


Dr. Bohen

	KING SAUD UNIVERSITY College of Science/Department of Mathematics (Math 151) Discrete Mathematics Summer Semester 1430/1431	Score
<b>Final Exam</b>		
Date: 16/09/1431	Time: 09:00 A.M – 12:00 A.M	Time allowed: 3 Hours

<b>STUDENT NAME (IN ENGLISH)</b>	
<b>Registration Number</b>	
<b>Lecture Time</b>	

- There are 14 multiple choice questions in part A and 7 questions in part B. The maximum score is 60 marks.
- Please do not forget to put your name and registration number on your paper.

Put your answers in the following table.

QUESTION	1	2	3	4	5	6	7
ANSWER	C	A	C	C	D	D	D
Question	8	9	10	11	12	13	14
Answer	C	A	B	C	D	B	D

## PART - A

2 × 14 = 28

Q1. The proposition  $((p \rightarrow q) \rightarrow q) \rightarrow (\neg(p \rightarrow q) \vee q)$  is

- A. logically equivalent to  $(p \wedge \neg q) \leftrightarrow (\neg p \rightarrow q)$
- B. Contradiction
- C. Tautology
- D. None of above

Q2. The proposition  $(p \rightarrow q) \rightarrow (\neg p \rightarrow r)$  is logically equivalent to

- A.  $p \vee r$                       B.  $p \rightarrow (r \vee q)$   
C.  $p \wedge r$                       D.  $(p \wedge q) \rightarrow r$

Q3. The consistent proposition is:

- A.  $\{p \rightarrow (q \wedge \neg r), p \wedge \neg q\}$                       B.  $\{p \rightarrow q, r \rightarrow \neg q, \neg r \rightarrow s, p \wedge \neg s\}$   
C.  $\{p \wedge q \rightarrow r, p \wedge \neg r\}$                       D. None of above

Q4. The relation T defined on  $\mathbb{R}$  by  $aTb \Leftrightarrow |a-b| \leq 5$  is

- A. Reflexive and transitive                      B. Symmetric and antisymmetric  
C. Symmetric but not transitive                      D. not reflexive and not transitive

Q5. If R is an equivalence relation on  $A = \mathbb{Z} \times \mathbb{Z}$  defined as  $(a,b)R(c,d) \Leftrightarrow a+c$  is even and  $b+d$  is even, then  $|A/R|$  is:

- A. 2                      B.  $\infty$                       C. 6                      D. 4

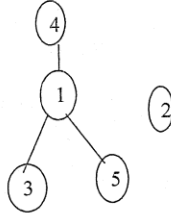
Q6. Let R, S be two relations defined on  $\mathbb{Z}^+ = \{1, 2, 3, \dots\}$  by  $aRb \Leftrightarrow ab$  is a perfect square and  $aSb \Leftrightarrow a$  divides  $b$ . Then

- A. S is an equivalence relation and R is a partial ordering relation.  
B. S and R are both equivalence relation.  
C. S and R are both partial ordering relation.  
D. S is a partial ordering relation and R is an equivalence relation.

Q7. The transitive closure of the relation  $R = \{(a,c), (b,b), (c,b)\}$  defined on  $A = \{a,b,c\}$  is

- A.  $\{(a,c), (b,b), (c,a)\}$                       B.  $A \times A$   
C.  $\{(a,b), (b,c), (c,c)\}$                       D.  $\{(a,b), (a,c), (b,b), (c,b)\}$

Q8. If S is a partial ordering on the set  $A = \{1, 2, 3, 4, 5\}$  and has for Hasse diagram, the following diagram:



Then,

- A.  $S = \{(1,1), (2,2), (3,3), (4,4), (5,5)\}$   
 B.  $S = \{(3,1), (5,1), (1,4)\}$   
 C.  $S = \{(1,1), (2,2), (3,3), (4,4), (5,5), (3,1), (1,4), (5,1), (3,4), (5,4)\}$   
 D.  $S = \{(1,1), (2,2), (3,3), (4,4), (5,5), (3,1), (1,4), (5,1)\}$

Q9. If  $B$  is a Boolean algebra and  $x, y \in B$  such  $xy = y$ , then

- A.  $x + y = x$   
 B.  $x + y = y$   
 C.  $x = 1$   
 D. None of above

Q10. If  $f(x, y, z, w) = (y' + z)w' + xy'z' + x'y'z$  then  $CSP(f)$  is equal to

- A.  $xy'z'w + xy'z'w' + x'y'zw + x'y'zw' + xy'zw' + x'y'z'w'$   
 B.  $xy'z'w + xy'z'w' + x'y'zw + x'y'zw' + xy'zw' + x'y'z'w' + xyzw' + x'yzw'$   
 C.  $xy'z'w + xy'z'w' + x'y'zw + x'y'zw' + xy'zw' + x'y'z'w' + xyzw'$   
 D.  $xy'z'w + xy'z'w' + x'y'zw + x'y'zw' + xy'zw' + x'y'z'w' + x'yzw' + x'yzw'$

Q11. The  $MSP$  form of the Boolean function  $f$  in Q10 is

- A.  $zw' + y'w' + x'y'z$   
 B.  $zw' + y'w' + xy'z'$   
 C.  $zw' + y'w' + x'y'z + xy'z'$   
 D.  $zw' + y'w'$

Q12. The  $MPS$  form of the Boolean function  $f$  in Q10 is

- A.  $(x' + z' + w')(x + z + w')$   
 B.  $(x' + z' + w')(x + z + w')(y' + z)$   
 C.  $(x' + z' + w')(x + z + w')(y' + w')$   
 D.  $(x' + z' + w')(x + z + w')(y' + z)(y' + w')$

Q13. If  $G$  is a simple graph with  $n$  vertices and 56 edges and  $\bar{G}$  has 80 edges then  $n$  is equal to

- A. 16  
 B. 17  
 C. 18  
 D. None of above

Q14. If a connected planar simple graph  $G$  has 11 vertices with degrees 2, 2, 2, 2, 3, 3, 3, 3, 4, 4, 6 then the number of regions in a planar representation is :

- A. 25  
 B. 22  
 C. 6  
 D. 8

PART-B

3 Marks

Q1. Show that  $\neg(p \vee (\neg p \wedge q))$  and  $\neg p \wedge \neg q$  are logically equivalent by developing a series of logical equivalences.

$$\begin{aligned} \neg(p \vee (\neg p \wedge q)) &\equiv \neg p \wedge (\neg(\neg p) \vee \neg q) && \textcircled{1} \\ &\equiv \neg p \wedge (p \vee \neg q) \\ &\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q) && \textcircled{1} \\ &\equiv F \vee (\neg p \wedge \neg q) \\ &\equiv \neg p \wedge \neg q && \textcircled{1} \end{aligned}$$

4 Marks

Q2. Show that  $(3n+2)$  is odd number if and only if  $(9n+5)$  is even number for an integer number  $n$ .

$$(3n+2) \text{ is odd } (\Leftrightarrow) (9n+5) \text{ is even}$$

" $\Rightarrow$ " We suppose that  $(3n+2)$  is odd.

Then there exists an integer  $k \in \mathbb{N}$  such  $3n+2 = 2k+1$ ,

$$9n+6 = 6k+3$$

$$9n+5 = 6k+2 = 2(3k+1) = 2m$$

$$(m = 3k+1 \in \mathbb{N})$$

So  $(9n+5)$  is even.

" $\Leftarrow$ " We suppose that  $(9n+5)$  is even. Then there exists

$l \in \mathbb{N}$  such  $9n+5 = 2l$

$$3n+2 = 2l - 6n - 3 = 2(l - 3n - 2) + 1$$

Now we put  $j = l - 3n - 2 \in \mathbb{Z}$ , we get

$$3n+2 = 2j+1 \quad \therefore \text{That means } (3n+2) \text{ is odd.}$$

4 Marks

Q3. If sequence  $\{a_n\}_{n=1}^{\infty}$  is defined as follows:

$$a_1 = 1, a_2 = 5$$

$$a_n = 2a_{n-1} + 3a_{n-2} \text{ for } n \geq 3,$$

By using mathematical induction, show that  $a_n < 2 \times 3^{n-1}$  for all integers  $n \geq 1$ .

We put  $P_n$  : " $a_n < 2 \times 3^{n-1} \forall n \geq 1$ "

Base step :

$n=1$	$a_1 = 1 < 2 \times 3^0$	$P_1$ is true
$n=2$	$a_2 = 5 < 2 \times 3$	$P_2$ is true

①

Inductive step : Let  $k \geq 1$ . We suppose that  $P_1, P_2, \dots, P_k$  are true and we will show that  $P_{k+1}$  remains true.

① As  $a_{k+1} = 2a_k + 3a_{k-1}$  and

①  $a_k < 2 \times 3^{k-1}$  ( $P_k$  is true)

①  $a_{k-1} < 2 \times 3^{k-2}$  ( $P_{k-1}$  is true)

① then  $a_{k+1} < 2 \times 3^{k-1} + 3 \times 2 \times 3^{k-2}$ , so  $a_{k+1} < 2 \times 3^k$   
 $< 2 \times 3^{k-1} [2+1]$  That means  $P_{k+1}$  is true.

Conclusion  $\forall n \geq 1, a_n < 2 \times 3^{n-1}$  6 Marks

Q4. Let S be the relation defined on the set  $\mathbb{Z} - \{0\}$  of non zero integers numbers as follows

$$xSy \Leftrightarrow \exists k \in \{0, 1, 2, 3, \dots\} \text{ such } x = y^{2k+1}.$$

i) Show that S is a partial ordering relation on  $\mathbb{Z} - \{0\}$ .

① S is a partial ordering relation if S is reflexive, antisymmetric and transitive.

① S is reflexive : if we take  $k=0$ ,  $x = x^{2 \times 0 + 1}$  then  $xSx$ .

S is antisymmetric : we suppose that  $xSy$  and  $ySx$ , we will show that  $x=y$ .

As  $xSy$  then  $\exists k \in \mathbb{N} / x = y^{2k+1}$  (1)

As  $ySx$  then  $\exists m \in \mathbb{N} / y = x^{2m+1}$  (2)

By substitution (1) into (2), we get  $y = (y^{2k+1})^{2m+1} = y^{2(2km+k+m)+1}$

①  $\Rightarrow y (1 - y^{2(2km+k+m)}) = 0$

As  $y \neq 0$  ( $y \in \mathbb{Z} - \{0\}$ ) then  $2km+k+m=0$ . As  $k, m \in \mathbb{N}$ , we get  $k=m=0$ . We deduce that  $x=y$ .

$S$  is transitive: we suppose  $x S y$  and  $y S z$ , we will show that  $x S z$ .

①

As  $x S y$  then  $\exists k \in \mathbb{N}$  such  $x = y^{2k+1}$ . Also  $y S z$  then  $\exists m \in \mathbb{N}$  such  $y = z^{2m+1}$ . By substitution, we get  $x = z^{2(2km+m+k)+1}$ .

We take now  $l = 2km + m + k \in \mathbb{N}$ , we have  $x = z^{2l+1} \Rightarrow x S z$ .

ii) Is it  $S$  a totally ordering relation on  $\mathbb{Z} - \{0\}$  or not?

We take for example  $x = 2$  and  $y = 3$ .

②

As  $2 \not S 3$  and  $3 \not S 2$  then 2 and 3 are not comparable.

Therefore  $S$  is not a totally ordering relation on  $\mathbb{Z} - \{0\}$ .

8 Marks

Q5. Construct the least and & or logic network for

$$f(x, y, z, w) = xyz'w' + xyz'w + xy'z'w' + xy'z'w + x'y'z'w' + x'y'z'w + x'yz'w' + x'yz'w$$

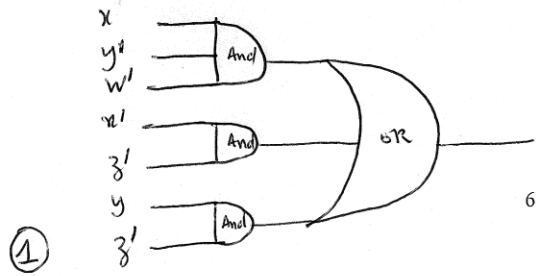
Karnaugh map

	$zw$	$zw'$	$z'w'$	$z'w$	
$xy$	0	0	1	1	②
$xy'$	0	1	1	0	
$x'y'$	0	0	1	1	
$x'y$	0	0	1	1	

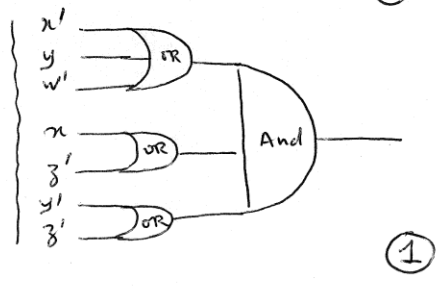
MSP (f) =  $yz' + x'z' + xy'w'$  ②

As MSP (f') =  $xy'w + x'z + yz$

then we deduce MPS (f) =  $(x' + y + w')(x + z')(y' + z')$  ②



4 gates

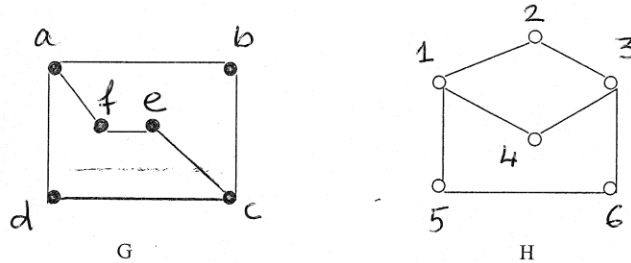


4 gates

The two cases are the least and/or logic network for

4 Marks

Q6. Show whether the graphs G and H are isomorphic or not?



②

$x$	a	b	c	d	e	f
$f(x)$	1	2	3	4	6	5

As  $f$  is bijective then  $G \cong H$  (are isomorphic)

②

3 Marks

Q7. If  $G$  is a simple graph with  $n$  vertices, show that  $|E| + |\bar{E}| = \frac{n(n-1)}{2}$  such  $\bar{E}$  is the set of edges for the complementary graph  $\bar{G}$ .

$G = (V, E)$  a simple graph with  $|V| = n$

$\bar{G} = (\bar{V}, \bar{E})$  its complementary.

② So  $G \cup \bar{G} = (V, E \cup \bar{E})$  is  $(n-1)$ -regular graph.

① Then  $|E \cup \bar{E}| = |E| + |\bar{E}| = \frac{n(n-1)}{2}$ .