Department of Statistics &	**Theory of Reliability&**
College of Science	**Operations Research**
King Saud University	**Life Testing**
Final Exam	**OR 563**
Time: Three hours	**Date: 10- 4-1425H**

Q1. **a. Find the structure function $f(\underline{x})$ and draw the block diagram without repeated components for the coherent system with minimal path sets {1, 2, 4}, {1,3,4} and {1,5}.**

b. Find the structural importance of every component in this system.

c. Find the dual structure $f^D(\underline{x})$ and draw the corresponding block diagram.

Q2. **a. Assume that the coherent system $f(\underline{x})$, given in Q1.a. , has independent components with reliabilities p_i, i=1, 2, 3, 4, 5, then find the reliability of the system conditioned on state of component 1.**

b. Evaluate the reliability importance of every component in this system for different and identical reliabilities of the components.

Q3. **a. prove that the dual of a coherent system is a coherent system.**

b. Let $f(\underline{x})$ be the structure function of a coherent system of n components then prove, for any state vectors $\underline{x}$ nd $\underline{y}$, that

$$f[1-(1-x_1)(1-y_1), \ldots, 1-(1-x_n)(1-y_n)] \geq 1-[1-f(\underline{x})][(1-f(\underline{y})],$$

and the equality holds for parallel structure function.

c. **Propose a reasonable definition for monotone multi-state coherent system** $f(\underline{x})$**:** $S^n --> S$**, where** $S = \{1, 2, ..., M\}$ **and M is a finite integer.**

Q4. a. Prove that if the life distribution F has increasing failure rate (IFRA) then it satisfies the new better than used (NBU) aging property.

b. Verify that the life distribution F(.) with survival function S(.) has the aging property used as good as new if and only if $S(x) = \exp(-l x)$**, where** $x \geq 0,\ l ::> 0$ **.**

b. Given the mean remaining life of the life random variable T by $m(t) = E(T - t / T = t)$ **, find the density function, the distribution function, the survival , the failure rate and the average failure rate in terms of** $m(t)$**.**

Q5. a. A random sample of size n, i.e. $\chi_1, \chi_2, ..., \chi_n$ **, is withdrawn from the population with life distribution F, density f, failure rate h and remaining life distribution** $m(t)$ **, show how one can estimate empirically these function.**

b. If it is known that the distribution of the population, in Q5.a., is exponential with $S(x) = \exp(-l x)$**,** $x \geq 0,\ l ::> 0$ **and** $l :$ **has a prior distribution** $f(l) = a \exp(-a l)$**,** $l \geq 0,\ a ::> 0$ **, for known** $a :$**, then find the maximum likelihood estimate (mle) and the Bayes estimate of the parameter** l **and the corresponding reliability functions of the population.**

Q6. a. Assume that the distribution of a lifetime is exponential with a random parameter that has a gamma distribution that is

$S(t) = \exp(-l t)$**,** $t \geq 0,\ l ::> 0$ **and**

$$f(l) = [\Gamma(k)]^{-1} b\,(b\,l)^{k-1} \exp(-l\,b), \quad l > 0,\ b > 0,\ k > 0.$$

Find the unconditional density function of T.

b. Three independent risks are competing on a component. The net lives T_i is exponential with mean ($1/l_i$), i=1, 2, 3. Find the net probability of failure $q_i(a,b)$ and the crude probability of failure $Q_i(a,b)$, for risk i. during the time interval [a,b).