BER for M-QAM with Space-Time Transmit Diversity in Nakagami and Rician Fading Channels

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Abstract-**This paper presents simple bit error rate results for M-ary Quadrature Amplitude Modulation (M-QAM) transmitted over slow, flat, identically independently distributed (i.i.d) Nakagami and Rician fading channels and using space-time transmit diversity (STTD) to enhance coverage and capacity. A single exponential function is used to approximate the symbol error rates (SER) for M-QAM modulation over a Gaussian channel. This approximation is bounded within 1 dB for the signal alphabet M ≥ 4 and 0 ≤ SNR ≤ 30 dB. Error rates for M-QAM with STTD is then obtained by averaging the conditional SER over the probability density function (pdf) of the received SNR per bit for i.i.d. Nakagami fading channels with STTD with two transmit antennas and one receive antenna. Then using the known relation between the Rician and Nakagami fading families we obtain a similar, simple, expression for the error rate of M-QAM with STTD over Rician fading channels. Our error rate expressions contain only simple exponential functions and are within one dB from exact results.**

***Keywords*- Error-rate, diversity reception, error probability performance, MQAM systems, Rician-fading channels.**

I. INTRODUCTION

The most adverse propagation effect from which wireless communications systems suffer is the multipath fading. One of the common methods used by wireless communications engineers to combat multipath fading is the antenna diversity technique. A classical combining technique is maximum-ratio combining (MRC) [1], where the signals from the received antenna elements are weighted such that the signal-to-noise ratio (SNR) of their sum is maximized. In recent years, space diversity has been increasingly popular since it is a means of improving the reliability without having to sacrifice spectral efficiency [2-4]. Alamouti [5] has presented the well known simple space time transmit diversity (STTD) analysis which has been well received by the scientific community around the world. His work has shown that optimum transmit diversity can be accomplished without the channel state information at the transmitter (CSIT). The results were presented for two transmit antennas over Rayleigh fading channel. The results have been extended to the case of Nakagami fading channels, and exact bit error rate expressions under i.i.d Nakagami fading channels with STTD were obtained in [6] for MPSK and for M-QAM with rectangular constellations and $M= 4^k$, k even. The expressions include the summation terms of hyper-geometric functions. Unfortunately, its use leads to numerical un-stability. In this paper, we have used an approximate expression for the SER of M-QAM over Gaussian channels and obtained a simple closed form expression for the SER for dual branch STTD over Nakagami and Rician channels. On comparing the exact and the approximate SER of M-QAM scheme under i.i.d Rician fading with STTD, we found that the maximum difference between the two is 0.8 dB at SER of 10^{-5}.

II. OVERVIEW OF THE PAPER

The paper is organized as follows: Section III presents an approximation for the BER for M-QAM modulation over AWGN channel. SER for M-QAM over Nakagami and over Rician fading channels with STTD with two transmit antennas and one receive antenna are presented in section IV, and section V, respectively, followed by conclusions in section VI

III. APPROXIMATE BER FOR M-QAM MODULATION OVER AWGN CHANNEL

For rectangular signal constellation in which $M=2^k$, where k is even, the exact symbol error rate for the M-QAM under Gaussian channel is given as [7]

$$P_{AWGN}(\gamma) = 1 - \left(1 - 2(1 - \frac{1}{\sqrt{M}})Q(\sqrt{\frac{3}{M-1}\gamma})\right)^2 \qquad (1)$$

The approximate BER for the M-QAM square constellation with gray coding under Gaussian channel bounded within 1 dB for M ≥ 4 and 0 ≤ SNR ≤ 30 dB is given as [8]

$$P_b(\gamma) \leq 0.2\exp[-\frac{1.5\ \gamma}{(M-1)}] \quad (2)$$

In terms of the instantaneous SNR γ, the conditional probability of symbol error for M-QAM is given as follows:

$$P(E/\gamma) = 0.\ 2\ \exp[-\ \frac{1.5\ \gamma}{(M-1)}]\ \log_2 M \quad (3)$$

IV. SER FOR M-QAM OVER NAKAGAMI FADING CHANNELS WITH STTD

The probability density function (*pdf*) of γ of the received SNR per bit for i.i.d. Nakagami fading channels with STTD with two transmit antennas and one receive antenna is given as [6]

$$p_\gamma(\gamma) = \frac{1}{E_b/N_o}\, p_\gamma(\frac{\gamma}{E_b/N_o}) = \frac{2\sqrt{\pi}m^{2m}\gamma^{2m-1}}{\Gamma(m)\Gamma(\frac{1}{2}+m)\bar{\gamma}^{2m}}\exp[\frac{-2m\gamma}{\bar{\gamma}}] \quad (4)$$

The SER under fading can be obtained simply by averaging the SER in AWGN channel over the fading signal statistics, that is

$$P(E) = \int_0^\infty P(E/\gamma)p_\gamma(\gamma)d\gamma \quad (5)$$

After substituting (3) and (4) into (5), we get

$$P(E) \approx \int_0^\infty \{\frac{2\sqrt{\pi}m^{2m}\ \gamma^{2m-1}}{\Gamma(m)\Gamma(\frac{1}{2}+m)\ \bar{\gamma}^{2m}}\exp[\frac{-2m\gamma}{\bar{\gamma}}]\}.\ 0.2\exp[-\frac{1.5\ \gamma}{(M-1)}]\ \log_2 M\, d\gamma \quad (6)$$

Using the following relation [9]

$$\int_0^\infty x^n.e^{-xa}d(x) = \begin{cases} \frac{\Gamma(n+1)}{a^{n+1}}, n > -1, a > 0 \\ \frac{n!}{a^{n+1}} a > 0, n: positive \end{cases} \quad (7)$$

The SER for M-QAM for i.i.d. Nakagami fading channels with STTD is given as follows:

$$P(E) \approx 0.4\sqrt{\pi}(\frac{m\ (M-1)}{(2m\ (M-1)+1.5\ \bar{\gamma})})^{2m} .\frac{\Gamma(2m)}{\Gamma(m)\Gamma(\frac{1}{2}+m)}\log_2 M \quad (8)$$

Comparison with exact results

Figure 1 depicts SER for 16-QAM with STTD using 2 transmit and one receive antennas over Nakagami fading channels. Exact performance based on analysis in [6] and approximate performance given by (8), are both shown. Table 1 summarizes the comparison for SEP=10^{-2} and 10^{-3} for values of m=0.5, 1 and 2. It can be seen can see that at m=0.5 the results coincide, and for m=1 the maximum difference between the two is 0.7 dB and, at m=2 the maximum difference is 0.8 dB

Table 1: Exact vs. Approximate SER for 16-QAM with STTD over Nakagami fading channels at SER=10^{-2} and 10^{-3}

Fading factor	*m*=1 S/N dB			*m*=2 S/N dB		
SER	Exact	Approx	Difference	Exact	Approx	Difference
10^{-2}	15.4	16	0.6	12.2	12.9	0.7
10^{-3}	20.6	21.3	0.7	15.6	16.4	0.8

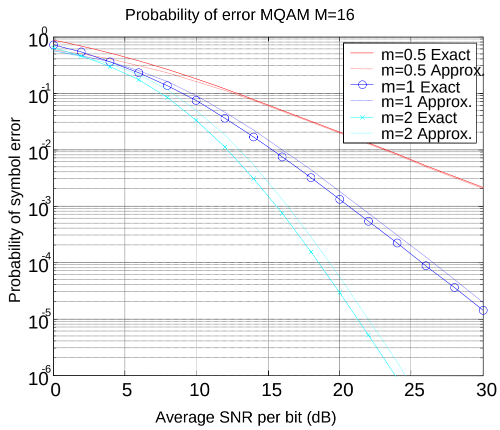

Figure 1: Comparison of SER for 16-QAM with STTD: Exact vs. Approximate under Nakagami Fading channels

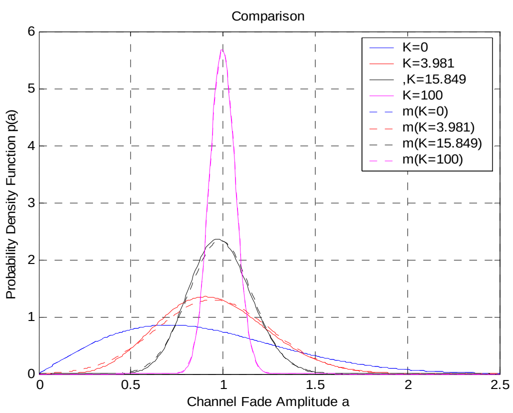

Figure 2: Nakagami-m and Rician distributions with equal AF "Amount of Fading" (9)

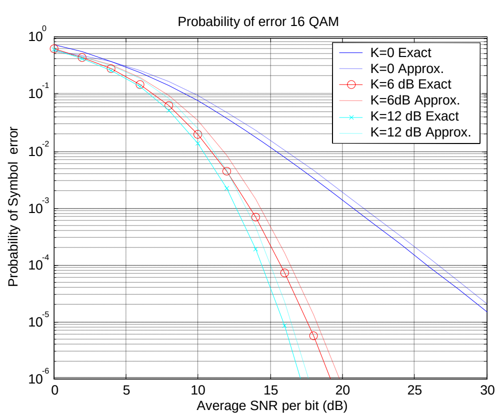

Figure 3: Comparison of SER for 16-QAM with STTD: Exact vs. Approximate under Rician Fading channels

V. SER FOR M-QAM OVER RICIAN FADING CHANNELS WITH STTD

A. Relation between Nakagami-m and Rician distribution

The relation between Nakagami-*m* and Rician distribution can be obtained by equating the amount of fading (AF), or "fading figure," associated with the fading PDF is defined as [10]

$$AF=\frac{\mathrm{var}(\alpha^2)}{(E([\alpha^2])^2}=\frac{E(\alpha^2-\Omega)^2]}{\Omega^2}=\frac{E(\gamma^2)-(E[\gamma])^2}{(E[\gamma])^2} \qquad (9)$$

And we obtain

$$m=\frac{(1+K)^2}{1+2.K} \qquad (10)$$

Figure 2 shows the pdf of the Rician and Nakagami distributions, with values of K and m satisfying (10). From the figure we see that the Rician and Nakagami-m distributions can closely approximate each other.

B. Symbol error rate

The *pdf* of received SNR per bit under Rician fading with STTD is obtained by substituting from (10) into (4) as follows

$$p_\gamma(\gamma)=\frac{2.\sqrt{\pi}\left(\frac{(1+K)^2}{1+2K}\right)^{\frac{2.(1+K)^2}{1+2K}}\gamma^{\frac{2.(1+K)^2}{1+2K}-1}}{\Gamma\left(\frac{(1+K)^2}{1+2K}\right)\Gamma\left(\frac{1}{2}+\frac{(1+K)^2}{1+2K}\right)\bar{\gamma}^{\frac{2.(1+K)^2}{1+2K}}} \qquad (11)$$

$$.\exp\left[\frac{-\frac{2(1+K)^2}{1+2K}\gamma}{\bar{\gamma}}\right]$$

Substituting from (3) and 11) into (5), we get

$$P(E)\approx 0.4\sqrt{\pi}\left[\frac{\left(\frac{(1+K)^2}{1+2K}\right)(M-1)}{2\frac{(1+K)^2}{1+2K}(M-1)+1.5\bar{\gamma}}\right]^{2.\frac{(1+K)^2}{1+2K}} \qquad (12)$$

$$.\frac{\Gamma\left(2\frac{(1+K)^2}{1+2K}\right)}{\Gamma\left(\frac{(1+K)^2}{1+2K}\right)\Gamma\left(\frac{1}{2}+\frac{(1+K)^2}{1+2K}\right)}\log_2 M$$

Equation (12) presents the probability of symbol error for M-QAM modulation for i.i.d Rician fading channels with space-time transmit diversity (STTD). As *K* goes to zero, the probability of bit error for M-QAM over i.i.d Rayleigh fading channels with STTD is:

$$P(E)\approx 0.8\left[\frac{(M-1)}{2(M-1)+1.5\bar{\gamma}}\right]^2\log_2 M \qquad (13)$$

C. Comparison with exact results

Figure 3 depicts SER for 16-QAM with STTD using 2 transmit and one receive antennas over Rician fading channels. Exact performance based on numerical integration and approximate performance given by (12), are both shown at SER=$10^{-4}$and 10^{-5} for different values of *K* and Table 2 summarizes the comparison.

VI. CONCLUSIONS

This paper has presented simple closed form SER expressions for M-QAM transmitted over slow, flat, identically independently distributed (i.i.d) Nakagami and Rician fading channels and using space time transmit diversity. A single exponential function is used to approximate symbol error rate (SER) for M-QAM modulation over a Gaussian channel. This approximation is bounded within 1 dB for $M \geq 4$ and $0 \leq SNR \leq 30$ dB. Two cases are considered: the first is Nakagami-m fading channel using simple space-time transmit diversity (STTD) with no channel side information available at the transmitter (CSIT). Secondly, the relation between the Nakagami and Rician families of fading channels is invoked and SER is obtained for Rician fading channels using STTD. Our results have been compared with exact results for Nakagami and for Rician channels, and the maximum difference between the two is 0.8 dB.

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Table 2: Exact Vs. Approximate SER for M-QAM with STTD under Ricain at SER =10^{-4} and 10^{-5}

K=0, 10^{-4}			K=6 dB, 10^{-4}			K=12 dB, 10^{-4}		
Exact dB	Approx. dB	Difference dB	Exact dB	Approx. dB	Difference dB	Exact dB	Approx. dB	Difference dB
25.8	26.4	0.7	15.7	16.3	0.6	14.8	15	0.2
K=0, 10^{-5}			K=6 dB, 10^{-5}			K=12 dB, 10^{-5}		
Exact dB	Approx. dB	Difference dB	Exact dB	Approx. dB	Difference dB	Exact dB	Approx. dB	Difference dB
30.8	31.4	0.6	18	18.3	0.3	15.9	16.2	0.3

REFERENCES

[1] D. G. Brennan, "Linear Diversity Combining Techniques," *Proc. IEEE* vol. 91, no. 2, Feb 2003.

[2] J. Sun and I. Reed, "Linear diversity analyses for M-PSK in Rician fading channels," *IEEE Trans. Commun*., vol. 51, no. 11, pp. 1749-1753, Nov. 2003.

[3] Annamalai, C. Tellambura, and V. Bhargava, "A general method for calculating error probabilities over fading channels", *IEEE Trans. Commun.,* vol.53, no. 5, pp. 841-852, May 2005

[4] S. Seo, C. Lee and S. Kang, "Exact performance analysis of *M*-ary QAM with MRC diversity in Rician fading channels," *IEE Electronics Letters,* vol. 40, no. 8, Apr. 2004

[5] S. M. Alamouti "A Simple Transmit Diversity Technique for Wireless Communications," *IEEE J. Select. Areas commun*., vol. 16, pp. 1451-1458, Oct. 1998

[6] R. He and J. Xei, "BER Performance of M-QAM and MPSK Nakagami fading channel with STTD," *IEEE Inter. Sympo. On personal indoor and mobile Radio commun*., 2003.

[7] J. G. Proakis, *Digital Communications*. McGraw-Hill, Singapore, Fourth edition, 2001.

[8] Goldsmith and S. G. Chua, "Variable-rate variable-power M-QAM for fading channels," *IEEE Trans. Commun*., vol. 45, pp. 1218–1230, Oct. 1997.

[9] S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*. San Diego, Fifth edition, CA: Academic, 1994.

[10] M. K. Simon and M. S. Alouini, *Digital Communications over Fading Channels*. John Wiley & Sons, Inc., New York, 2000.