Capillary Pressure / Permeability Relations

Single Capillary Tube

- Previously discussion showed that for a single capillary:

$$P_c = \frac{2\,\sigma\,\cos(\theta)}{r}$$

  – consistent units required
    - P_c dyne/cm^2
    - σ dyne/cm
    - r cm

Bundle of Capillary Tubes Model

- Purcell (ABW page 169) developed a model which considers the porous media to be a "bundle of capillary tubes" of varying sizes
- This model was used to develop a relationship for predicting permeability from the drainage capillary pressure curve
  - for a discrete set of capillary sizes, in consistent units:
    - k cm^2
    - σ dyne/cm
    - P_c dyne/cm^2

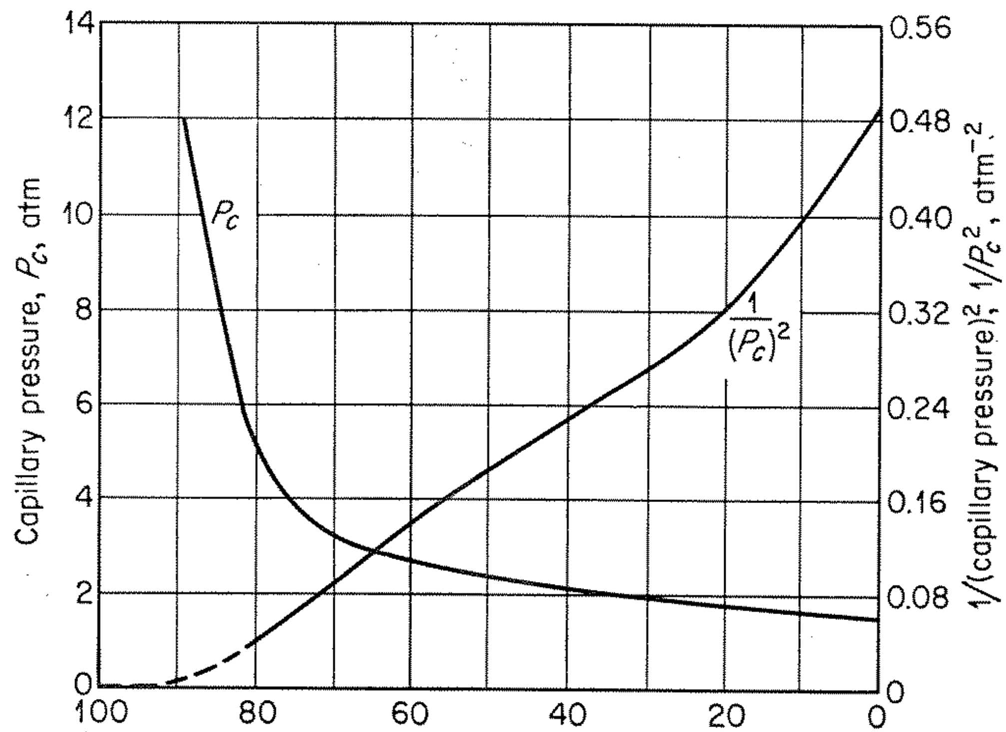
$$k = \frac{(\sigma \cos(\theta))^2}{2} \phi \sum_{i=1}^{n} \frac{(S_w)_i}{(P_c^2)_i}$$

Purcell's Equation

- For a continuous distribution of capillary sizes, determined from the $P_c(S_w)$ function (consistent units)

$$k = \frac{(\sigma \cos(\theta))^2}{2} \phi \alpha \int_{S_w=0}^{S_w=1} \frac{1}{(P_c)^2} dS_w$$

  - α is an dimensionless, empirical "lithology factor" (Purcell used symbol, λ)
  - The function $1/P_c^2$ could be integrated numerically
  - OR, we could use the S_w* model for P_c
  - Note that the function $1/P_c^2$ has value of zero if $S_w<S_{wi}$



Purcell's Equation

- For oilfield units,

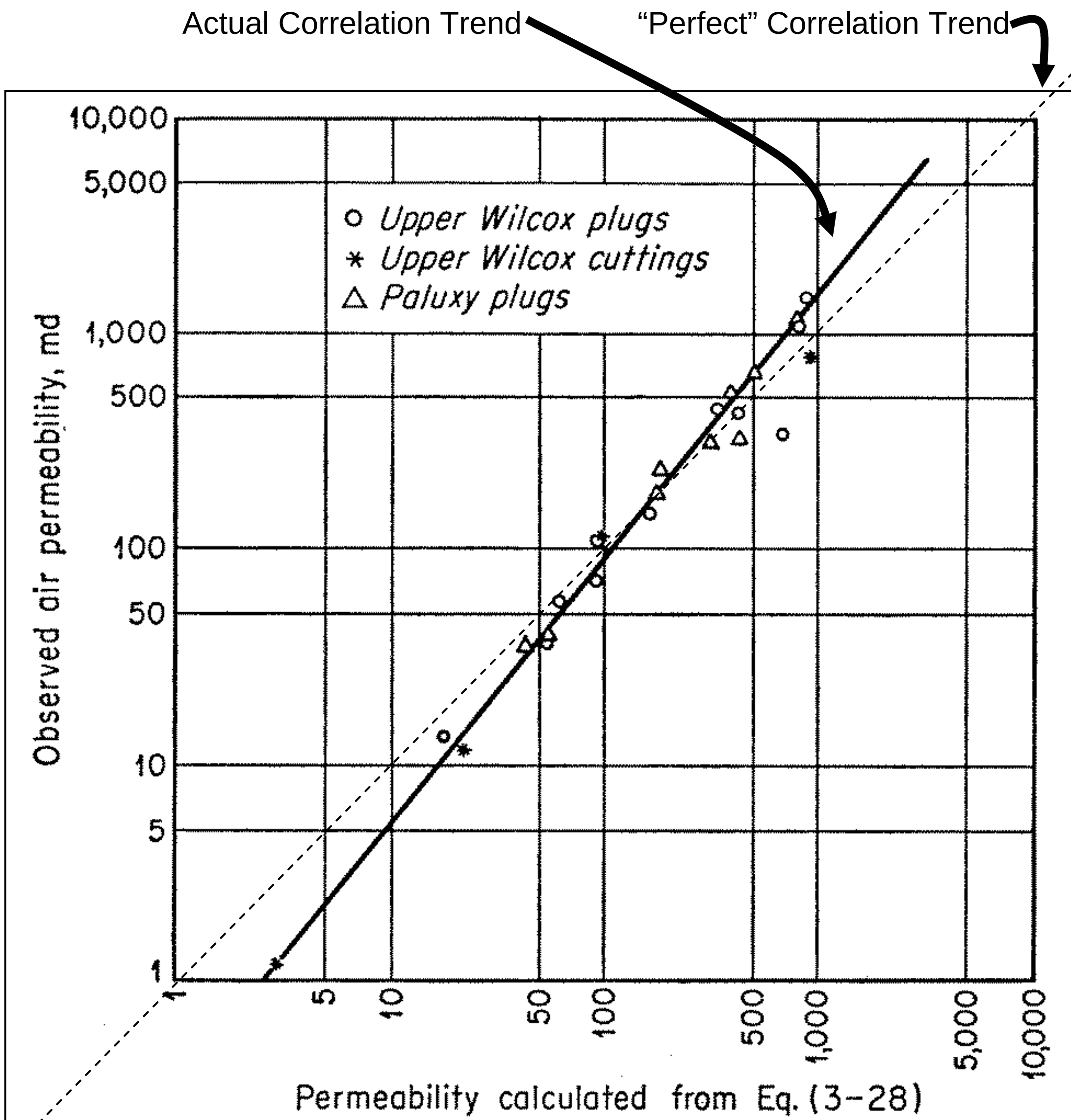
$$k = 10.66\left(\sigma \cos(\theta)\right)^2 \ \phi \alpha \int_{S_w=0}^{S_w=1} \frac{1}{\left(P_c\right)^2} \, dS_w$$

  - k md
  - σ dyne/cm
  - P_c psia
  - Note: Handwritten notes in margin of ABW photocopy is missing a squared symbol on conversion from dyne/cm^2 per atmosphere

- Self Study: Be able to derive this conversion constant, 10.66

Purcell's Equation

- Using an average lithology factor, α=0.216, Purcell showed that permeability can be estimated from the drainage capillary pressure curve
  - For permeability, estimates that are the correct order of magnitude are considered fairly accurate



Application of S_w^* Model

- To integrate the function $1/P_c^2$, we consider piecewise integration

$$\int_{S_w=0}^{S_w=1} \frac{1}{(P_c)^2}\, dS_w = \int_{S_w=0}^{S_w=S_{wi}} \frac{1}{(P_c)^2}\, dS_w + \int_{S_w=S_{wi}}^{S_w=1} \frac{1}{(P_c)^2}\, dS_w$$

$$= 0 + \int_{S_w=S_{wi}}^{S_w=1} \frac{1}{(P_c)^2}\, dS_w$$

Application of $S_w{}^*$ Model

- From our definition of $S_w{}^*$: $$S_w^* = \frac{S_w - S_{wi}}{1 - S_{wi}}$$

- Integral requires dS_w:

$$dS_w^* = \frac{1}{1 - S_{wi}}\, dS_w \quad \therefore \quad dS_w = (1 - S_{wi})\, dS_w^*$$

- S_w* model for P_c: $$P_c = P_d \left(S_w^*\right)^{-1/\lambda}$$

Application of Sw* Model

- Substituting $dS_w{}^*$ for dS_w in integrand changes limits

$$\int_{S_w = S_{wi}}^{S_w = 1} \frac{1}{\left(P_c\right)^2} dS_w = \int_{S_w^* = 0}^{S_w^* = 1} \frac{\left(1 - S_{wi}\right)}{\left(P_c\right)^2} dS_w^*$$

Application of Sw* Model

- Substituting S_w^* model for P_c in integrand

$$\int_{S_w^*=0}^{S_w^*=1} \frac{\left(1-S_{wi}\right)}{\left(P_c\right)^2}\, dS_w^* = \int_{S_w^*=0}^{S_w^*=1} \frac{\left(1-S_{wi}\right)}{\left(P_d\,(S_w^*)^{(-1/\lambda)}\right)^2}\, dS_w^*$$

$$= \frac{\left(1-S_{wi}\right)}{P_d^2} \int_{S_w^*=0}^{S_w^*=1} S_w^{*\,(2/\lambda)}\, dS_w^*$$

$$= \frac{\left(1-S_{wi}\right)}{P_d^2} \left[\frac{\lambda}{\lambda+2} S_w^{*\,((\lambda+2)/\lambda)}\right]_{S_w^*=0}^{S_w^*=1}$$

$$= \frac{\left(1-S_{wi}\right)}{P_d^2} \left[\frac{\lambda}{\lambda+2}\right]$$

Purcell's Equation with S_w^* Model

- Finally, for oilfield units:

$$k = 10.66\left(\sigma \cos(\theta)\right)^2 \phi \alpha \frac{\left(1 - S_{wi}\right)}{P_d^2}\left[\frac{\lambda}{\lambda + 2}\right]$$