King Saud University
College of Engineering
Electrical Engineering Department

EE-205: Electrical Circuit Lab Experiment No.7

Magnetically Coupled Circuits

Objectives

To get acquainted with the basic principles of magnetic coupling, self inductance, mutual inductance, coupling coefficient.

Material Required

Coils: Two coupled coils (500 and 1000 tuns) connected as a transformer.
Equipment: Two Ammeters, three Voltmeters, Variable AC Power Supply (Variac).

Background

I. Self Inductance

Consider a single coil wound around a magnetic core as shown in Fig. 1.

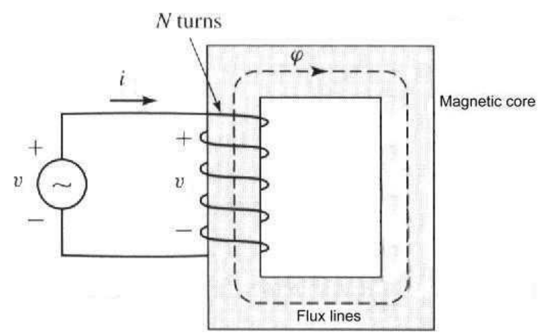


Fig. 1: A single coil wound around a magnetic core.

A current i flowing in this coil will produce a flux ϕ in the magnetic core. If the current is varying with time, the produced flux will also be varying with time. According to Faraday's law of induction, an induced voltage v will be generated across the coil terminals. V is given by the relation,

$$v = N \frac{d\phi}{dt} \quad . \tag{1}$$

The self inductance of a coil is defined as the ratio between the flux linkage, N ϕ, linking the coil and the current, i , producing the flux, or:

$$L = \frac{N\Phi}{i} \tag{2}$$

Substitution from equation (2) into (1) gives

$$v = L\frac{di}{dt} \tag{3}$$

In this equation L is assumed constant, otherwise the following general equation has to be used.

$$v = \frac{d}{dt}(Li) \tag{4}$$

II. Mutual Inductance

If a second coil is wound around the magnetic core of Fig.1 as shown in Fig.2, the two coils become magnetically coupled. The flux produced by the first links the second. The mutual inductance, M_{12}, between coil 1 and coil 2 is defined as the ratio between the flux linkage, $N_2\,\phi$, linking coil 2 and the current i_1 in coil 1 producing this flux, or:

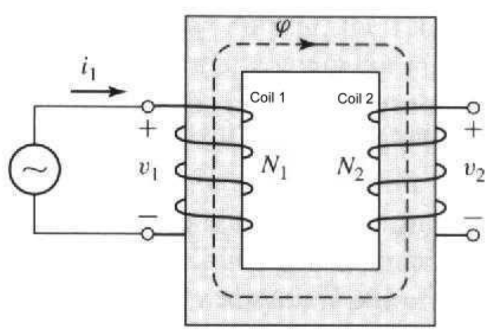


Fig. 2: Two magnetically-coupled coils.

$$M_{12} = \frac{N_2\Phi}{i_1} \tag{5}$$

When the terminals of coil 2 are open-circuited, i.e. no current flows in this coil, the following equations can be written.

$$v_1 = \frac{d}{dt}(N_1\Phi) = L_1\frac{di_1}{dt} \tag{6}$$

$$v_2 = \pm\frac{d}{dt}(N_2\Phi) = \pm M_{12}\frac{di_1}{dt} \tag{7}$$

v_2 is the voltage induced in coil 2 due to the current i_1 in coil 1. It can be shown that $M_{12} = M_{21,}$ subscripts are unimportant and we can write:

$$M_{12} = M_{21} = M \qquad (8)$$

The polarity of the induced voltage v_2 depends on the relative method of coiling. To explain this point look at the 3 coils of Fig. 3, which are wound on the same magnetic core.

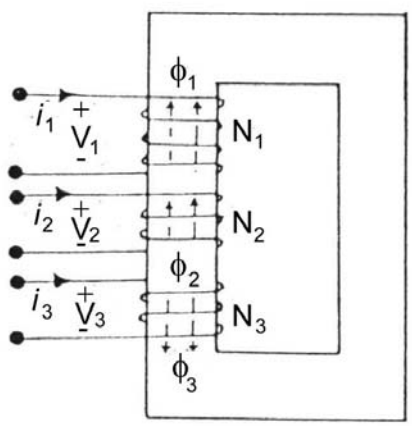
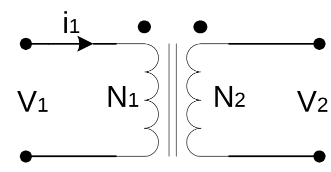


Fig. 3: Three magnetically-coupled coils.

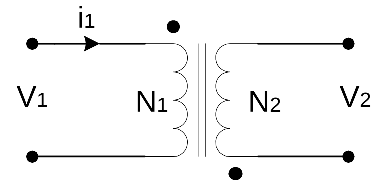
From this figure, it can be seen that coil 3 is wound differently. It follows that v_1 and v_2 have the same polarity, while v_3 has a different polarity. Notice also that a current flowing in coil 1 in the direction shown will set a magnetic field in the same direction as that produced by coil 2. The magnetic field produced by coil 3 when a current flows into it in the direction shown will be opposing that of coils 1 or 2.

III. The Dot Rule

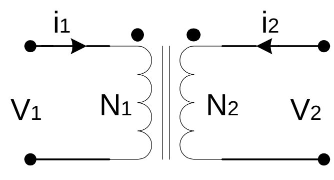
The polarity of the induced voltage is very important when writing the equations of an electric circuit. To indicate this, a dot is put at one of the terminals of each coil to show its relative polarity as shown in Fig.4.



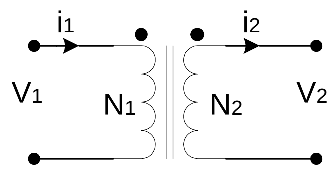
(a) V_2 has the same polarity as V_1.



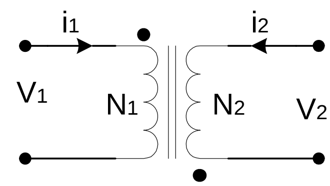
(b) V_2 has an opposite polarity to that of V_1.



(c) mmf of coil 2 adds to that of coil 1.



(d) mmf of coil 2 opposes that of coil 1.



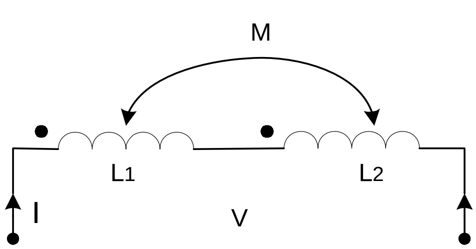
(e) mmf of coil 2 opposes that of coil 1.

Fig. 4: Application of dot rule.

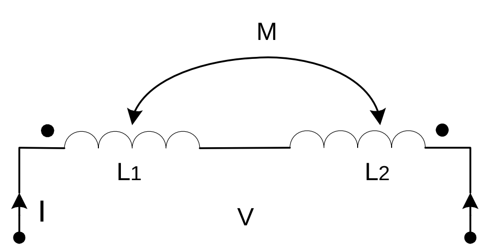
IV. Mathematical Relations

The mathematical relations describing mutually coupled circuits depend on the method of connection.

For series connected coils, two situations are met. The first is the addition situation while the second is the opposition situation . Both cases are shown in Fig.5.



(a) Addition



(a) Opposition

Fig. 5: Two magnetically-coupled coils connected in series.

The corresponding voltage equation for each case, when coil resistance are neglected is

$$v = (L_1 + L_2 + 2M)\frac{di}{dt} \qquad \text{(Additive)} \qquad (9)$$

$$v = (L_1 + L_2 - 2M)\frac{di}{dt} \qquad \text{(Opposition)} \qquad (10)$$

For parallel connected circuits as that shown in Fig.6, the following relations can be written:

$$v = L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \qquad (11)$$

$$v = L_2\frac{di_2}{dt} + M\frac{di_1}{dt} \qquad (12)$$

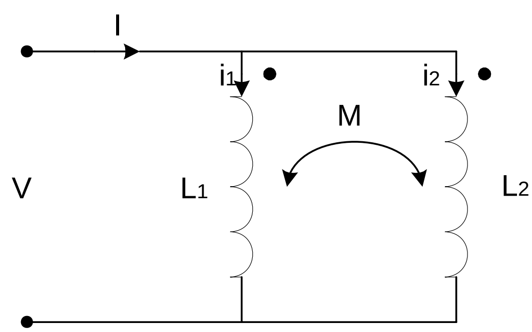


Fig. 6: Parallel connected magnetically coupled circuit.

It can be easily proved that the equivalent inductance of this circuit is given by:

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} \qquad (13)$$

V. <u>Coefficient of Coupling</u>

Not all the flux linking a coil flows inside the magnetic core. A small percent of the flux produced by a coil leaks away from the core. This is known as the leakage flux. It follows that the self inductance of a coil has a value higher than that of the mutual inductance between it and an identical coil. The measure of the leakage flux is the coupling coefficient k.

$$k = \frac{M}{\sqrt{L_1 L_2}} \qquad (14)$$

The maximum value of the coupling coefficient is 1 for the ideal case of no leakage. In this case:

$$M = \sqrt{L_1 L_2} \qquad (15)$$

VI. AC Excitation

For sinusoidal applied voltage of the form V = V_m cos ωt the steady state currents will be also sinusoidal. To take care of this, the operator d/dt in equations (9-12) is replaced by jω and the following relation can be written:

$$\overline{V} = \left[(r_1 + r_2) + j\omega\{L_1 + L_2 + 2M\}\right]\overline{I} \qquad (16)$$

for series connection (addition) and

$$\overline{V} = \left[(r_1 + r_2) + j\omega\{L_1 + L_2 - 2M\}\right]\overline{I} \qquad (17)$$

for series connection (subtraction). V and I are, respectively, the phasor voltage and the phasor current.

Notice that we have considered here the coil resistance; r_1 for coil 1 and r_2 for coil 2.

For a single coil, following equation applies:

$$\overline{V} = (r + j\omega L)\overline{I} = \overline{Z}\,\overline{I} \qquad (18)$$

where $\overline{Z}$ = r + jωL is the impedance of the coil at the frequency ω rad/s.

Procedure I - Determination of Self Inductance

1. Measure the resistance R_{dc} of coil 1 with the help of multimeter. This is dc resistance of coil 1
2. Determine the coil 1 ac resistance R_1 by allowing 15% increase due to skin effect, such that

$$R_1 = 1.15\, R_{dc}$$

3. Similarly find out R_2, the ac resistance of coil 2.
4. Connect coil 1 to a variable 60 Hz voltage source as shown in Fig. 7.

Coil 1: 1000 turns.
Coil 2: 500 turns.

5. Adjust the magnitude of the applied voltage to 2 Vrms and then take the readings of both the ammeter and voltmeter.
6. Calculate the coil (self) inductance as follows:

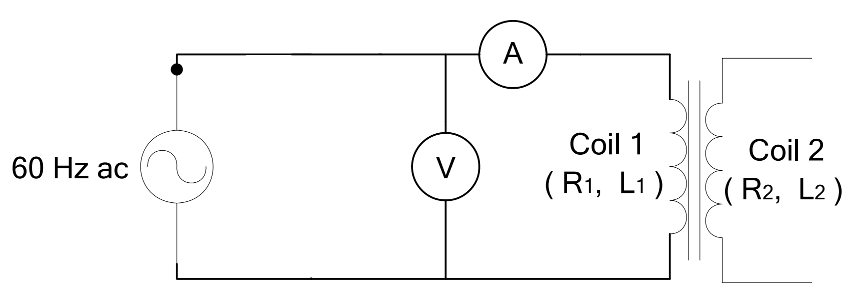


Fig. 7: Measuring the self inductance of a coil.

$$Z_1 = \frac{V}{I} = \sqrt{R_1^2 + X_1^2}$$

$$X_1 = \sqrt{Z_1^2 - R_1^2}$$

$$L_1 = \frac{X_1}{\omega}$$

7. Repeat for coil 2 to find L_2.

Procedure II - Determination of Mutual Inductance

1. Connect the circuit as shown in Fig. 8, with coil 2 magnetically coupled to coil 1.

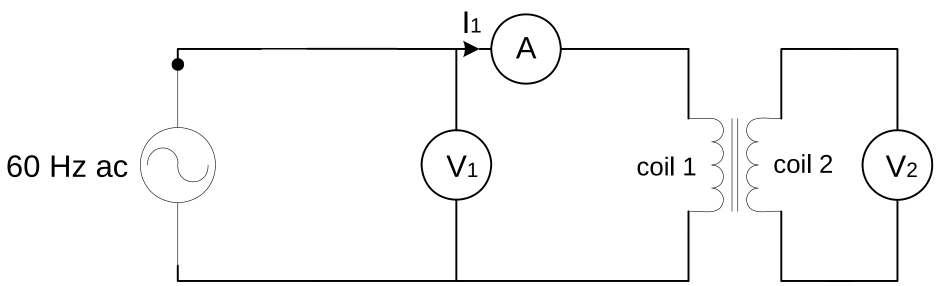


Fig. 8: Determination of the mutual inductance.

2. Adjust the magnitude of the applied voltage to 2 Vrms, and take the readings of V_1, I_1 and V_2.
3. Calculate the mutual inductance as follows:

$$X_M = V_2 / I_1$$

$$M = X_M / \omega$$

Procedure III - Determination of Polarity of Magnetically Coupled Coils

1. Connect the electric circuit as shown in Fig. 9.

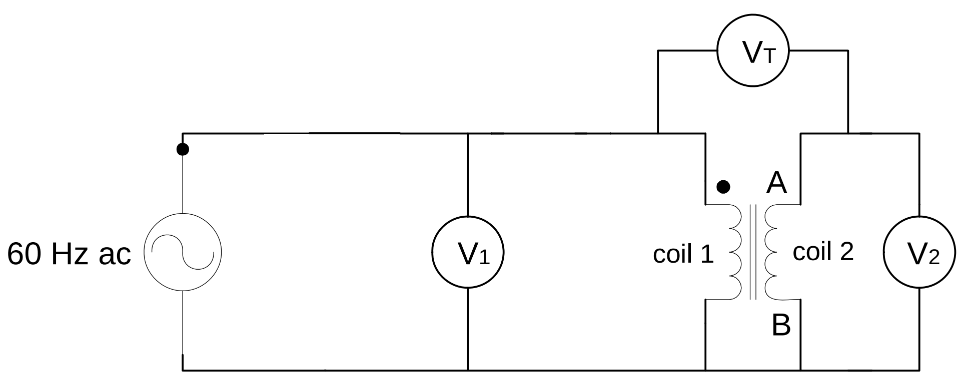


Fig. 9: Determination of the polarity.

2. With specified voltage applied to coil 1, take the reading of V_T.
3. If V_T is the difference between V_1 and V_2, the dot is to be put at point A, otherwise it is to be put at point B.

Procedure IV - Series Connected Magnetically Coupled Coils

1. According to the polarity determined above, connect the electric circuit as shown in Fig.10.

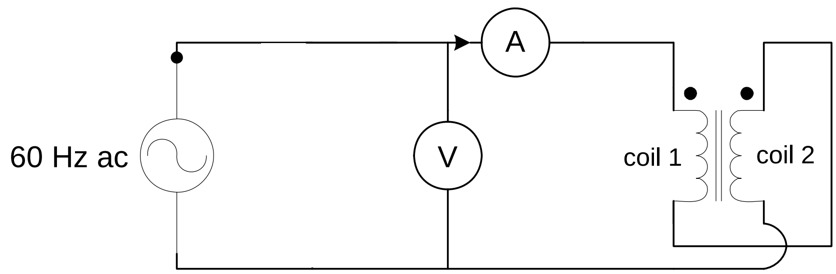


Fig. 10: Series-connected magnetically -coupled coils (adding).

2. Adjust the supply voltage to 2 Vrms and take the readings of the voltage and current.
3. Calculate the equivalent impedance in this case as follows:

$$X_{eq1} = \sqrt{(V/I)^2 - (R_1 + R_2)^2}$$
$$= \omega\,(L_1 + L_2 + 2M)$$

4. Repeat steps 1 and 2 but with the two coils connected in opposition

$$X_{eq2} = \sqrt{(V/I)^2 - (R_1 + R_2)^2}$$
$$= \omega\,(L_1 + L_2 - 2M)$$

5. Determine the mutual inductance M_{12} using the relation:

$$M_{12} = \frac{X_{eq1} - X_{eq2}}{4\omega}$$

6. Check with the value obtained before and comment on any discrepancy between these results.
7. Determine the coupling coefficient k. Where,

$$k = \frac{M_{12}}{\sqrt{L_1 L_2}}$$