



KING SAUD UNIVERSITY
College of Science
Department of Mathematics

M-106

Second Semester (1430/1431)

Solution Badeel Second Mid-Exam

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 20

Time: 90 minutes

Marks:

Multiple Choice (1-10)	
Question # 11	
Question # 12	
Question # 13	
Question # 14	
Total	

Multiple Choice

Q.No:	1	2	3	4	5	6	7	8	9	10
$\{a, b, c, d\}$	b	a	c	d	b	b	a	b	c	a

Q. No: 1 $\lim_{x \rightarrow \infty} (e^x + x)^{\frac{1}{x}}$ is equal to:

- (a) e^2 (b) e (c) 1 (d) ∞

Q. No: 2 The partial fraction decomposition of $\frac{x^3}{(x-1)^2}$ takes the form:

- (a) $x + \frac{A}{x-1} + \frac{B}{(x-1)^2}$ (b) $\frac{A}{x-1} + \frac{B}{(x-1)^2}$ (c) $x + \frac{A}{(x-1)^2}$ (d) $\frac{A}{x+1} + \frac{Bx+C}{(x-1)^2}$

Q. No: 3 To evaluate the integral $\int \frac{x^3}{\sqrt{9-x^2}} dx$, we use the substitution:

- (a) $x = 3 \tan \theta$ (b) $\theta = 3 \sin x$ (c) $x = 3 \sin \theta$ (d) $x = 3 \sec \theta$

Q. No: 4 The value of the integral $\int_0^{\frac{\pi}{3}} \sin^4(3x) \cos^3(3x) dx$ is equal to:

- (a) 1 (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) 0

Q. No: 5 The substitution $u = \tan\left(\frac{x}{2}\right)$ transforms the integral $\int \frac{1}{1 + \sin x + \cos x} dx$ into:

- (a) $\int \frac{1}{1+2u} du$ (b) $\int \frac{1}{1+u} du$ (c) $\int \frac{1}{1-u} du$ (d) $\int \frac{1}{u^2+u+1} du$

Q. No: 6 To evaluate the integral $\int \frac{\sec x}{\cot^5(x)} dx$, we use the substitution:

- (a) $u = \tan x$ (b) $u = \sec x$ (c) $u = \cot x$ (d) $u = \sin x$

Q. No: 7 The improper integral $\int_0^1 \frac{1}{x^{2\alpha}} dx$ converges if

- (a) $\alpha < \frac{1}{2}$ (b) $\alpha = \frac{1}{2}$ (c) $\frac{1}{2} < \alpha < \frac{3}{2}$ (d) $\alpha \geq \frac{3}{2}$

Q. No: 8 The value of the integral $\int \frac{1}{\sqrt{x^2 + 2x + 2}} dx$ is equal to:

- (a) $\sinh^{-1}(x-1) + c$ (b) $\sinh^{-1}(x+1) + c$ (c) $\sin^{-1}(x+1) + c$
 (d) $\sin^{-1}(x-1) + c$

Q. No: 9 The improper integral $\int_0^{\infty} x e^{-x^2} dx$

(a) diverges (b) converges to 1 (c) converges to $\frac{1}{2}$ (d) converges to -1

Q. No: 10 The value of the integral $\int \sec^4 x \tan^2 x dx$ is equal to:

(a) $\frac{1}{3} \tan^3 x + \frac{1}{5} \tan^5 x + c$ (b) $\frac{1}{3} \tan^3 x - \frac{1}{5} \tan^5 x + c$

(c) $\frac{-1}{3} \tan^3 x + \frac{1}{5} \tan^5 x + c$ (d) $\frac{-1}{3} \tan^3 x - \frac{1}{5} \tan^5 x + c$

Full Questions

Question No: 11 **Evaluate** $\int \frac{\sin^{-1}(\ln x)}{x} dx$ [3]

Solution:

Let $u = \ln x \Rightarrow du = \frac{1}{x} dx$

$$\begin{aligned} \int \frac{\sin^{-1}(\ln x)}{x} dx &= \int \sin^{-1}(u) du \\ &= u \sin^{-1} u + \sqrt{1-u^2} + c \quad (\text{By Integration by parts}) \\ &= (\ln x) \sin^{-1}(\ln x) + \sqrt{1-(\ln x)^2} + c \end{aligned}$$

Question No: 12 **Evaluate** $\int \frac{1}{x\sqrt{1+x^2}} dx$ [2]

Solution:

Let $x = \tan \theta \Rightarrow dx = \sec^2 \theta d\theta$

$$\int \frac{1}{x\sqrt{1+x^2}} dx = \int \csc \theta d\theta = \ln |\csc \theta - \cot \theta| + c = \ln \left| \frac{\sqrt{1+x^2}}{x} - \frac{1}{x} \right| + c$$

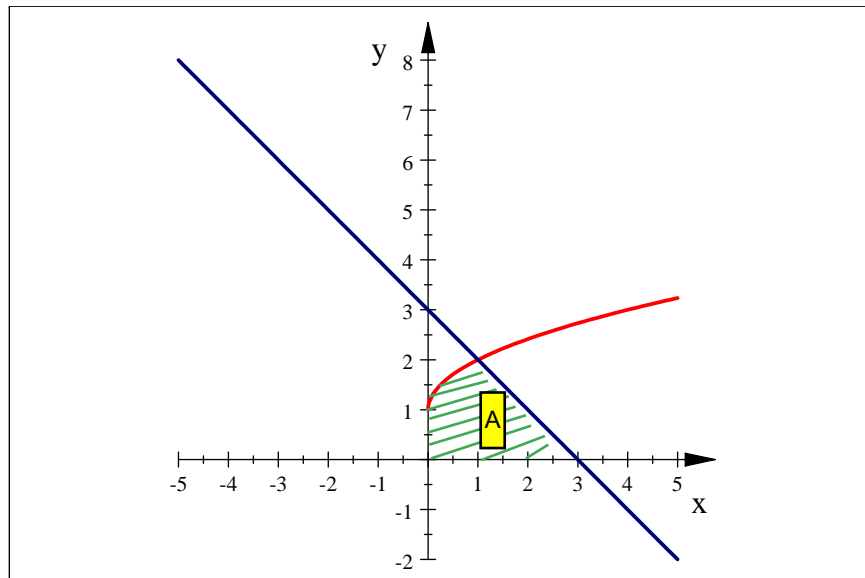
Question No: 13 **Evaluate** $\int \frac{x-3}{x^3-1} dx$ [3]

Solution:

$$\begin{aligned} \int \frac{x-3}{x^3-1} dx &= \int \frac{\frac{-2}{3}}{x-1} dx + \int \frac{\frac{2}{3}x + \frac{7}{3}}{x^2+x+1} dx \\ &= \frac{-2}{3} \ln|x-1| + \frac{1}{3} \ln|x^2+x+1| + \frac{4}{\sqrt{3}} \tan^{-1}\left(\frac{2}{\sqrt{3}}\left(x+\frac{1}{2}\right)\right) \end{aligned}$$

Question No: 14 Let R be the region bounded by the graphs of $y = 1 + \sqrt{x}$, $y = 3 - x$, x -axis and y -axis. **Sketch** the region R and **set up** (Do not evaluate) an integral that can be used to find its **area**. [2]

Solution:



$$y = 1 + \sqrt{x} \text{ and } y = 3 - x$$

$$A = \int_0^1 (1 + \sqrt{x}) dx + \int_1^3 (3 - x) dx$$