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New exact travelling wave solutions for some famous nonlinear partial differential equations using the improved tanh-function method

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The improved tanh-function method is a powerful tool for obtaining exact travelling wave solutions. The method is used for constructing exact travelling wave solutions and new kinds of solutions for the modified dispersive water wave equation, the Abrahams–Tsuneto reaction diffusion system, and for a class of reaction diffusion models. The solutions obtained are different from those reported in the literature.

Keywords: Tanh-function method; Travelling wave solutions; Reaction diffusion; Modified dispersive water wave; Abrahams–Tsuneto reaction diffusion system

1. Introduction

The exact solutions for nonlinear evolution equations and nonlinear systems of equations play an important role in the study of nonlinear physical, biological and economic phenomena. A large number of methods have been reported to find exact solutions to nonlinear evolution equations and nonlinear systems of equations, such as the inverse scattering method [1], the homogeneous balance method [2], the Jacobi elliptic function method [3–6], the tanh-function method [7, 8], the extended tanh-function method [9–12], etc.

The purpose of this study was to find more exact solutions for the modified dispersive water wave (MDWW) [13, 14] using the improved tanh-function method [15]. Moreover, the method will be used to find more and different solutions that require less effort to apply for a class of reaction diffusion models [16–18] and the Abrahams–Tsuneto reaction diffusion system [19,20]. In section 2 we summarize the improved tanh-function method. Solutions of the MDWW, Abrahams–Tsuneto reaction diffusion system and a class of reaction diffusion models are presented in sections 3, 4 and 5, respectively. Section 6 presents the conclusions.

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2. The improved tanh-function method

It is useful to summarize the main steps when using the improved tanh-function method [15].

1. Consider the general form of a nonlinear partial differential equation (PDE),

$$N(u, u_t, u_x, u_{xx}, \ldots) = 0. \tag{1}$$

2. To find the travelling wave solution of equation (1), we introduce the wave variable $\zeta = k(x + \omega t)$ so that

$$u(x, t) = U(\zeta), \tag{2}$$

where k and ω are the wave number and the wave speed, respectively. Thus, we use the following changes:

$$u_t = k\omega U'(\zeta), \quad u_{tt} = k^2\omega^2 U''(\zeta), \quad u_x = kU'(\zeta), \quad u_{xx} = k^2 U''(\zeta), \ldots, \tag{3}$$

where U' indicates the derivative of U with respect to ζ. Using (3) we reduce the PDE (1) to an ordinary differential equation (ODE), given by

$$N(U, U', U'', \ldots) = 0. \tag{4}$$

3. If all the terms of (4) contain derivatives in ζ, then by integrating this equation, and taking the constant of integration to be zero, we obtain a simplified ODE.
4. Introduce the ansatz

$$U(\zeta) = \sum_{i=0}^{n} a_i F^i(\zeta), \tag{5}$$

where n is a positive integer that can be determined by balancing the highest linear term with the nonlinear term in equation (1), a_j, $j = 1, 2, \ldots, n$, are parameters to be determined and $F(\zeta)$ is a solution of the following Riccati equation:

$$F' = CF^2 + A, \tag{6}$$

where A and C are constants. The relations between some values of A and C and the corresponding $F(\zeta)$ in (6) are given in table 1 [15].
5. Introducing (6) into (5) and then substituting (5) into equation (4) yields a set of algebraic equations involving a_j ($j = 1, 2, \ldots, n$), k and ω because all coefficients of $F_j(\zeta)$ have to vanish. Having determined these parameters, we obtain an analytic solution in closed form.
6. When the balancing number n is not a positive integer, a transformation formula is used to change equation (1) into another equation, the balancing number of which will be a positive integer [7].

Table 1. Relations between values of A and C and the corresponding $F(\zeta)$ in (6).

Case	A	C	$F(\zeta)$
1	1/2	−1/2	$\coth\zeta \pm \operatorname{csch}\zeta$, $\tanh\zeta \pm i\,\operatorname{sech}\zeta$, $i=\sqrt{-1}$
2	1/2	1/2	$\sec\zeta + \tan\zeta$, $\zeta - \cot\zeta$
3	−1/2	−1/2	$\sec\zeta - \tan\zeta$, $\zeta + \cot\zeta$
4	1	−1	$\tanh\zeta$, $\coth\zeta$
5	−1	−1	$\cot\zeta$
6	1	1	$\tan\zeta$

3. Solutions of the modified dispersive water wave equations

The MDWW equations are given by [13, 14]

$$u_t = -\frac{1}{4}v_{xx} + \frac{1}{2}(uv_x + vu_x), \quad v_t = -u_{xx} - 2uu_x + \frac{3}{2}vv_x. \tag{7}$$

In order to obtain travelling wave solutions of (7), we set

$$u(x,t) = U(\zeta), \quad v(x,t) = V(\zeta), \quad \zeta = (x + \omega t). \tag{8}$$

Substituting (8) into (7), we find that

$$-\frac{1}{4}V'' + \frac{1}{2}(UV' + VU') - \omega U' = 0, \quad -U'' - 2UU' + \frac{3}{2}VV' - \omega V' = 0. \tag{9}$$

Balancing V'' with UV' and U'' with VV' in equation (9) gives $n = 1$. Therefore, we may choose the following ansatz:

$$U(\zeta) = a_0 + a_1 F(\zeta), \quad V(\zeta) = b_0 + b_1 F(\zeta). \tag{10}$$

Now substituting (10) into (9) along with equation (6) and using Mathematica yields a system of equations with respect to F^j. Setting the coefficients of F^j in the resulting system of equations to zero, we can deduce the following set of algebraic equations with respect to unknowns a_0, a_1, b_0, b_1 and ω:

$$\begin{aligned}
-A\omega a_1 + \frac{1}{2}Aa_1b_0 + \frac{1}{2}Aa_0b_1 &= 0,\\
-C\omega a_1 + \frac{1}{2}Ca_1b_0 + \frac{1}{2}Ca_0b_1 &= 0,\\
-\frac{1}{2}ACb_1 + Aa_1b_1 &= 0,\\
-\frac{1}{2}C^2b_1 + Ca_1b_1 &= 0,\\
-2Aa_1a_0 - A\omega b_1 + \frac{3}{2}Ab_0b_1 &= 0,\\
-2Ca_1a_0 - C\omega b_1 + \frac{3}{2}Cb_0b_1 &= 0,\\
-2ACa_1 - 2Aa_1^2 + \frac{3}{2}Ab_1^2 &= 0,\\
-2C^2b_1 - 2Ca_1^2 + \frac{3}{2}Cb_1^2 &= 0.
\end{aligned}$$

Using Mathematica, we obtain a family of solutions corresponding to the different cases listed in table 1. We obtain the following families of solutions taking into consideration $\zeta = (x + \omega t)$.

Case (1). $A = 1/2$, $C = -1/2$:

$$u_{11} = \mp\frac{\omega}{2} - \frac{1}{4}(\coth\zeta \pm \operatorname{csch}\zeta), \quad v_{11} = \omega \pm \frac{1}{2}(\coth\zeta \pm \operatorname{csch}\zeta),$$

$$u_{12} = \mp\frac{\omega}{2} - \frac{1}{4}(\tanh\zeta \pm i\operatorname{sech}\zeta), \quad v_{12} = \omega \pm \frac{1}{2}(\tanh\zeta \pm i\operatorname{sech}\zeta).$$

Case (2). $A = C = 1/2$:

$$u_{21} = \pm\frac{\omega}{2} + \frac{1}{4}(\sec\zeta + \tan\zeta), \quad v_{21} = \omega \pm \frac{1}{2}(\sec\zeta + \tan\zeta),$$

$$u_{22} = \pm\frac{\omega}{2} + \frac{1}{4}(\csc\zeta - \cot\zeta), \quad v_{22} = \omega \pm \frac{1}{2}(\csc\zeta - \cot\zeta).$$

Case (3). $A = C = -1/2$:

$$u_{31} = \pm\frac{\omega}{2} - \frac{1}{4}(\sec\zeta + \tan\zeta), \quad v_{31} = \omega \pm \frac{1}{2}(\sec\zeta + \tan\zeta),$$

$$u_{32} = \pm\frac{\omega}{2} - \frac{1}{4}(\csc\zeta - \cot\zeta), \quad v_{32} = \omega \pm \frac{1}{2}(\csc\zeta - \cot\zeta).$$

Case (4). $A = 1$, $C = -1$:

$$u_{41} = \pm\frac{\omega}{2} - \frac{1}{2}\tanh\zeta, \quad v_{41} = \omega \pm \tanh\zeta,$$

$$u_{42} = \pm\frac{\omega}{2} - \frac{1}{2}\coth\zeta, \quad v_{42} = \omega \pm \coth\zeta.$$

Case (5). $A = C = -1$:

$$u_{51} = \pm\frac{\omega}{2} + \frac{1}{2}\tan\zeta, \quad v_{51} = \omega \pm \tan\zeta.$$

Case (6). $A = C = 1$:

$$u_{61} = \pm\frac{\omega}{2} - \frac{1}{2}\cot\zeta, \quad v_{61} = \omega \pm \cot\zeta.$$

4. Solutions for the Abrahams–Tsuneto reaction diffusion system

In superconductivity, the Abrahams–Tsuneto reaction diffusion system is given by [19, 20]

$$u_t = u_{xx} + (1 - u^2 - v^2)u, \tag{11}$$

$$v_t = v_{xx} + (1 - u^2 - v^2)v. \tag{12}$$

In order to obtain travelling wave solutions of (11) and (12) we set

$$u(x,t) = U(\zeta), \quad v(x,t) = V(\zeta), \quad \zeta = k(x + \omega t). \tag{13}$$

Substituting (13) into (11) and (12) we obtain

$$-\frac{1}{4}V'' + \frac{1}{2}(UV' + VU') - \omega U' = 0, \quad -U'' - 2UU' + \frac{3}{2}VV' - \omega V' = 0. \tag{14}$$

Balancing V'' with UV' and U'' with VV' in equation (14) gives $n = 1$. Therefore, we may choose the following ansatz:

$$U(\zeta) = a_0 + a_1 F(\zeta), \quad V(\zeta) = b_0 + b_1 F(\zeta). \tag{15}$$

Now substituting (15) into (14) along with equation (6) and using Mathematica yields a system of equations with respect to F^j. Setting the coefficients of F^j in the resulting system of equations to zero, we can deduce the following set of algebraic equations with respect to the unknowns $a_0, a_1, b_0, b_1, \omega$ and k:

$$\begin{aligned}
-a_0 + a_0^3 + \omega k a_1 + a_0 b_0^2 &= 0,\\
-a_1 - 2k^2 a_1 + 3a_0^2 a_1 + a_1 b_0^2 + 2a_0 b_0 b_1 &= 0,\\
\omega k a_1 + 3a_0 a_1^2 + 2a_1 b_0 b_1 + a_0 b_1^2 &= 0,\\
-2k^2 a_1 + a_1^3 + a_1 b_1^2 &= 0,\\
-b_0 + a_0^2 b_0 + b_0^3 + k\omega b_1 &= 0,\\
-2a_0 a_1 b_0 - 2k^2 b_1 + a_0^2 b_1 + 3b_0^2 b_1 &= 0,\\
a_1^2 b_0 + k\omega b_1 + 2a_0 a_1 b_1 + 3b_1^2 b_0 &= 0,\\
-2k^2 b_1 + a_{10}^2 b_1 + b_1^3 &= 0.
\end{aligned}$$

Using Mathematica we found a family of solutions corresponding to the different cases listed in table 1.

Case (1). $A = 1/2$, $C = -1/2$, where $F(\zeta) = \coth\zeta + \operatorname{csch}\zeta$:

$$\begin{aligned}
u_{11} &= \pm\frac{\sqrt{3}}{2}i\left[1 + \coth\left[\frac{1}{4}(3t - \sqrt{2}x)\right]\right],\\
v_{11} &= \pm\left(1 + \coth\left[\frac{3t}{2} - \frac{x}{\sqrt{2}}\right] + \operatorname{csch}\left[\frac{3t}{2} - \frac{x}{\sqrt{2}}\right]\right),\\
u_{12} &= \pm\frac{\sqrt{3}}{2}i\left[1 + \coth\left[\frac{1}{4}(3t + \sqrt{2}x)\right]\right],\\
v_{12} &= \pm\left(1 + \coth\left[\frac{3t}{2} + \frac{x}{\sqrt{2}}\right] + \operatorname{csch}\left[\frac{3t}{2} + \frac{x}{\sqrt{2}}\right]\right).
\end{aligned}$$

Case (2). $A = 1/2$, $C = 1/2$, where $F(\zeta) = \csc\zeta - \cot\zeta$:

$$\begin{aligned}
u_{21} &= \pm\frac{\sqrt{3}}{2}i\left[1 + \tanh\left[\frac{1}{4}(3t - \sqrt{2}x)\right]\right],\\
v_{21} &= \pm\left(1 + \tanh\left[\frac{1}{4}(3t - \sqrt{2}x)\right]\right),\\
u_{22} &= \pm\frac{\sqrt{3}}{2}i\left[1 + \tanh\left[\frac{1}{4}(3t + \sqrt{2}x)\right]\right],\\
v_{22} &= \pm\left(1 + \tanh\left[\frac{1}{4}(3t + \sqrt{2}x)\right]\right).
\end{aligned}$$

Case (3). $A = -1/2$, $C = -1/2$, where $F(\zeta) = \csc\zeta + \cot\zeta$:

$$u_{31} = \pm\frac{\sqrt{3}}{2}i\left(1 + \coth\left[\frac{1}{4}(3t - \sqrt{2}x)\right]\right),$$

$$v_{31} = \pm\left(1 + \coth\left[\frac{1}{4}(3t - \sqrt{2}x)\right]\right),$$

$$u_{32} = \pm\frac{\sqrt{3}}{2}i\left(1 + \coth\left[\frac{1}{4}(3t + \sqrt{2}x)\right]\right),$$

$$v_{32} = \pm\left(1 + \coth\left[\frac{1}{4}(3t + \sqrt{2}x)\right]\right).$$

Case (4). $A = 1$, $C = -1$, where $F(\zeta) = \tanh\zeta$:

$$u_{41} = \pm\frac{\sqrt{3}}{2}i\left(-1 + \tanh\left[\frac{1}{4}(-3t + \sqrt{2}x)\right]\right),$$

$$v_{41} = \pm 1 + \tanh\left[\frac{1}{4}(\pm 3t \mp \sqrt{2}x)\right],$$

$$u_{42} = \pm\frac{\sqrt{3}}{2}i\left(1 + \tanh\left[\frac{1}{4}(3t + \sqrt{2}x)\right]\right),$$

$$v_{42} = \pm\left(1 + \tanh\left[\frac{1}{4}(3t + \sqrt{2}x)\right]\right).$$

Case (5). $A = -1$, $C = -1$:

$$u_{51} = \pm\frac{\sqrt{3}}{2}i\left(-1 + \coth\left[\frac{1}{4}(-3t + \sqrt{2}x)\right]\right),$$

$$v_{51} = \pm 1 + \coth\left[\frac{1}{4}(\pm 3t \mp \sqrt{2}x)\right],$$

$$u_{52} = \pm\frac{\sqrt{3}}{2}i\left(1 + \coth\left[\frac{1}{4}(-3t + \sqrt{2}x)\right]\right),$$

$$v_{52} = \pm 1 + \coth\left[\frac{1}{4}(\mp 3t \pm \sqrt{2}x)\right].$$

Case (6). $A = 1$, $C = 1$:

$$u_{61} = \pm\frac{\sqrt{3}}{2}i\left(-1 + \tanh\left[\frac{1}{4}(-3t + \sqrt{2}x)\right]\right),$$

$$v_{61} = \pm 1 + \tanh\left[\frac{1}{4}(\pm 3t \mp \sqrt{2}x)\right],$$

$$u_{62} = \pm\frac{\sqrt{3}}{2}i\left(1 + \tanh\left[\frac{1}{4}(3t + \sqrt{2}x)\right]\right),$$

$$v_{62} = \pm 1 \pm \tanh\left[\frac{1}{4}(3t + \sqrt{2}x)\right].$$

5. Exact solutions for a class of reaction diffusion models

Reaction diffusion models appear in many branches of science [21–23]. One of the important classes of such reaction diffusion models is [21]

$$u_t = u_{xx} + u^{q+1}(1 - u^q), \quad q \in \mathbb{N}. \tag{16}$$

To find exact solutions for this class, let $u(x,t) = U(\zeta)$, where $\zeta = k(x + \omega t)$, then equation (16) reduces to

$$k\omega U' - k^2 U'' + U^{q+1} - U^{2q+1} = 0. \tag{17}$$

Balancing U'' with U^{2q+1} gives $n = 1/q$, which is not an integer since $q \neq 1$. This means that the improved tanh-function method is not appropriate for any positive integer $q \neq 1$.

To find travelling wave solutions of equation (16) we use the following transformation:

$$V = U^{1/q}. \tag{18}$$

Then equation (17) reduces to

$$k\omega V V' - k^2 V V'' + \left(1 - \frac{1}{q}\right) k^2 (V')^2 - (V^3 - V^4) = 0. \tag{19}$$

Now, balancing $(V')^2$ with V^4 in (19), we find $n = 1$. So we seek its travelling wave solution of the form

$$V = a_0 + a_1 F(\zeta). \tag{20}$$

Substituting (20) into (19) along with equation (6) and using Mathematica yields a system of equations with respect to F^j. Setting the coefficients of F^j in the resulting system of equations to zero we can deduce the following set of algebraic equations with respect to unknowns a_0, a_1, k and ω:

$$\begin{aligned}
-a_0^3 + a_0^4 + A\omega k a_0 a_1 + A^2 k^4 a_1^2 - \frac{A^2 k^4 a_1^2}{q} &= 0,\\
-2ACk^2 a_0 a_1 - 3a_0^2 a_1 + 4a_0^3 a_1 + A\omega k a_1^2 &= 0,\\
k\omega C a_0 a_1 - 2ACk^2 a_1^2 + 2ACk^4 a_1^2 - 3a_1^2 a_0 + 6a_0^2 a_1^2 - \frac{2ACk^4 a_1^2}{q^2} &= 0,\\
-2C^2 k^2 a_0 a_1 + \omega C k a_1^2 - a_1^3 + 4a_1^3 a_0 &= 0,\\
-2C^2 k^2 a_1^2 + C^2 k^4 a_1^2 + a_1^4 - \frac{C^2 k^4 a_1^2}{q} &= 0.
\end{aligned}$$

Solving the above system of equations using Mathematica we can find the values of a_0, a_1, k and ω and, in a similar manner as introduced above, we will obtain only two families of solutions corresponding to $q = 1$ and $q = 2$.

First groups ($q = 1$).

Case (1). $A = 1/2, C = -1/2$:

$$u_{11} = \frac{1}{2}\left[1 + \coth\left(\frac{1}{2}(t - \sqrt{2}x)\right) \pm \operatorname{csch}\left(\frac{1}{2}(t - \sqrt{2}x)\right)\right],$$
$$u_{12} = \frac{1}{2}\left[1 + \coth\left(\frac{1}{2}(t + \sqrt{2}x)\right) \pm \operatorname{csch}\left(\frac{1}{2}(t + \sqrt{2}x)\right)\right],$$
$$u_{13} = \frac{1}{2}\left[1 - i\operatorname{sech}\left(\frac{1}{2}(t \pm \sqrt{2}x)\right) + \tanh\left(\frac{1}{2}(t \pm \sqrt{2}x)\right)\right],$$
$$u_{14} = \frac{1}{2}\left[1 + i\operatorname{sech}\left(\frac{1}{2}(t \pm \sqrt{2}x)\right) + \tanh\left(\frac{1}{2}(t \pm \sqrt{2}x)\right)\right].$$

Case (2). $A = C = 1/2$:

$$u_{21} = \frac{1}{2}\left[1 + i\operatorname{sech}\left(\frac{1}{2}(t \pm \sqrt{2}x)\right) + \tanh\left(\frac{1}{2}(t \pm \sqrt{2}x)\right)\right],$$
$$u_{22} = \frac{1}{2}\left[1 - i\operatorname{sech}\left(\frac{1}{2}(t \pm \sqrt{2}x)\right) + \tanh\left(\frac{1}{2}(t \pm \sqrt{2}x)\right)\right],$$
$$u_{23} = \frac{1}{2}\left[1 + \coth\left(\frac{1}{2}(t \pm \sqrt{2}x)\right) - \operatorname{csch}\left(\frac{1}{2}(t \pm \sqrt{2}x)\right)\right].$$

Case (3). $A = C = -1/2$:

$$u_{31} = \frac{1}{2}\left[1 + i\operatorname{sech}\left(\frac{1}{2}(t \pm \sqrt{2}x)\right) + \tanh\left(\frac{1}{2}(t \pm \sqrt{2}x)\right)\right],$$
$$u_{32} = \frac{1}{2}\left[1 - i\operatorname{sech}\left(\frac{1}{2}(t \pm \sqrt{2}x)\right) + \tanh\left(\frac{1}{2}(t \pm \sqrt{2}x)\right)\right],$$
$$u_{33} = \frac{1}{2}\left[1 + \coth\left(\frac{1}{2}(t \pm \sqrt{2}x)\right) + \operatorname{csch}\left(\frac{1}{2}(t \pm \sqrt{2}x)\right)\right].$$

Case (4). $A = 1, C = -1$:

$$u_{41} = \frac{1}{2}\left(1 + \tanh\left(\frac{1}{4}(t \pm \sqrt{2}x)\right)\right),$$
$$u_{42} = \frac{1}{2}\left(1 + \coth\left(\frac{1}{4}(t \pm \sqrt{2}x)\right)\right).$$

Case (5). $A = C = -1$:

$$u_{51} = \frac{1}{2}\left(1 - \coth\left(\frac{1}{4}(t \pm \sqrt{2}x)\right)\right).$$

Case (6). $A = C = 1$:

$$u_{61} = \frac{1}{2}\left(1 - \tanh\left(\frac{1}{4}(t \pm \sqrt{2}x)\right)\right).$$

Second groups ($q = 2$).

Case (1). $A = 1/2, C = -1/2$:

$$u_{11} = \left[\frac{1}{2}(1 - \coth\zeta - \operatorname{csch}\zeta)\right]^{1/2}, \quad \zeta = t - \sqrt{2}t + \sqrt{2 - \sqrt{2}}x,$$

$$u_{12} = \left[\frac{1}{2}(1 - \coth\zeta \mp \operatorname{csch}\zeta)\right]^{1/2}, \quad \zeta = t + \sqrt{2}t \mp \sqrt{2 + \sqrt{2}}x,$$

$$u_{13} = \left[\frac{1}{2}(1 \mp \coth\zeta \pm \operatorname{csch}\zeta)\right]^{1/2}, \quad \zeta = t - \sqrt{2}t \mp \sqrt{2 + \sqrt{2}}x,$$

$$u_{14} = \left[\frac{1}{2}(1 \mp \coth\zeta + \operatorname{csch}\zeta)\right]^{1/2}, \quad \zeta = t + \sqrt{2}t \mp \sqrt{2 + \sqrt{2}}x,$$

$$u_{15} = \left[\frac{1}{2}(1 \mp i\operatorname{sech}\zeta - \tanh\zeta)\right]^{1/2}, \quad \zeta = t - \sqrt{2}t + \sqrt{2 - \sqrt{2}}x,$$

$$u_{16} = \left[\frac{1}{2}(1 \mp i\operatorname{sech}\zeta + \tanh\zeta)\right]^{1/2}, \quad \zeta = t - \sqrt{2}t + \sqrt{2 - \sqrt{2}}x,$$

$$u_{17} = \left[\frac{1}{2}(1 \mp i\operatorname{sech}\zeta - \tanh\zeta)\right]^{1/2}, \quad \zeta = t + \sqrt{2}t - \sqrt{2 - \sqrt{2}}x,$$

$$u_{18} = \left[\frac{1}{2}(1 \mp i\operatorname{sech}\zeta + \tanh\zeta\right]^{1/2}, \quad \zeta = t + \sqrt{2}t + \sqrt{2 - \sqrt{2}}x.$$

Case (2). $A = C = 1/2$:

$$u_{21} = \left[\frac{1}{2}(1 \pm i\operatorname{sech}\zeta - \tanh\zeta)\right]^{1/2}, \quad \zeta = (\sqrt{6} - 3)t + \sqrt{\sqrt{6} - 2}x,$$

$$u_{22} = \left[\frac{1}{2}(1 \pm i\operatorname{sech}\zeta - \tanh\zeta)\right]^{1/2}, \quad \zeta = (\sqrt{6} - 3)t - \sqrt{\sqrt{6} - 2}x,$$

$$u_{23} = \left[\frac{1}{2}(1 \pm i\operatorname{sech}\zeta + \tanh\zeta)\right]^{1/2}, \quad \zeta = (\sqrt{6} + 3)t + i\sqrt{\sqrt{6} + 2}x,$$

$$u_{24} = \left[\frac{1}{2}(1 \pm i\operatorname{sech}\zeta + \tanh\zeta)\right]^{1/2}, \quad \zeta = (\sqrt{6} + 3)t - i\sqrt{\sqrt{6} + 2}x,$$

$$u_{25} = \left[\frac{1}{2}(1 - \coth\zeta + \operatorname{csch}\zeta)\right]^{1/2}, \quad \zeta = (\sqrt{6} - 3)t + \sqrt{\sqrt{6} - 2}x,$$

$$u_{26} = \left[\frac{1}{2}(1 + \coth\zeta - \operatorname{csch}\zeta)\right]^{1/2}, \quad \zeta = (\sqrt{6} + 3)t \pm \sqrt{\sqrt{6} + 2}x,$$

$$u_{27} = \left[\frac{1}{2}(1 + \tanh\zeta)\right]^{1/2}, \quad \zeta = \frac{1}{2}(\sqrt{6} + 3)t \pm i\sqrt{\sqrt{6} + 2}x.$$

Case (3). $A = C = -1/2$:

$$u_{31} = \left[\frac{1}{2}(1 \pm i(\operatorname{sech}\zeta - \tanh\zeta)\right]^{1/2}, \quad \zeta = (\sqrt{6} - 3)t + \sqrt{\sqrt{6} - 2}x,$$

$$u_{32} = \left[\frac{1}{2}(1 \pm i(\operatorname{sech}\zeta - \tanh\zeta)\right]^{1/2}, \quad \zeta = (\sqrt{6} - 3)t - \sqrt{\sqrt{6} - 2}x,$$

$$u_{33} = \left[\frac{1}{2}(1 \pm i(\operatorname{sech}\zeta + i\tanh\zeta)\right]^{1/2}, \quad \zeta = (\sqrt{6}+3)t + i\sqrt{\sqrt{6}+2}x,$$

$$u_{34} = \left[\frac{1}{2}(1 \pm i(\operatorname{sech}\zeta + i\tanh\zeta)\right]^{1/2}, \quad \zeta = (\sqrt{6}+3)t - i\sqrt{\sqrt{6}+2}x,$$

$$u_{35} = \left[\frac{1}{2}(1 - (\coth\zeta + \operatorname{csch}\zeta)\right]^{1/2}, \quad \zeta = (\sqrt{6}-3)t \pm \sqrt{\sqrt{6}-2}x,$$

$$u_{36} = \left[\frac{1}{2}(1 + (\coth\zeta + \operatorname{csch}\zeta)\right]^{1/2}, \quad \zeta = (\sqrt{6}+3)t \pm i\sqrt{\sqrt{6}+2}x,$$

$$u_{37} = \left[\frac{1}{2}(1 + \coth\zeta)\right]^{1/2}, \quad \zeta = (\sqrt{6}+3)t \pm i\sqrt{\sqrt{6}+2}x.$$

Case (4). $A = 1, C = -1$:

$$u_{41} = \left[\frac{1}{2}\left(1 - \tanh\left[\frac{1}{2}\left((7-2\sqrt{14})t \pm \sqrt{8-2\sqrt{14}}x\right)\right]\right)\right]^{1/2},$$

$$u_{42} = \left[\frac{1}{2}\left(1 - \tanh\left[\frac{1}{2}\left((7+2\sqrt{14})t \pm \sqrt{8+2\sqrt{14}}x\right)\right]\right)\right]^{1/2},$$

$$u_{43} = \left[\frac{1}{2}\left(1 - \coth\left[\frac{1}{2}\left((7-2\sqrt{14})t \pm \sqrt{8-2\sqrt{14}}x\right)\right]\right)\right]^{1/2},$$

$$u_{44} = \left[\frac{1}{2}\left(1 - \coth\left[\frac{1}{2}\left((7+2\sqrt{14})t \pm \sqrt{8+2\sqrt{14}}x\right)\right]\right)\right]^{1/2}.$$

Case (5). $A = C = -1$:

$$u_{51} = \left[\frac{1}{2}\left(1 + \coth\left[\frac{1}{2}\left((9+6\sqrt{2})t \pm i\sqrt{6\sqrt{2}+8}x\right)\right]\right)\right]^{1/2},$$

$$u_{52} = \left[\frac{1}{2}\left(1 + \coth\left[\frac{1}{2}\left((9-6\sqrt{2})t \pm \sqrt{6\sqrt{2}-8}x\right)\right]\right)\right]^{1/2}.$$

Case (6). $A = C = 1$:

$$u_{61} = \left[\frac{1}{2}\left(1 + \tanh\left[\frac{1}{2}\left((9+6\sqrt{2})t \pm i\sqrt{6\sqrt{2}+8}x\right)\right]\right)\right]^{1/2},$$

$$u_{62} = \left[\frac{1}{2}\left(1 + \tanh\left[\frac{1}{2}\left((9-6\sqrt{2})t \pm \sqrt{6\sqrt{2}-8}x\right)\right]\right)\right]^{1/2}.$$

6. Conclusions

We have used the improved tanh-function method to obtain many different and new exact solutions for the modified dispersive water wave equation and the Abrahams–Tsuneto reaction diffusion system, presented in [24, 25]. Also, the method was used to find more new and simple forms of exact solutions for an important class of nonlinear reaction diffusion equations [17].

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