CABLE-STAYED BRIDGE MODEL UPDATE USING DYNAMIC MEASUREMENTS

Eng. H. M. Abozeid
Ph.D. Student, Faculty of Engineering, Ain Shams University, Cairo, Egypt.
E-Mail: hossam_abozeid@yahoo.com

Dr. M. N. Fayed
Prof., Faculty of Engineering, Ain Shams University, Cairo, Egypt.

Dr. S. M. Mourad
Assoc. Prof., Faculty of Engineering, Ain Shams University, Cairo, Egypt.

Dr. A. H. Khalil
Assoc. Prof., Faculty of Engineering, Ain Shams University, Cairo, Egypt.

ABSTRACT

SHM "Structural Health Monitoring" and damage detection at the earliest possible stage is considered one of the most interesting issues of the civil engineering community, especially for those structures with long design life, life-safety implications and high capital expenditures like cable-stayed bridges. The EMA "Experimental Modal Analysis" gives the required measurements that are used in model update and needed for those damage detection techniques based on changes in modal properties.

This paper presents the technology of determining the structural properties of the cable-stayed bridges using dynamic measurements. A proposed computerized analysis tool is introduced that uses the EMA output data to extract the modal properties of the measured structure. The Suez-Canal cable-stayed bridge is assumed as a case study to perform the model update operation using the dynamic measurements. The paper addresses the EMA test setup, the modal parameters extracting technique and the model update strategy.

Keywords: Experimental Modal Analysis; Mode shapes; Natural Frequencies; Model Update.

INTRODUCTION

Commonly in the design process, a mathematical model is needed for structural analysis of the designed structure. The model is usually created with assumed dimensions and material properties that certainly differ from those of the real structure after construction.

In order to obtain a realistic mathematical model to be used for design, prediction, simulation, diagnosis and monitoring, a heavily computer-based technology, with emphasis on using computer models is needed to predict the performance of the structures in question.

Experimental testing could play a major and vital role in design process, especially when it is properly integrated with analytical processes. Experimentation serves two important functions in such design activities. The first is to obtain measured data with which to check the accuracy of theoretical predictions and the second is to check their completeness.

Experience gained in recent years recommends the Experimental Modal Analysis "EMA" as the most economic, accurate and effective non-destructive tool for inspection and health monitoring of the existing structures. [1]

In an EMA test, the dynamic response of the structure is measured using special sensors "accelerometers" that record the response versus time intervals, called time-domain response. Using a special signals analyzer and FFT technique "Fast Fourier Transform" [2], the time-domain response is transferred to frequency-domain response "response versus the frequency" as shown in figure (1). Some of the appeared peaks in the frequency-domain response for the different tested joints may represent one of the natural frequencies of the bridge, so they should be checked. The assumed peaks may be chosen by guidance of the theoretical modal analysis results "finite elements analysis". The peak is considered one of the natural frequencies, when it appears in most frequency-domain responses of the observed joints and the joints response plot simulate one of the tested structure mode shape[3,4], so the accelerometers stations should be verified accurately[5,6]. In addition, an appropriate method of excitation should be chosen according to the type of the tested structure, the frequency resolution and the required mode shapes [7]. For bridges, two main major methods of excitation may be used. The first is the ambient excitation techniques like traffic, wind and earthquakes. The second is the measured input techniques like hammer, shaker and step relaxation [8].
The extracted dynamic properties of a structure could be established in terms of modal parameters "mode shapes and natural frequencies" that are used to update the mathematical model. There are two major techniques for model update. The first is the direct matrix methods in which, the individual elements in the system matrices are adjusted directly from comparison between test data and initial analytical model prediction using special and advanced softwares. The second is the indirect, physical property adjustment methods in which changes are made to specific physical or elemental properties in the model (using the engineering sense) in a search for an adjustment, which brings measured and predicted data closer together[9].

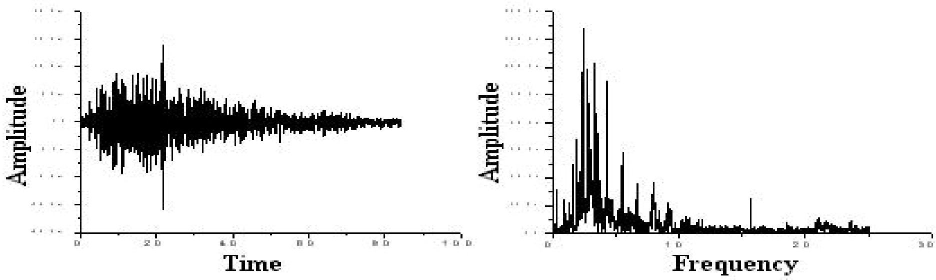


Fig (1): The Time-Domain response and the frequency-domain Response

THE PROPOSED ANALYSIS TECHNIQUE

Usually the number of accelerometers is limited with respect to the large scale of the bridge like in this case study, so the test may be carried out through several subtests. Also the measurement stations may be taken for a quarter of the bridge at the cables line on the deck. In addition extra two stations should be taken. The first is assumed mirror of the nearest one to mid span to distinguish between symmetric and non symmetric modes. The second is opposite to the first in the transverse direction to distinguish between torsion and bending modes.
The test output data gives the acceleration response versus time "Time Domain" in the three directions in text file. Then the acceleration response versus frequency "Frequency Domain" is computed using any FFT analyzer as shown in figure (2). the compatible peaks that appear in all FRF graphs of the whole stations of the same subtest should be selected. The mode shapes could be plotted using the response of all the stations for each frequency value. they should be compared with those extracted from the finite element analysis to find out the natural frequencies and the corresponding mode shapes of the bridge.
A proposed data analysis tool was created using Microsoft Excel to simplify the extraction operation of modal data "mode shapes and corresponding natural frequencies".

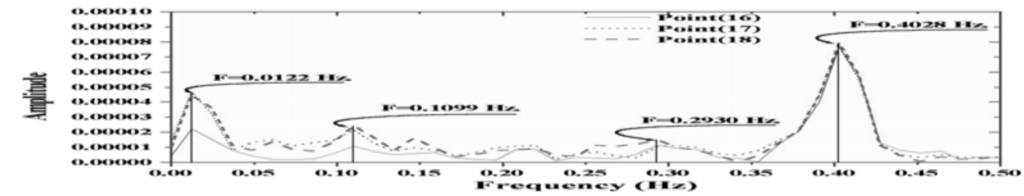


Fig (2): Acceleration response in Z-direction subtest (7)
For frequency range 0-0.5 Hz

THE ANALYSIS TOOL

The FRF data is imported from the FFT analyzer to the Excel File, each subtest in a separate sheet as shown in table (1). The analysis tool performs two main functions, the first is peaks extraction and the second is mode shapes plotting [10].

Table (1): FFT sample data for subtest (7)

Freq. Hz.	Point (16)		Point (17)		Point (18)		Criteria (2)				Criteria (1)			
	R16 m/sec^2	Φ16 deg.	R17 m/sec^2	Φ 17 deg.	R18 m/sec^2	Φ 18 deg.	R16	R17	R18	S2	R16	R17	R18	S1
0.000	4.6E-06	360	1.4E-05	0	1.0E-05	360	0	0	0	0	0	0	0	0
0.012	2.2E-05	240	4.7E-05	-133	4.4E-05	231	1	1	1	3	1	1	1	3
0.024	1.5E-05	401	3.2E-05	39	3.6E-05	402	0	0	0	0	0	0	0	0
0.037	8.0E-06	203	8.1E-06	169	1.1E-05	213	0	0	0	0	0	0	0	0
0.049	4.2E-06	367	1.3E-05	279	8.2E-06	281	0	0	0	0	0	0	0	0
0.061	2.1E-06	465	1.5E-05	446	1.4E-05	453	0	1	1	2	0	1	1	2
0.073	2.2E-06	596	1.1E-05	296	8.8E-06	297	0	0	0	0	0	0	0	0
0.085	2.2E-06	484	1.3E-05	146	7.5E-06	150	0	0	0	0	0	0	0	0
0.098	6.1E-06	259	1.5E-05	264	1.4E-05	254	0	0	0	0	0	0	0	0
0.110	1.1E-05	422	2.2E-05	419	2.4E-05	417	1	1	1	3	1	1	1	3

1- Peaks Selection Criteria

Since any peak value is bigger than the value before and the value after, so it is needed to compare the response amplitude of each record with the two adjacent values (one after and one before) as criteria (1) or with the four adjacent values (two after and two before) as criteria (2).
By the help of Microsoft Excel computational functions, MAX function used for this comparison on a separate column for each point, and with IF function, if the check is true a flag value (1) appear in this cell, else flag value (0) appear, by using one extra column for each criteria containing the sum of the three flag values, then if the sum is equal to the number of points, it means that this record corresponds to a natural frequency as shown in table (1), and by using the data filter tool in the sum column equal to the number of points we can get all records of the compatible peaks and corresponding natural frequencies, By repeating the same operations for all subtests, all peaks of all points were extracted.
Another check was done to assure the appearance of each peak in the all subtests taking into consideration which direction is being studied and the point locations to minimize the selected peaks and its corresponding frequencies to use them to draw the mode shapes, this is the final check of considering the extracted frequency as a natural one of the bridge (true if the drawn mode has a logical shape with respect to the studied direction and the mathematical model analysis results that are explained before), so it is therefore recommended to draw the mode shapes corresponding to the extracted peaks of the all points.

2- Plotting of Mode Shapes

Plotting of any mode shape corresponding to a certain frequency is very important to check the reality of this frequency as one of the natural frequencies of the bridge, so it is needed to find out an automated, efficient and fast technique to be used in plotting of the different mode shapes corresponding to the extracted large number of peaks frequencies. A Microsoft Excel macro is used to do the following operations:

1. Get the response amplitude and the phase angle of all points in all the subtests to a table form for the selected frequency value.
2. Normalize or rescale the response of the all joints with respect to the subtest (1) using the relative response values of the overlap joints.
3. Calculate the relative sign of the joint response for all joints with respect to the subtest (1) using the relative sine value of the phase angles of the overlap joints.
4. Check the mode symmetry by comparing the response sign of joint 16 and joint 17.
5. Check the mode type (bending or torsion) by comparing the response sign of joint 17 and joint 18.
6. Draw the normalized mode shape ordinates of the bridge along its longitudinal axis using the symmetry check in step 4.

All the above operations is being done automatically by just selecting a frequency value from those extracted in the previous step in the calculation sheet, three files containing the same technique are created for the three response directions X, Y and Z. Figure (3) shows the flow chart of the proposed analysis tool.

FEA MODEL UPDATE STRATEGY

The model update operation follows one of a two major methods. The first is the direct matrix method, which needs especial and advanced softwares that are not available in the local market. The second is "the indirect physical property adjustment method" [9], in which some changes are made to specific elemental properties in the model (using the engineering sense) searching for an adjustment, which brings FEA and EMA modal data closer together. The second method was used for the bridge FEA model update as follow:

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \tag{1}$$

Where: ***f*** is the natural frequency, **ω** is the angular frequency, **K** is the stiffness and **m** is the mass.

And: $$k = \lambda \frac{EI}{L} \tag{2}$$

$$m = \frac{w}{g} \tag{3}$$

Where: **λ** is a factor depends on boundary conditions of the member, ***E*** is the modulus of elasticity, ***I*** is the inertia of the member and ***L*** is the length of the member.

So to get a higher value of frequency, one should to either increase the stiffness or decrease the mass. To increase the mass, own weight should be increased, for the main girder and cables own weights, they were perfectly calculated according to the physical statistical data of the main bridge and any load changes makes the model lose its reality especially for static analysis, however for concrete elements own weights, the concrete specific unit weight may be changed slightly within normal figures because it was cast in site. It is preferable mainly to update the stiffness to compensate any neglected factors or elements of the boundary conditions of the different members like variations in materials properties, variations in sections dimensions, variations in supports and fixation conditions and local stiffeners of the main girder "box girder modeling achieved the real stiffness as a whole section but it is not necessary to achieve the real stiffness of the different elements like webs, flanges and diaphragms". To change the stiffness according to equation (2), it is not possible to change the member length and it is also hard to find out the required parameters that governs the changes of **λ** coefficient and member inertia due to the large number of members, so the only choice is to use the modulus of elasticity to change the stiffness.

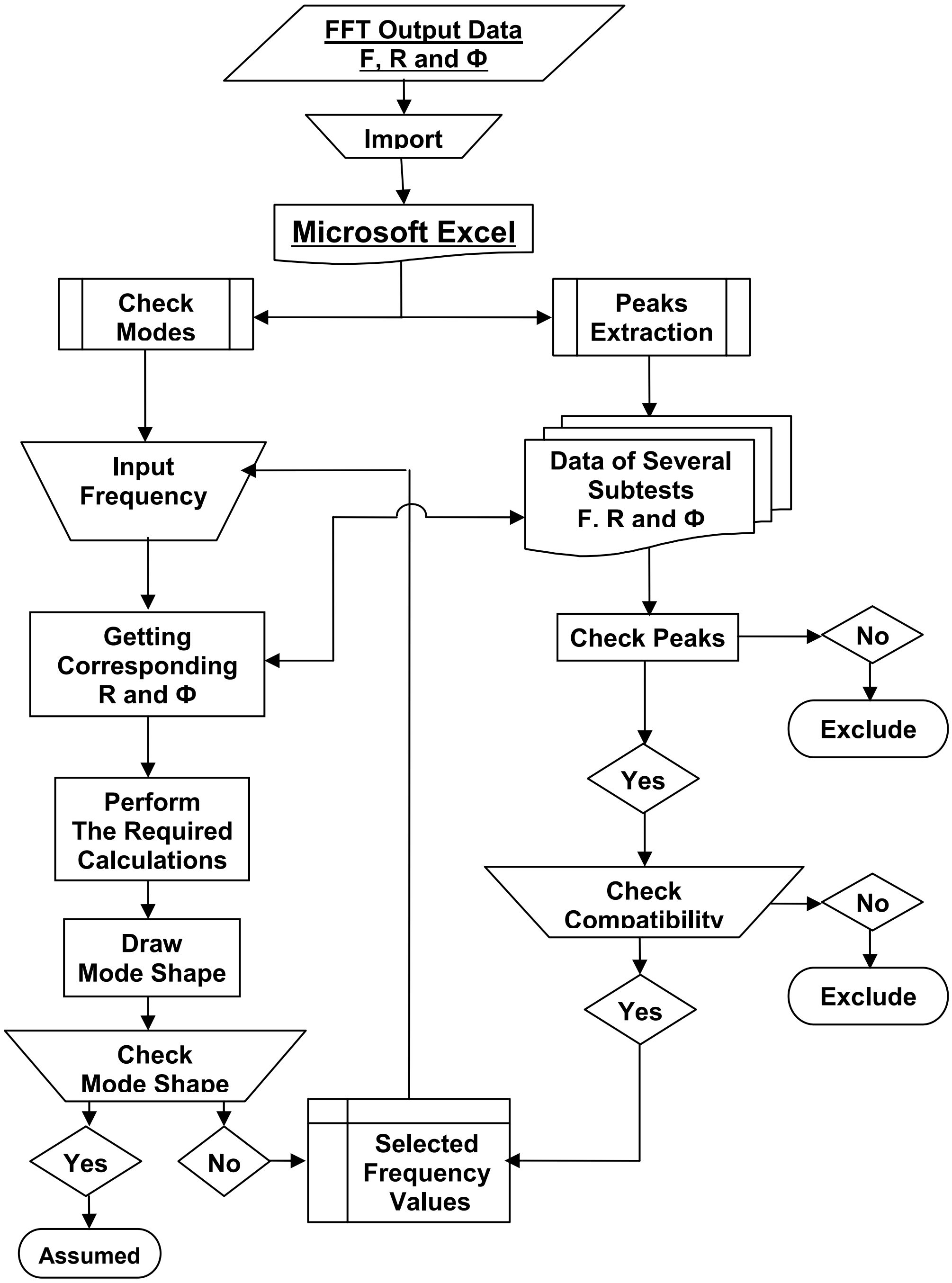


Fig (3): the flow chart of the proposed analysis tool

The engineering sense governs the selection of the updated elements to change the natural frequency of a certain mode based on the assumption that those elements are the main resistance elements to the motion described by this mode. For the longitudinal modes “sway modes”, they are mainly affected by the stiffness of towers and girder bearing elements. The transverse bending modes are affected mainly by main girder stiffness (especially the upper and lower decks). The vertical bending modes are affected mainly by the stiffness of both the main girder and the cables. So the model update strategy depends on changing the modulus of elasticity value of the different materials that assigned to the different bridge elements to change their stiffnesses by trials and errors to adjust the model to give the required natural frequencies values similar to those of the EMA selected mode shapes.

THE CASE STUDY

The Suez-Canal cable-stayed bridge is assumed as a case study to implement the proposed technique. An EMA test had been done for the bridge and the data was analyzed and the analysis results used to update the finite element model of the bridge as follow.

THE SUEZ-CANAL CABLE-STAYED BRIDGE

The Suez-Canal cable stayed bridge links Africa and Eurasia, crossing the Suez-Canal in Qantara city, which is located about 50-km south of the Mediterranean Sea. The bridge construction was completed in autumn of 2001. The bridge has total length of about 3900 meters with two lanes for each direction and with maximum vertical grade 3.3% for smooth traffic flow.
The main bridge is a steel cable-stayed bridge with girder length of 730 meters and central span of 404 meters “clear span is 384 meters” and two side spans of 163 meters with a vertical clearance of 70 meters above high water level to assure free navigation on the canal.
The concrete pylons are H-shaped R.C with height of about 160 meters. The tower cross-section is a variable box section of dimensions 7.6 m x 7.8 m x 0.7 m at base level that reduce gradually with average slope 1:35 till it reaches 2.5 m x 4.5 m x 0.5 m at top level.
The stay cables are 128 cables of 16 steps in double plane for each side, and for each plane the cables were arranged using four types in cross-section, where the first two cables at tower side are of 27.51 cm^2, the next three cables are of 42.51 cm^2, the next two cables are of 54.79 cm^2 and the last nine cables are of 67.06 cm^2 as shown in figure (4).
The main girder is a single-cell steel box girder of 20.8m wide “4 traffic lanes, 0.8m side walks and 1.2m median strip” and variable depth of 1.2m at edges and 2.6m at the middle, the upper and lower decks are made of orthotropic plates with closed ribs, the outer sides and the two longitudinal stiffeners are made of solid plates of thickness 16mm and 11mm respectively, solid steel cross diaphragms of 10mm thickness are added at cables locations and in mid-distance between cables to strengthen the section as shown in figure (5).
The reinforced concrete used for pylons and piers has a cubic compressive strength of 500 kg/cm^2, the steel reinforcement is of grade 36/52 with Young’s modulus of elasticity 2100 t/cm^2, the stay cables ultimate strength is 18000 kg/cm^2, with Young’s modulus of elasticity 2000 t/cm^2.

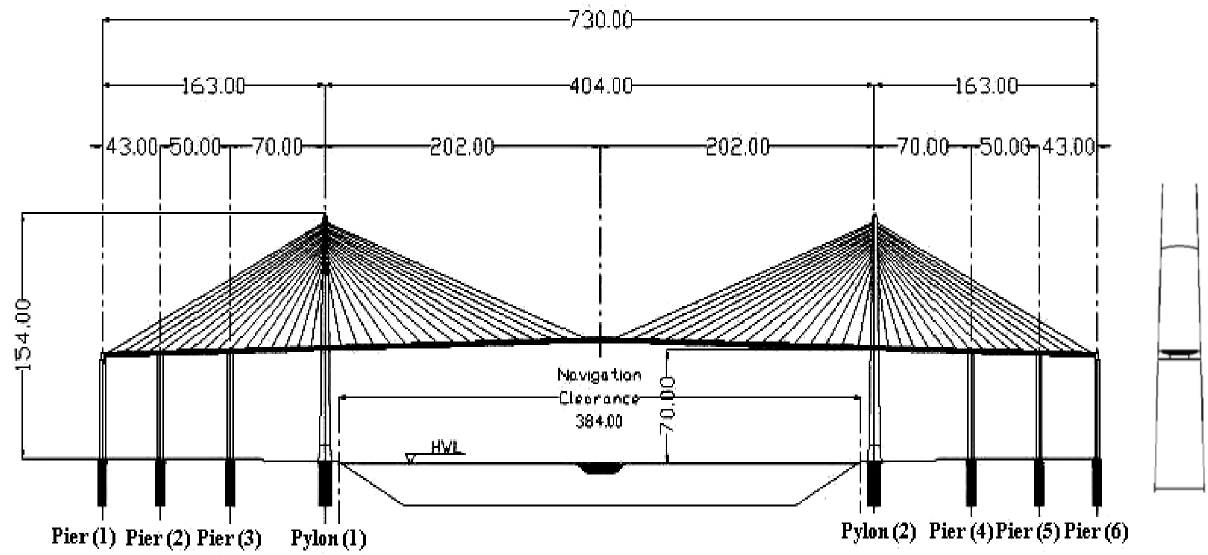


Fig (4): Schematic elevation and side view of The Suez-Canal Bridge

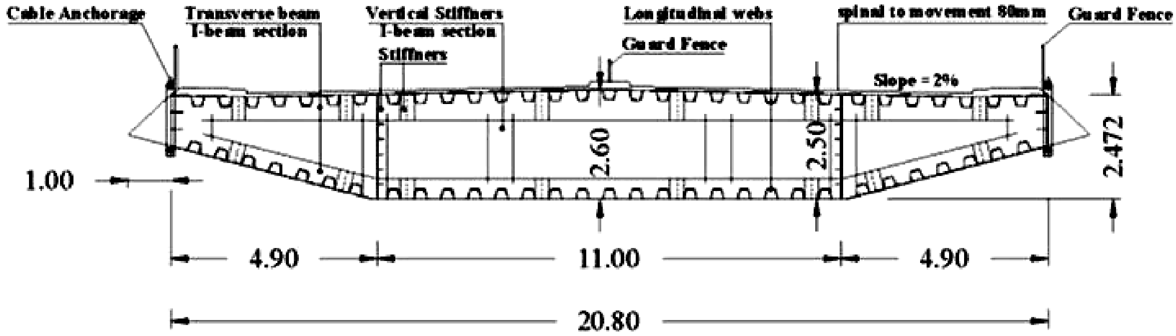


Fig (5): Main Girder Cross Section

FEA MODEL OF THE BRIDGE

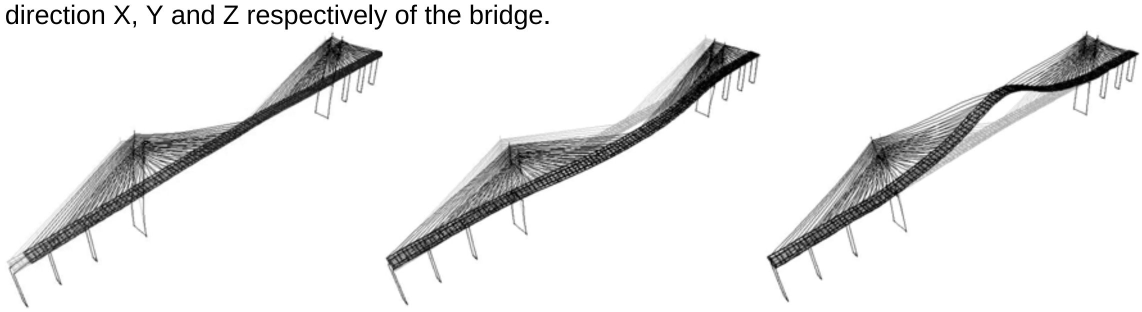
A finite elements model was created using “SAP2000” by the help of the original bridge documentations [11,12]. The model was verified also for both static and dynamic responses using “Staad Pro 2003”.

Pylons, piers, cables and bearings members were modeled using frame elements while the main girder was modeled using the shells elements with equivalent thicknesses that gives the

same properties of the original section. All Pylons and piers supports were assumed totally fixed. Main girder bearings at piers were released for displacement and rotation in the longitudinal direction in addition to rotation about their axes (torsion). Main girder vertical bearings at towers were released for displacement and rotation in transverse direction in addition to torsion, but were partially restrained for displacement in the longitudinal direction (V2=1000 t/m). Main girder horizontal bearings at towers were restrained only for axial displacement (transverse direction). The compression limit of cables was assumed zero and their post tension forces were modeled using p-delta initial force program option. Masses were modeled by the program as lumped masses at joints using the different materials mass density. The total mass of the model was checked by that reported in the original bridge documentation.

MODAL ANALYSIS RESULTS

Geometric non-linear analysis with ten iteration steps assuming large deformation into consideration were performed using SAP2000 Model of the bridge and the resultant stiffness matrix was used for modal analysis to extract the first thirty mode shapes and their corresponding natural frequencies. Figure (6) shows the first mode shape in each global direction X, Y and Z respectively of the bridge.



1St longitudinal mode F=0.160 Hz

1St transverse mode F=0.216 Hz

1St vertical mode F=0.361 Hz

Fig (6): The first modal mode shape in the three directions

THE BRIDGE EMA TEST

An experimental modal analysis "EMA" test was carried out on the bridge using ambient excitation by a truck. The test consists of seven subtests, for each, three or four accelerometers were used to record the bridge response. Only response of half of the bridge was recorded due to limited number of accelerations. Figure (7) shows the subtest layout and locations of the used accelerometers, where 1, 2, 3 and 4 are the accelerometer reference number. All accelerometers were located at cables connections with the deck in the southern side of the bridge "right side" except the accelerometer no 4 of subtest 7 was located at the northern side "left side" of the bridge to distinguish between torsion and bending mode shapes. The accelerometer no 3 of subtest 7 that located at the west side indicates the mode shape symmetry. The subtests were carried out by a sampling rate Δt =0.02 sec for the half of the bridge only due to the lack of accelerometers depending on the symmetry of the bridge.

Test Records Analysis

The EMA test measurements were recorded in seven text files for the seven subtests each contains the acceleration response versus time intervals in the three directions. The locations of the accelerometers were renumbered and rearranged as shown in figure (8), in order to simplify the coordination between the accelerometers locations and their response, hence to draw the mode shapes easily, taking into consideration that the bold circled points are representing the subtest overlap locations.

A FFT analysis were performed using the Microcal Origin version 6.0 program using Hanning windowing technique to transfer data from time-domain form to the frequency-domain form. The FFT output records of the different subtests have 0.01 Hz frequency resolution (the minimum accurately resolved frequency shift) and 25 Hz folding frequency range (Nyquist Frequency- the maximum accurately resolved frequency), which is enough to get the master mode shapes within the available excitation conditions.

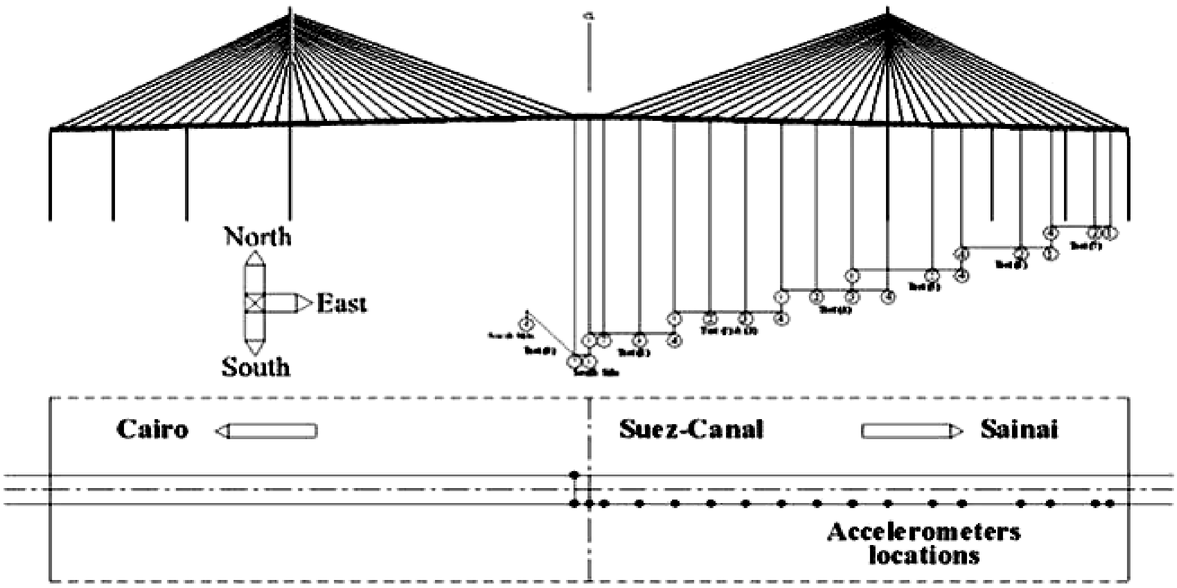


Fig (7): Accelerometers locations for the different subtests

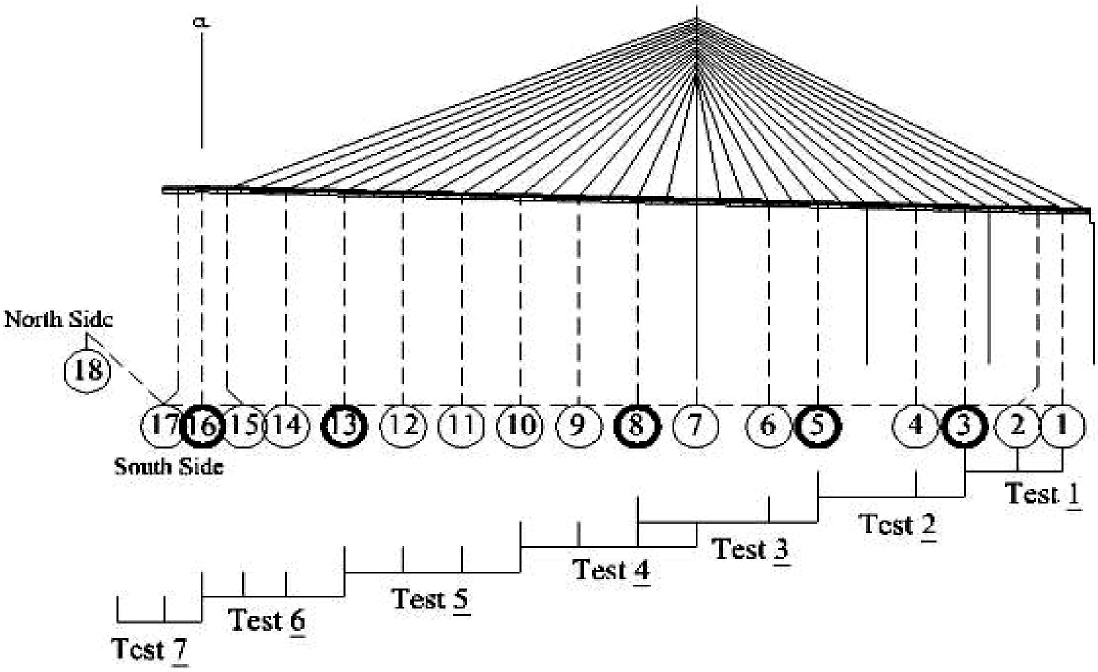


Fig (8): Accelerometers Locations reference numbers
For the different subtests

The FFT analyzer gives the opportunity to plot the FFR (frequency response function) of each joint to select some of the appeared peaks to be checked. To minimize the peaks selection, it is beter to plot the FFR of the whole joints of each subtest in one graph to select the compatible peaks that appear in all the FFRS as shown in figure (2).

It should be noted that each graph represents the response of only one point during the excitation, and some the resultant peaks may appear due to resonance representing natural frequencies of the bridge, and some others may appear due to noise or error in sampling at the observed point, which is not needed, so to find out a peak corresponding to a natural frequency, it should appear in all graphs of all points of the same subtest regardless the response value.

To check if the selected peak is one of the natural frequencies of the bridge, relative response ordinate of all observed points should be plotted and checked to ensure they are similar to one of the mode shapes that were calculated theoretically using the FEA. The selection of the frequency peaks may be governed by those extracted theoretically using the FEA.

Normally, the FRF peaks is extracted using visual judgment for the function graph, which is impractical due to the large numbers of recorded peaks and the required complicated calculations and procedures to draw the corresponding mode shape, so it is essential to use the proposed analysis tool that can perform those procedures automatically to give the opportunity to check the large number of peaks accurately and quickly.

Data Analysis Results

For such structures like cable-stayed bridges it is enough to study the FRF up to 1 Hz for global mode shapes, while the higher frequency values represent some members' local mode shapes, so the studied peaks were selected within this range. The extracted theoretical modes guided the selected peaks.

From the bridge EMA test data analysis, six mode shapes and their corresponding natural frequencies were extracted using the proposed analysis tool as shown in table (2) and in figures (9), (10) and (11).

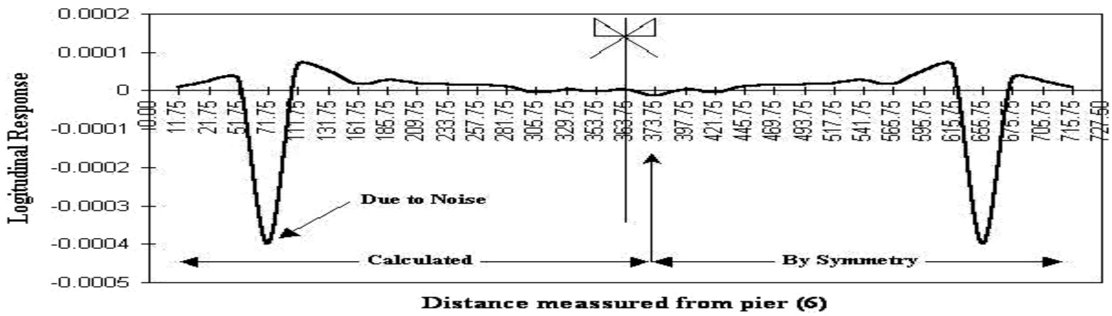
Table (2): FEA and EMA natural frequencies and mode shapes

Mode No.	Mode Direction	Mode Description	FEA Freq. (Hz.)	EMA Freq. (Hz)
1	Longitudinal	Symmetric Pylon Bending "Sway"	0.160	0.256
2	Transverse	Symmetric Girder/Pylon Bending "**1**st Transverse Bending of Girder"	0.216	0.269
3	Transverse	Anti-symmetric Pylon Bending "**2**nd Transverse Bending of Girder"	0.254	0.281
N.A.	Transverse	"**3**rd Transverse Bending of Girder"	-	0.293
5	Vertical	Symmetric Girder Bending "**1**st Vertical Bending of Girder"	0.361	0.403
22	Vertical	Symmetric Girder/Pylon Bending "**3**rd Vertical Bending of Girder"	0.570	0.830

FEA RESULTS EVALUATION WITH RESPECT TO EMA RESULTS

By comparing the extracted modal parameters using the FEA model with those extracted from the EMA test results, some differences in the natural frequencies values, where the mathematical values are less than the experimental values that means that the assumed bridge stiffness in the FEA model is less than it should be. Table (2) shows the FEA modal frequencies values versus those of the EMA.

So the FEA model should be updated to match the EMA model as possible as it could be and gives the same modal parameters of the EMA test. This updating operation will introduce the FEA model as a good representative mathematical model for the bridge to be used for further studies.



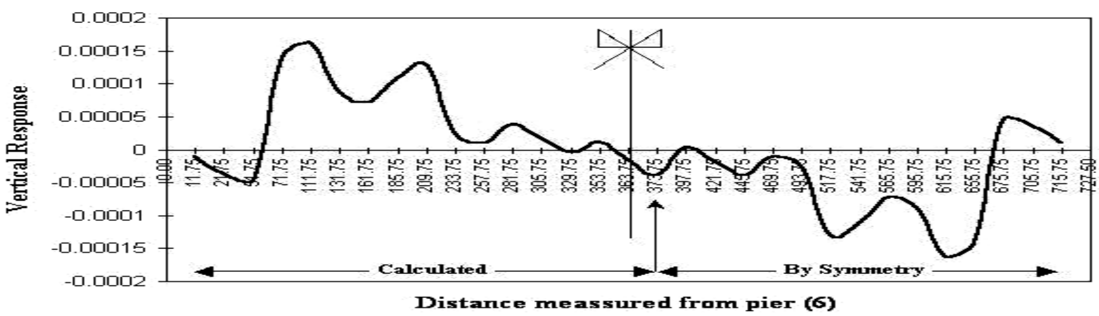


Fig (9): The 1st Longitudinal "Sway" mode and the corresponding mode shape respectively

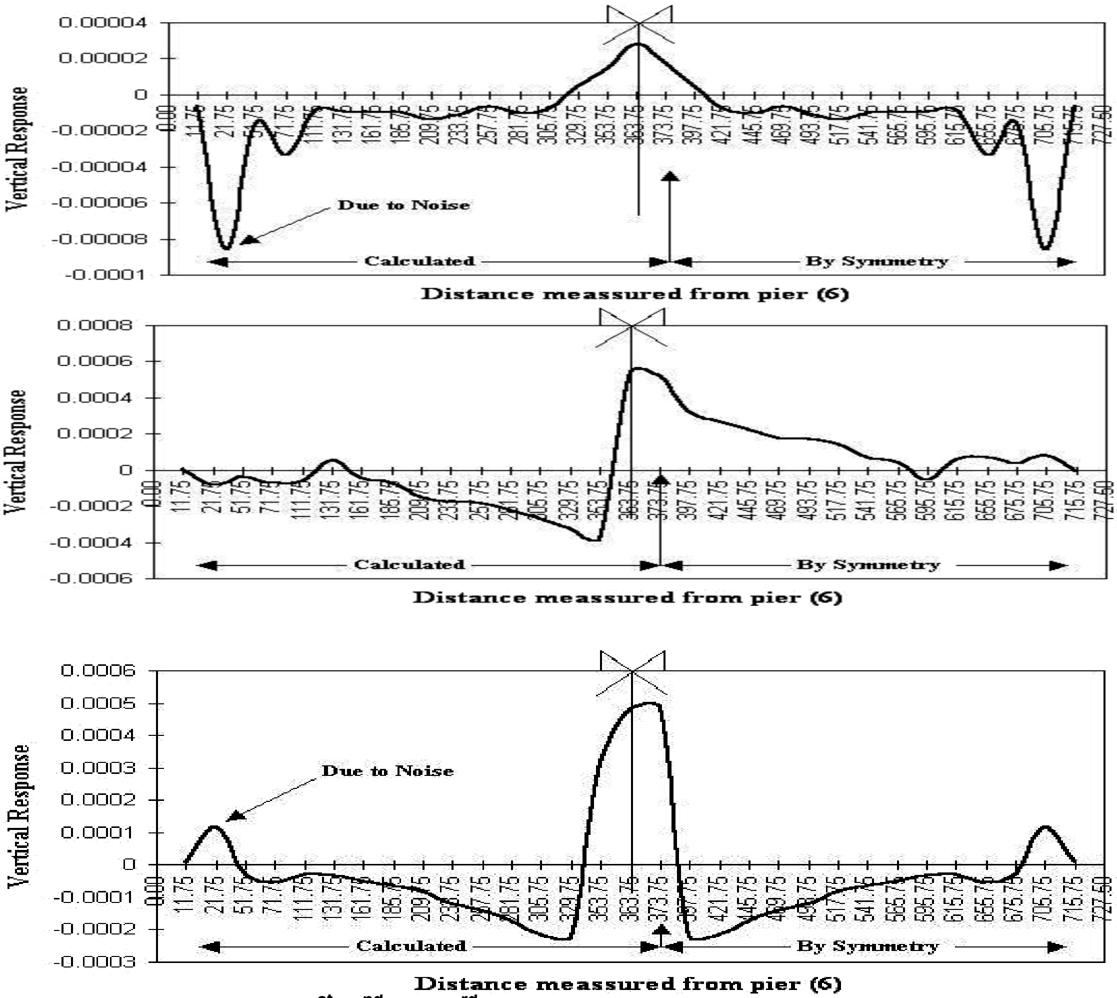


Fig (10): The 1st, 2nd and 3rd transverse bending modes respectively

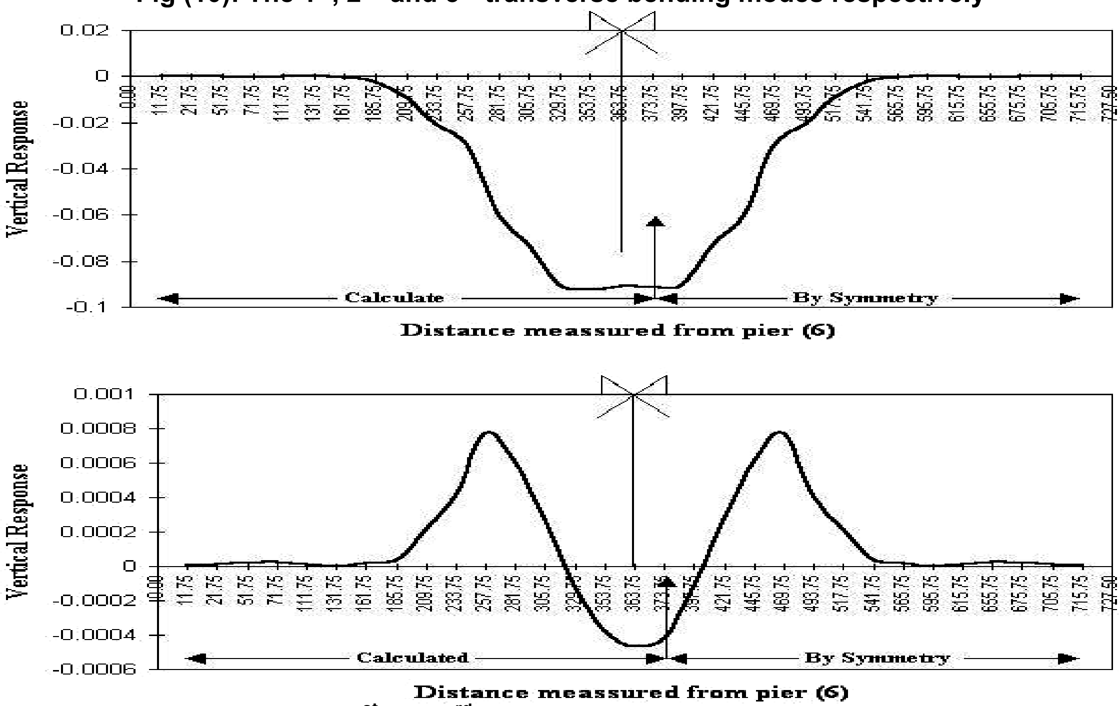


Fig (11): The 1st and 3rd vertical bending modes respectively

EMA Selected Modes

The low frequency modes (early modes) are always representing the global behavior of the structure, while the high frequency modes are representing the local behavior of the different elements.
The first mode in each direction is enough to represent the motion in this direction, so only the first three EMA modes that were used in FEA model update. The selected modes are the shaded modes in table (2).

UPDATE OPERATION DETAILS

The FEA model update operation required more than fifty trials of model modifications until adjustment was achieved by applying the following:
The bridge bearing elements on piers and towers were partially restrained as shown in table (3) to increase the stiffness of the bridge against the sway mode, where the main bridge and the two approach bridges are attached, which restrain the bridge motion partially in the longitudinal direction.
The specific unit weight of the concrete material of towers was decreased and the modulus of elasticity was increased as shown in table (4) to increase the stiffness of towers and bridge consequently against both longitudinal and transverse directions. The specific unit weight of the concrete material of piers was decreased as shown in table (4) to increase the bridge stiffness against transverse motion.
The modulus of elasticity of the different steel materials used for main girder elements and cables was increased as shown in table (4) to increase the stiffness of the bridge against bending modes in both transverse and vertical directions.

Table (3): Bearing partial restraining values in the longitudinal direction

Bearing Element	Elements Number	Original V_2 Value (Ton/m.)	Modified V_2 Value (Ton/m.)
Towers Bearings	4	1000	1500
Piers Bearings	12	0	300

Table (4) Original and modified properties of the used materials

Material Name	Material Description	Unit Mass Ton-s^2/m^4		Unit Weight Ton/m^3		E Ton/m^2	
		Original	Mod.	Original	Mod.	Original	Mod.
CONC	Towers Concrete	0.25	0.20	2.50	2.00	2100000	3350000
Conc_P	Piers Concrete	0.25	0.22	2.50	2.20	2100000	2100000
STEEL	Diaphragms Steel	1.10	1.10	10.76	10.76	21000000	27500000
Steel_C	Cables Steel	0.80	0.80	7.85	7.85	21000000	25000000
STEEL_LD	Lower Deck Steel	1.10	1.10	10.76	10.76	21000000	27500000
STEEL_UD	Upper Deck Steel	1.16	1.16	11.43	11.43	21000000	27500000
STEEL_WB	Webs Steel	1.10	1.10	10.76	10.76	21000000	27500000
Z_Conc	Zero Wt. Concrete	0.00	0.00	0.00	0.00	2100000	2100000
Z_Steel	Zero Wt. Steel	0.00	0.00	0.00	0.00	21000000	21000000

CONCLUSION

The EMA could be considered the best available non-destructive testing tool for cable-stayed bridges, by which the bridge modal properties can be extracted to be used powerfully in the bridge model update. Then the updated model would simulate the real bridge and could be used in monitoring its structural health by repeating the EMA test and using the appropriate SHM techniques. A proposed computerized analysis tool was introduced that could be used powerfully in analysis of any EMA test results especially incase of using a limited number of accelerometers. For the case-study of the Suez-Canal cable-stayed bridge, the experimentally extracted mode shapes and corresponding natural frequencies were very near to those extracted theoretically using FE analysis. The extracted modal properties were used to update the FE model of the bridge using the indirect physical property adjustment method that used later to study the appropriate SHM techniques for cable-stayed bridges.

ACKNOWLEDGEMENT

The EMA test team supervisors consisted of Dr. Ayman Hussein for the test plan, Dr. Adel El-Attar and Dr. Mashhour Ghuniem for providing the test equipments and supervising the excution.
Many thanks for all team members for test data supply.

REFERENCES

1. Ewins, D., Modal Testing: Theory and Practice, John Wiley and Sons, Inc., New York, 1984.
2. Morrison, N., Introduction to Fourier analysis, John Wiley and Sons, Inc., New York, 1994.
3. Bendat, J.S.; Piersol, A.G., Random Data: Analysis and Measurement Procedures, John Wiley and Sons, Inc., New York, 1971.
4. Ren, W., X. and Z. H. Zong, Output-only modal parameter identification of civil engineering structures, Structural Engineering and Mechanics, Vol. 17, No. 3-4 (2004).
5. Ewins, D., Basics and state-of-the-art of modal testing, Sadhana ,Vol. 25, Part 3, pp. 207-220, India, June 2000.
6. DTA 1996 Hand Book of Best Practice, Vol. 3, Model testing.
7. Khalil, A., Aspects in Nondestructive Evaluation of Steel Plate Girder Bridges, PH. D. Dissertation, Iowa State University, Iowa, USA, 1998.
8. Charles R. Farrar, Thomas A. Duffey, Phillip J. Cornwell, Scott W. Doebling, “Excitation Methods for Bridge Structures”, in Proceedings of the 17th International Modal Analysis Conference Kissimmee, FL, Feb 1999.
9. Ewins, D., Adjustment or Updating of Models , Sadhana ,Vol. 25, Part 3, pp. 235-245, India, June 2000.
10. Abozeid, H., Dynamic Response of Cable-Stayed Bridges, M. Sc.Dissertation, Ain Shams University, Cairo, Egypt, 2005.
11. Pacific Consultants International, Chodai Co., LTD., “Final Design Drawings of The Suez-Canal Bridge, The Shop Drawings of The Main Bridge”, August 1997.
12. Kajema-NKK-Nippon Steel Consortium, “Suez-Canal Bridge – Construction brochure”, October 2001.