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DEVELOPMENT OF STATISTICAL THEORY IN RELIABILITY ENGINEERING: A SURVEY

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ABSTRACT

Statistical theory and its applications have played a significant role in the advancement of methodology and techniques for solving various problems in reliability engineering. Different criteria of aging have been introduced for modelling and characterizations of wear properties. Some real life engineering reliability operations arising in maintenance and replacement policies formulations are investigated as convolution, mixture and formation of coherent systems. Shock models are studied for shocks arriving according to Poisson process and causing damages with specific form of aging. Cumulative damage models have been also studied by many researchers. Statistical hypotheses testing are used to test whether a certain, censored or uncensored data has exponential or a specific aging propety.

In this paper we present a unified state of the art type of survey for the research and developments in these problems. In particular we expose the main recent contributions, trace the trend of research and highlight the possible direction of further research.

1. INTRODUCTION

There have been a significant developments, in the last two decades, of ways and means of statistical theory to solve or model different problems in reliability engineering. Statisticians have introduced various life criteria to represent the aging or wear of units or modules in a system or the system itself. Bryson and Siddiqui (1969) have given the interrelationships between various classes of life distributions, with aging properties existing at that time, and construcuted some counter-examples for impossible implications. Some additional concepts of positive or negative aging have been introduced by many authors see Barlow and Proschan (1981).

In Section 2 we give a full accounts of the main existing classes. Concepts of partial ordering between two life distributions or their respective random variables (r.vs.) have been used to characterize aging criteria when the ordering is made with respect to exponential. For example convex ordering was introduced by Van Zwet (1964) and star-shaped ordering was discussed by Barlow and Proschan (1966) and Marshal, Olkin and Proschan (1967). These concepts of partial orderings and other related ones are discussed in section 3.

In section 4 we review most classes of life distributions presented in section 2 with respect to closure properties under some reliability operations. These operations, namely, are

(i) formation of coherent systems,

(ii) addition of lifelengths or convolution, and

(iii) random or probabilistic selection of an item(s) from a set of distributions (population) or mixture.

In section 5 we give discrete version of these classes of life distributions. We, in addition, discuss life distributions of a device, modules or systems which are subjected to homogeneous, nonhomogeneous Poisson processes or accumulated shock models where damages occur according to a specific aging criterion. Esary, Marshal and Proschan (1973) studied shock model of homogeneous Poisson and cumulative damage model for all aging concepts in Bryson and Siddiqui (1969).

In section 6 we briefly review contributions in testing statistical hypotheses for life data. The survey included testing based on complete data and random censoring. Total time on test transform, see Barlow and Compo (1975), is also discussed.

Note that throughout the paper increasing and decreasing are used for nondecreasing and nonincreasing, respectively.

It should be noted that topics such as structural properties of systems, allocation of spares, relevancy conditions and measures of importance in either binary and multistate structures are not discussed here. These topics are too wide to be included and they deserve a separate survey.

In this paper we do not intend to present merely the existing results in exhustive manner but we present a unified state of art type of survey of the main contributions in these problems.

2. CRITERIA OF AGING

Let T be a nonnegative r.v. with (life) distribution $F(t) = P(T \leq t)$ where $F(t) = 0$ for $t < 0$ and $F(0)$ may not be zero. The corresponding survival distribution is $\overline{F}(t) = 1 - F(t)$, for $t \geq 0$ and the density function (if exists) is $f(t) = \frac{d}{dt} F(t)$. The instantaneous conditional failure rate, or simply the failure rate, at time t is defined by

$$r(t) = \lim_{\delta t \to 0} \frac{1}{\delta t} \frac{P(t < T \leq t + \delta t)}{P(T > t)}$$

$$= f(t) / \overline{F}(t), \quad t \geq 0 . \tag{2.1}$$

r(t) in (2.1) is also known by hazard rate, force of mortality or intensity rate. The cumulative failure rate $R(t) = \int_0^t r(u)\,du$ can be derived from (2.1) to give

$$\overline{F}(t) = \exp\left[- \int_0^t r(u) du \right]$$

$$= \exp\left[- R(t) \right] .$$

Any aging property may be associated to the lifelength T, its life distribution F, its survival $\overline{F}$ or its density function f(t).

Definition 2.1 The r.v. T is said to have an increasing failure rate, denoted by IFR, if

$$\overline{F}(t + x) \,|\, \overline{F}(t) \nearrow \text{in } t, \qquad t \geq 0, \tag{2.2}$$

where $\nearrow$ ($\searrow$) stands for increasing (decreasing).

If f(t) exists then Definition (2.1) is equivalent to stating that

$$r(t) \nearrow \text{in } t, \quad \text{for } t \geq 0 . \tag{2.3}$$

Note that (2.3) implies that $\frac{1}{t} R(t) \nearrow$ in t.

Definition 2.2 The life distribution has increasing failure rate average (IFRA) property if

$$t^{-1}\int_0^t r(u)\,du \nearrow \text{ in } t, \qquad t > 0. \tag{2.4}$$

Relation (2.3) means that $t^{-1}\log\overline{F}(t) \nearrow$ in t or

$$\frac{s}{t}\log\overline{F}(t) \le \log\overline{F}(s), \qquad t \ge s > 0.$$

Assume that

$$\overline{F}(s+t) > \overline{F}(s)\,\overline{F}(t)$$

or

$$\log\overline{F}(s+t) > \log\overline{F}(s) + \log\overline{F}(t)$$

$$\ge \frac{s+t}{t}\log\overline{F}(t).$$

Therefore

$$(s+t)^{-1}\log\overline{F}(s+t) \ge t^{-1}\log\overline{F}(t).$$

This contradicts (2.3) which means that

$$\overline{F}(s+t) \le \overline{F}(t)\,\overline{F}(s). \tag{2.5}$$

Definition 2.3 F is new better than used (NBU) if

$$\overline{F}(t \mid s) \le \overline{F}(t \mid 0), \quad \forall\, t,\ s \ge 0. \tag{2.6}$$

Integrating both sides of relation (2.6) or equivalently (2.5) over $[0,\infty]$ implies that

$$\int_0^\infty \overline{F}(t+s)ds \le \overline{F}(t)\int_0^\infty \overline{F}(s)ds$$

This gives

$$\int_t^\infty \overline{F}(u)du \le \mu\overline{F}(t), \tag{2.7}$$

where $\mu = ET = \int_0^\infty \overline{F}(s)ds$. In fact (2.7) means

$$E(T - t \mid T > t) \le ET.$$

Definition 2.4 F is new better than used in expectation (NBUE) if

$$\frac{\int_t^\infty \overline{F}(u)du}{\overline{F}(t)} \leq \mu \quad t \geq 0 . \tag{2.8}$$

In fact the L.H.S. of relation (2.8) is denoted by $\mu(t) = E(T - t \mid T > t)$, the mean remaining life of a unit at time t. One can see that if T is an IFR r.v. that

$$\mu(t) \searrow \text{in } t, \quad t \geq 0 . \tag{2.9}$$

Definition 2.5 F has decreasing mean remaining life (DMRL) property if

$$\mu(t) = \int_0^\infty \overline{F}(t+u)du \,/\, \overline{F}(t) \searrow \text{ in } t, \tag{2.10}$$

where $\overline{F}(t) > 0$ for $t \geq 0$.

For these classes of distributions with the previous ageing criteria we refer to Bryson and Siddiqui (1969) and Barlow and Proschan (1981). Rolski (1975) noted that relation (2.8) is the same as

$$[\mu(t)]^{-1} \geq \mu , \quad t \geq 0. \tag{2.11}$$

Integrating (2.11) over [0,t] implies that

$$\int_t^\infty \overline{F}(u)du \leq \mu e^{-t/\mu} \quad t > 0 . \tag{2.12}$$

Definition 2.6 F has harmonic new better than used in expectation (HNBUE) if relation (2.12) is satisfied.

Note that if $\mu(t) \nearrow$ in t and $\mu(0) = \mu$ exists then $\mu(t) \leq \mu$. Recall relation (2.5) one has

$$\overline{F}(t) - \overline{F}(s+t) \geq \overline{F}(t)\,[1 - \overline{F}(s)] .$$

Dividing by s and taking the limit as s tends to zero, we get

$$\frac{1}{\overline{F}(t)} \lim_{s\to 0} \frac{F(s+t) - F(t)}{s} \geq \lim_{s\to 0} \frac{F(s)}{s} .$$

This implies that

$$\frac{f(t)}{\overline{F}(t)} \leq f(0), \quad t \geq 0. \tag{2.13}$$

if f(t) and f(0) are defined.

Definition 2.7 F is new better than used failure rate (NBUFR) if

$$r(t) \leq r(0) \quad t \geq 0. \tag{2.14}$$

This concept was introduced by Deshpande et al. (1986) and Abouammoh and Ahmed (1988). Integrating both sides of (2.14) over [0,t] implies that

$$t^{-1} \int_0^t r(u)du \geq r(0), \quad t > 0. \tag{2.15}$$

Definition 2.8 F has the new better than averaged failure rate (NBAFR) if

$$-t^{-1} \log \overline{F}(t) \geq r(0), \quad t > 0. \tag{2.16}$$

Relation (2.16) was introduced and studied by Loh (1984). Integrating the NBU and the HNBUE properties, defined by relations (2.6) and (2.12) over [0,t] and [t,∞], respectively implies

$$\int_0^t \overline{F}(u \mid s)du \leq \int_0^t \overline{F}(u)du, \quad t \geq 0, \ s \geq 0. \tag{2.17}$$

and

$$\int_t^\infty \int_s^\infty \overline{F}(u)duds \leq \mu^2 e^{-t/\mu}, \quad t \geq 0, \tag{2.18}$$

and $\mu = ET \leq \infty$. Distributions satisfying (2.17) and (2.18) are called NBU(2) and HNBU(3) by Deshpande et al. (1986). Abouammoh and Ahmed (1989) have studied these classes of life distributiion under different names as follows.

Definition 2.9 F is the new better that used in averaged conditional survival (NBUAS) if (2.17) is satisfied.

Definition 2.10 F has the harmonic new better than used in expectation in the upper tail (HNBUET) if relation (2.18) is satisfied

Note that if t approaches ∞ as in (2.17), then the relation gives $\mu(s) \leq \mu$, for $s \geq 0$. Also, deviding both sides of (2.17) by μ $(\mu > 0)$ implies

$$\frac{\int_t^{\infty} \overline{F}(u+s)du/\mu}{\int_t^{\infty} \overline{F}(u+s)du/\mu} \leq \overline{F}(s)$$

or

$$\overline{W}(s \mid t) \leq \overline{F}(s), \qquad t \geq 0, s \geq 0,$$

where

$$\overline{W}(t) = \mu^{-1} \int_t^{\infty} \overline{F}(u)\, du, \;\; W(t) = \mu^{-1} \int_0^t \overline{F}(u)du,$$

$W(t) = \mu^{-1}\, \overline{F}(t)$ and μ is the mean of the parent distribution F since the mean of W is denoted by μ_w. The stationary renewal distribution for independent and identically distributed replaced units with distribution F is given by W(t).

Definition2.11 F is new better than renewal used (NBRU) if

$$\overline{W}(t \mid s) \leq \overline{F}(t), \quad t \geq 0, \; s \geq 0. \tag{2.19}$$

Cao and Wang (1991) have referred to this class by NBUC i.e. new better than used in convex ordering property. Integrating both sides of (2.19) w.r.t. to t over $[0,\infty]$ implies that

$$\int_s^{\infty} \overline{W}(t) \leq \overline{W}(s)\mu, \qquad t \geq 0. \tag{2.20}$$

This class of life distribution is introduced by Abouammoh, Ahmed and Barry (1992) .

Definition 2.12 F has new better than renewal used in expectation (NBRUE) if

$$E(T_w - t \mid T_w > t) \leq ET, \quad t \geq 0. \tag{2.21}$$

In fact relation (2.21) states that

$$\frac{\int_t^{\infty} \overline{W}(u)du}{\overline{W}(t)} \leq \int_0^{\infty} \overline{F}(u)du.$$

or

$$\mu_w(t) \leq \mu \qquad t \geq 0.$$

Note that $[\mu_w(t)]^{-1} \geq \mu^{-1}$ and integrating over (0,t) implies that

$$\int_0^t [\mu_w(u)]^{-1} du \geq t\mu^{-1}$$

$$\int_t^\infty \int_s^\infty \overline{F}(u)duds \leq e^{-t/\mu} \int_0^\infty \int_s^\infty \overline{F}(u)duds \qquad (2.22)$$

Definition 2.13 F is harmonic new better than renewal used in expectation (HNBRUE) if

$$t^{-1} \int_0^t \mu_w(u)du \geq \mu^{-1}, \quad t > 0. \qquad (2.23)$$

Recall the NBUFR property (2.14) or

$$f(u) \leq r(0)\,\overline{F}(u), \quad u \geq 0.$$

integrating for u over $[t,\infty]$ implies that

$$\overline{F}(t) \leq r(0) \int_t^\infty \overline{F}(u)du, \quad t \geq 0. \qquad (2.24)$$

Definition 2.14 F has new better than used renewal failure rate (NBURFR) if

$$r_w(t)du \leq r(0), \quad t \geq 0. \qquad (2.25)$$

Taking the average of both sides of (2.25) gives

$$\int_t^\infty \frac{w(u)}{\overline{W}(u)} \geq t\,r(0), \quad t > 0. \qquad (2.26)$$

Definition 2.15 F has new better than averaged (used) renewal failure rate (NBARFR) if

$$t^{-1} \int_0^t r_w(u)du \leq r(0), \quad t > 0. \qquad (2.27)$$

The last two criteria of aging are due to Abouammoh and Ahmed (1992).

The implications between these life criteria are summarized in Figure 1.

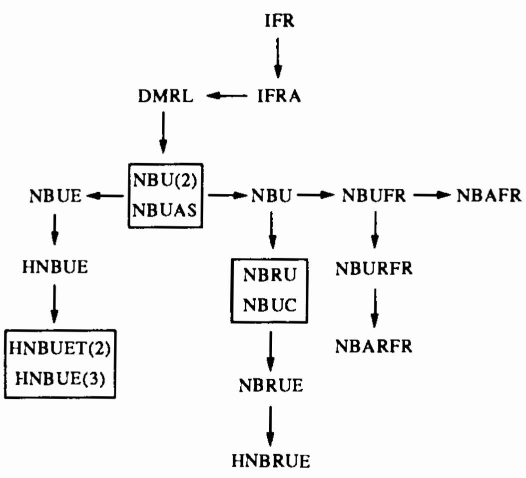

Fig. 1.

The corresponding dual criteria of the previous classes of life distributions are DFR, DFRA, NWU, NWUE, IMRL, HNWUE, NWAFR, NWUAS, HNWUET, NWRU, NWRUE, HNWRUE, NWURFR, NWARFR, respectively.

Note that D, W and I stand for decreasing, worse and increasing, respectively.

Counter examples for the impossible implications are constructed in some of the references mentioned earlier.

In addition to these classes authors have introduced classes of life distributions indexed by $t_0 (t_0 \geq 0)$ which mean for the aging property to hold either at specific point t_0 or specific interval $[0,t_0]$. For example NBU - t_0, DMRL - t_0 and IFR - t_0 are introduced and studied by Hollander et al. (1986), Kulaskera et al. (1987) and Abouammoh and Ahmed (1990). Barry (1992) have used this concept to generalize the NBU, NBUE and HNBUE for specific interval $[0,t_0]$.

3. PARTIAL ORDERINGS

The concept of ordering of two variables is used in reliability engineering to compare the tail heaviness which is reflected in comparisons of the variances, evaluating efficiencies and estimation. Here we restrict ourselves to partial orderings of nonnegative random variables or their life distributions.

Definition 3.1 An order " < " between the distributions F and G i.e. F < G, is called partial ordering if it is

(i) transitive, i.e. F < G and G < H imply F < H.
(ii) reflexive, i.e. F < F for each F, and
(iii) antisymmetric, i.e. F < G and G < F imply F = G up to scale factor.

In this section we briefly review the main partial orderings used in characterizing different aging concepts. The following partial orderings are presented in Barlow and Proschan (1981, p. 105-109).

Definition 3.2 Let F and G be continuous life distribution, G is strictly increasing on its support and F(0) = G(0) = 0. Then:

(i) F is convex with respect to (w.r.t.) G, F $\underset{c}{<}$ G, if $G^{-1}F(t)$ is convex in x.

(ii) F is star-shaped w.r.t. G, F $\underset{*}{<}$ G, if $G^{-1}F(t)$ is star-shaped that is $x^{-1}G^{-1}F(t)$ is increasing for x ,

(iii) F is super-additive w.r.t G, F $\underset{su}{<}$ G, if $G^{-1}F$ is super additive, or $G^{-1}F(s+t) \geq G^{-1}F(s) + G^{-1}F(t), \ \forall\, s, t > 0$.

Next we present some recent partial orderings.

Definition 3.3 Let F and G as definition 3.2, then

(i) F is less mean remaining life than G, F $\underset{mrl}{<}$ G, if $\mu_F(t) < \mu_G(t)$, see Deshpande et al. (1990).

(ii) F is less weakly to-ordered than G, F $\underset{wt}{<}$ G, if $L = \lim_{s \to 0} s^{-1}G^{-1}F(y)$ exists and $\frac{d}{ds}G^{-1}F(s) > L$, see Abouammoh and Newby (1989).

(iii) F is less t-ordered then G, F $\underset{t}{<}$ G, if L exists and $t^{-1}G^{-1}F(t) \geq L$, see Loh (1984) .

(iv) F is less failure rate ordered with respect to G, F $\underset{FR}{<}$ G, if $r_F(t) \geq r_G(t), t \geq 0$, see Ross (1983).

(v) F is less likelihood ratio than G, F $\underset{LR}{<}$ G, if $\overline{G}(t)/\overline{F}(t)$ is increasing in $t \geq 0$.

(vi) F is less variable than G, F $\underset{v}{<}$ G if $\int_t^\infty \overline{F}(u)du \geq \int_t^\infty \overline{G}(u)du$.

(vii) F is less virtual decreasing ordered than G, F $\underset{vd}{<}$ G , if $V_F(t) - V_G(t) \searrow$ in t, $t \geq 0$ where $V_F(t) = \mu_F(0) - \mu_F(t)$.

(viii) F is less new better than used virtual ordered than G,

$F \underset{NBUV}{<} G$, if $V_F(t) < V_G(t), \ \forall \ t \geq 0$.

(ix) F is less global memory ordered than G, $F \underset{G}{<} G$ if $m_g(F) < m_g(G)$ where $m_g(F) = 2 - \frac{2}{\mu_F^2} \int_0^\infty \mu_F(u) \, \overline{F}(u) du$.

(x) F is less virtual rate increasing ordered than G, $F \underset{VR}{<} G$, if $R_F(t) - R_G(t) \nearrow$ in t, $\forall \ t \geq 0$ where $R_F(t) = r_F(t) - r_F(0)$.

These results are reviewed or introduced in Abouammoh, Ahmed and Khalique (1990). Abouammoh (1990) has considered the discrete version of partial orderings. It is interesting to introduce new concepts of partial orderings such that if the ordered is made with respect to exponential distribution one might obtain other forms and criteria of aging. Interrelationships between some of the present forms of partial ordering require investigation. In fact some criteria of aging are deduced from the study of their parallel partial orderings.

4. CLOSURE PROPERTIES

In this section we consider three forms of reliability operations namely

(i) Formation of coherent system using independent components.
(ii) Addition of independent life lengths.
(iii) Random selection of a unit from a set of known units or mixture.

The inheritance of aging properties under these three operations is very important for design and maintenance engineers.

The system is called coherent if (a) every component is relevent and (b) the structure function, that represent the preformance of the system in terms with the performance of the component, is increasing. Design engineers give greater importance to coherency in building systems. Also replacement of a faild unit instantly by a similar independent unit put us infront of query whether the lifelength of both units has the same aging property as the parent unit or not. Random selection according to probability distribution $G(\alpha)$ of a unit from a set of units each has the distribution F_α gives the lifelength of the selected unit as

$$F(t) = \int_\alpha F_\alpha(t) \, dG(\alpha) . \qquad (3.1)$$

Note that the IFR class of life distribution is closed under convolution but is not closed under formation of a coherent structure of independent components. In table 3.1 we present some closure properties of different criteria of aging where C means closed, NC means not closed and NR means not resolved.

Table 3.1 Closure properties for classes of life distributions

Criteria	Formation of coherent system	Convolution	Mixtures of life distributions
IFR	NC	C	NC
IFRA	C	C	NC
NBU	C	C	NC
DMRL	NC	NC	NC
NBUE	NC	NC	NC
NBRU	NC	C	NC
NBUAS	NR	C	NC
NBUET	NR	C	NC
NBUFR	NR	C	NC
NBAFR	NR	C	NC
NBURFR	NR	C	NC
NBARFR	NR	C	NC

The dual classes for the aging criteria in Table 3.1 have similar but sometimes different closure behaviour. Some of these class are closed under formation of parallel system of i.i.d. components such as the DMRL and the NBUE classes, see Abouammoh and El-Neweihi (1986). It is interesting to check the closure under formation of coherent system of i.i.d. components for classes which are not closed in general. Almost all of the dual classes are not closed under formation of coherent system for i.i.d. component or even formation of parallel system. One may relaxe the identicality condition of component for the DMRL or the NBUE classes and seek answer for the closure under formation of coherent system.

Independence conditions is questionable especially in systems with associated components. The general mixture defined by relation (3.1) is reduced to the mixture of non crossing distributions if for every $\alpha_1 \neq \alpha_2$ either F_{α_1} is stochastically larger than F_{α_2} or F_{α_1} is stochastically smaller than F_{α_2}. It has been shown that most of the dual classes of criteria in Table (3.1) are closed under mixture of noncrossing life distributions.

5. DISCRETE AGING AND SHOCK MODEL

It is known that in most practical situations, failure data are measured by discrete units such as, number of runs, cycles, days or shocks. Also sample size might be too small and hence continuous approximation for modelling data is not justified. Now let p_i, $x \in z = \{0,1,2,...,\}$, be the probability mass function (p.m.f.) and $P_k = \sum_{i=0}^{k-1} p_i$ be the cumulative distribution function (c.d.f) and $\overline{P}_k = 1 - P_x = \sum_{i=k}^{\infty} p_i$ be the survival or reliability function (s.f.). Next we give the following definition of discrete aging properties.

Definition 5.1. Let p_k, P_k and $\overline{P}_k, k \in Z$ be the p.m.f., c.d.f. and the s.f. of the life random variable X, respectively. Then p_k, P_k or $\overline{P}_k$ is said to have the following properties:

(i) IFR if $\overline{P}_{k+1}/\overline{P}_k$. in $k \in Z$,

(ii) IFRA if $\overline{P}_k\ 1/k \searrow$ in $k \in Z$,

(iii) NBU if $\overline{P}_{k+j} \le \overline{P}_k \overline{P}_j$ for $k \in Z, j \in Z$,

(iv) NBUE if $\mu = \sum_{k=0}^{\infty} \overline{P}_k < \infty$ and $\sum_{i=k}^{\infty} \overline{P}_i \le \mu \overline{P}_k$,

(v) DMRL if $\mu < \infty$ and $\sum_{i=k}^{\infty} \overline{P}_i / \overline{P}_k \searrow$ in $k \in Z$,

(vi) NBUFR if $\overline{P}_{k+1} \le \overline{P}_1 \overline{P}_k$ for $k \in Z$,

(vii) NBAFR if $\overline{P}_k \le (\overline{P}_1)^k$ for $k \in Z$,

(viii) NBUAS if $\mu < \infty$ and $\sum_{i=0}^{j} \overline{P}_{k+i} \le \overline{P}_k \sum_{i=0}^{j} \overline{P}_i, j \in Z, k \in Z$,

(ix) NBUET if $\mu < \infty$ and $\sum_{j=k}^{\infty} \sum_{i=j}^{\infty} \overline{P}_i \le \mu^2 (1-1/\mu)^k, k \in Z$.

(x) NBURFR if $\sum_{i=k+1}^{\infty} \overline{P}_i \le \overline{P}_1 \sum_{i=k}^{\infty} \overline{P}_i$, $k \in Z$

(xi) NBARFR if $\mu < \infty$ and $\sum_{i=k}^{\infty} \overline{P}_i \le \mu (\overline{P}_1)^k$, $k \in Z$.

(xii) HNBUE if $\mu < \infty$ and $\sum_{i=k}^{\infty} \overline{P}_i \le \mu (1 - 1/\mu)^k$, $k \in Z$

Implications between these classes of discrete life distributions are explained by figure parallel to Figure 1. Also definitions for the dual life criteria are obtained by either reversing the monotonicity or the inequality directions in Definition 5.1 .

Closure properties under formation of coherent structures, convolution and mixture are expected to have similar behaviour. We do not know, to the best of our knoweledge, a comprehensive study of the discrete aging criteria.

Consider the distribution function H with H(0-) = 0 of a device which is subjected to a sequence of shocks occuring randomly in time. In particular let P_k be the probabilities of not surviving k shocks $k \in Z$, then $\overline{P}_k = 1 - P_k$ is the discrete survival probabilities. The survival probability $\overline{H}(t) = 1 - H(t)$ that the device survives untill time t is given by the form

$$\overline{H}(t) = \sum_{k=0}^{\infty} S_k(t)\, S_k\,, \tag{5.1}$$

where $S_k(t)$ represents the probability that the number of shocks during (0,t) is k.

Esary et al. (1973) considered model (5.1) for $S_k(t) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}$, i.e. shocks occur in time according to a homogeneous Poisson process. A-Hameed and Proschan (1973) have used this model by taking $\wedge(t)$ to be the mean value function of the underlying Poisson process where the event rate $\lambda(t) = \lambda \wedge(t)/t$ i.e. the nonhomogeneous Poisson process. A-Hameed and Proschan (1975) have considered model (5.1) under suitable conditions on the mean-value function of the process

$$h(t) = \sum_{k=0}^{\infty} S_k(t)\, \lambda_k\, \lambda(t)\, \overline{P}_{k+1}\,, \tag{5.2}$$

which is a pure birth shock model. In these shock models $\overline{H}(t)$ is investigated for the necessary conditions to inherits the discrete aging criteria of $\overline{P}_k, k \in Z$. In particular the IFR, IFRA, NBU, NBUE and DMRL properties are considered. Block and Savits (1978) have generalized (5.1) under less resrictive conditions on $\overline{P}_k, k \in Z$. Klefsjo (1981) has studied the earlier results for the HNBUE property. Vinogradov (1973) studied the IFR property in terms of Laplace transform. This result has motivated Block and Savits (1980) to consider similar approach for some life criteria. Abouammoh et al. (1988) and Abouammoh and Hendi (1991) have studied homogeneous and nonhomogeneous Poisson processes, and Laplace transform for the NBUFR, NBAFR, NBURFR and NBARFR. They have also considered the cumulative damage model for these classes. Note that shock models have not been considered for NBRU class and general pure birth explored.

6. TESTING OF EXPONENTIALITY VERSUS AGING

The main problem in testing reliability data is testing the null hypothysis

$$H_0 : F(t) = 1 - \exp(-\lambda t), \quad t \geq 0, \lambda > 0$$

versus the alternative

$$H_1 : F(t) = \nabla$$

where ∇ denotes to a class of positive or negative aging criteria. This problem is a recent one. Proschan and Pyke (1967), Barlow (1968), Bickel and Doksum (1969), Klefsjo (1982) considered this problem for ∇ = IFR. Barlow and Compo (1975), Bergman (1977) and Klefsjo (1983) considered other classes including ∇ = NBUE. Hollander and Proschan (1981) considered ∇ = DMKL, whereas Basu and Ebrahimi (1985) and Singh and Kochar (1986) have studied this problem for ∇ = HNBUE . Barlow and Compo (1975) have used the total time on test (TTT) - transform $H^{-1}(t)$ and the scaled TTT-transform $\phi(t)$ to study this problem. Note that

$$H^{-1}(t) = \int_0^{F^{-1}(t)} \overline{F}(x)dx ,$$

and

$$\phi_F(t) = H^{-1}(t) / H^{-1}(1) ,$$

where

$$\mu = H^{-1}(t) = \int_0^{\infty} \overline{F}(u)du < \infty .$$

Abouammoh and Khalique (1987) have considered the later procedure to test exponentiality versus IFR, IFRA, NBUE, DMRL, DMRLA and HNBUE properties. Some researchers have used the empirical procedure rather than the TTT-transform, see for example Abouammoh and Newby (1989) and Abouammoh and Ahmed (1990) and references there in. In many cases it turns out to be difficult to study the behaviour of the proposed test statistics. Simulation techniques are used to evaluate the critical values for the upper and lower percentiles. Tests of exponentiality under random censorship models has also received great deal of interest by researchers in reliability engineering. Kumazawa (1990) has given a full account of testing under random censorship and studied the behaviour of the corresponding test statistics for IFR, IFRA, NBU, NBUE, DMRL and HNBUE properties. In his (1990) thesis Kumazawa has reviewed this problem briefly and included all basic reference.

Tests of exponentiality versus the mean remaining life criteria have been studied by Kumazawa (1991) and for all remaining life classes presented in Abouammoh (1988). Testing of exponentiality under random censorship is not yet considered for other practical and wide classes of life distributions such as NBRU, NBUFR, NBAFR, NBURFR, NBARFR NBRU, NBUAS and HNBUET. Also one may investigate the problem when the censoring distribution is not necessarily a power of the parent exponential i.e. difference is based only on the scale parameter. The power of the proposed tests for some commonly used distributions in reliability engineering should be considered especially when the exact distributions of the test statistics are not obtainable.

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