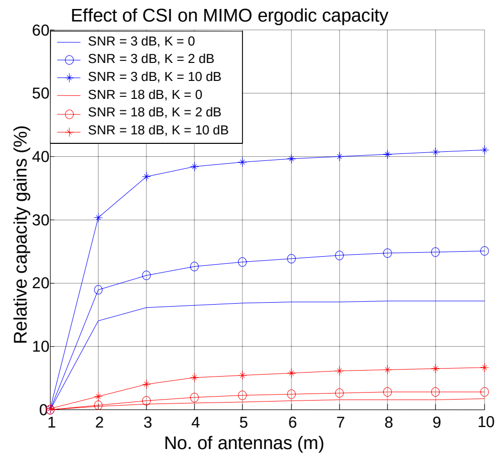
For Rician fading channels, the elements of **H** matrix consist of two parts [6]. The first part, $\mathbf{H}^{sp}$ is fixed while the second part consists of $\mathbf{H}^{sc}$, which is random. The elements of $\mathbf{H}_{\omega}$ are i.i.d. zero mean circularly symmetric complex Gaussian (ZMCSCG) random numbers with unit variance, generated using Matlab software. Here, in this section we present the effect of CSI on the capacity of $m \times m$ MIMO systems under different channel conditions. In all the cases discussed here, we have considered correlation at both ends. Fig. 1 presents the relative capacity gain in percentage due to the availability of CSI at the transmitter as function of the number of antennas (m) for various values of SNR, Rician fading parameter K, and correlation parameter (r).

In order to ascertain the correctness of our results, we compare our results with the results presented in [10] for a special case when $r = 0$. This comparison is presented in Fig.1-a. The following can be deduced from Fig.1:

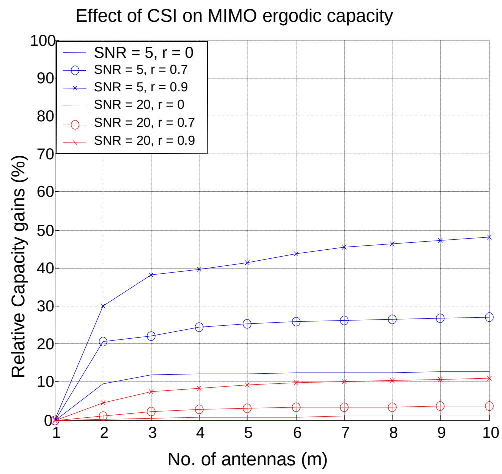
- It is interesting to note that the water-filling gains i.e., when the CSI is available at the transmitter compared to equal power i.e., when no CSI is available at the transmitter are significant at low SNR and reduces at high SNR. The fact that water-filling gains are reduced at high SNR levels can be intuitively explained by the fact that knowledge of the CSI provides array gain both at the transmitter and at the receiver. Since the relative importance of transmit array gain in boosting average SNR decreases in high SNR region, the benefit of CSI at the transmitter also reduces.

- Increasing the K factor reduces the capacity of the system, therefore, when the K factor increases, knowledge of CSI at the transmitter becomes relatively more important than when the K factor is small. This can be seen from the Fig.1-a.

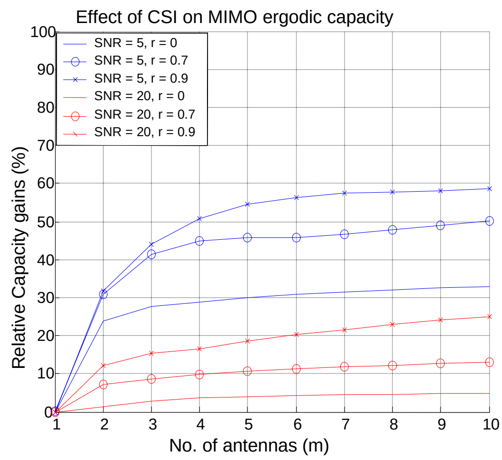
- Increasing channel correlation r reduces the capacity of the system. Hence, for highly correlated fading, knowledge of CSI at the transmitter becomes more important than when the correlation parameter is small. This trend is seen from the Fig. 1-b and Fig. 1-c.



(a) $r = 0$



(b) $K = 0$



(c) $K = 10$

Fig. 1: Relative Ergodic Capacity gain due to CSI at the transmitter vs. m

VI. CONCLUSIONS

For $m \times m$ MIMO systems, we found that for all values of m, as the value of K increases, the ergodic capacity decreases. However, the loss in the capacity is more at higher values of m. Also, for $m \times m$ MIMO systems, we found that for all values of m, as the value of correlation parameter (r) increases, capacity decreases. However, the loss in the capacity is more at higher values of m.

Water-filling gain i.e., when CSI is available at the transmitter compared to equal power, is significant at low SNR and reduces at high SNR. Relative capacity gain when CSI is available at the transmitter is more pronounced at higher values of K and r i.e., the improvement in capacity when CSI is available at the transmitter is more at higher values of K and r. Hence, CSI is more valuable for non ideal MIMO channels.

REFERENCES

[1] A. J. Paulraj, D. Gore, R. U. Nabar, H. Bolcskei, "An overview of MIMO communications–A key to gigabit wireless", *Proceedings of the IEEE* vol. 92, no. 2, pp. 198-218, Feb. 2004.

[2] L. Zheng, and D. Tse "Diversity and Multiplexing: A funadamental tradeoff in multiple- antenna channels" *IEEE Trans. Inform. Theory*, vol. 49, no. 5, pp. 1073–1096, May 2003.

[3] D.-S. Shiu, G. J. Foschini, M. J. Gans, and J. M. Kahn, "Fading correlation and its effect on the capacity of multi-element antenna systems," *IEEE Trans. Commun.*, vol. 48, no. 3, pp. 502–513, Mar. 2000

[4] S. L. Loyka, "Channel capacity of MIMO architecture using the exponential correlation matrix," *IEEE Commun. Lett.*, vol. 5, no. 9, pp. 369-371, Sept. 2001.

[5] M. Kang and M. Alouini, "Capacity of correlated MIMO Rayleigh channels", *IEEE*

Trans. Wireless Commun., vol. 5, no. 2, pp. 143–155, Jan. 2006.

[6] S. Jayaweera and H. Poor*, "*On the Capacity of Multiple-Antenna Systems in Rician Fading", *IEEE Trans. Wireless Commun.*, vol. 4, no. 3, pp. 1102–1111, May 2005.

[7] M. Kang and M. Alouini, *"*Capacity of MIMO Rician channels", *IEEE Trans. Wireless Commun.*, vol. 5, no. 2, pp. 112–122, Jan. 2006.

[8] J. Sun, and I. Reed "Linear diversity analysis for M-PSK in Rician fading channels", *IEEE Trans. Commun.,* vol. 51, no. 11, pp. 1749–1753, Nov. 2003.

[9] A. J. Paulraj, D. Gore, and R. U. Nabar, "Performance limits in fading MIMO channels", Globecome, 2002.

[10] D. Gesbert, M. Shafi, D.-S. Shiu, P. J. Smith, and A. Naguib "From theory to practice: An overview of MIMO space–time coded wireless systems", *IEEE J. Select. Areas Commun.*, vol. 21, no. 3, pp. 281–302, April. 2003.

[11] S. M. Qaseem and Adel A. Ali, "Capacity of correlated MIMO Rician channels" International Conference of Communication Computer and Power - ICCCP'05, Feb. 14-16, 2005, Muscat, Oman.

[12] I.E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Transactions on Telecommunications*, vol. 10, no. 6, pp. 585-595, 1999

[13] F. R. Farrokhi, G. J. Foschini, A. Lozano and R. A. Valenzuela, "Linkoptimal space-time processing with multiple transmit and receive antennas", *IEEE Commun. Lett.*, vol. 5, no. 3, pp. 85-87, March 2001.

[14] D. Gesbert, H. Bolcskei, D. J. Gore, A. J. Paulraj, "Outdoor MIMO wireless channels: Models and Performance prediction", *IEEE Trans. Commun.*, vol. 50, pp. 1926–1934, Dec. 2002.