



KING SAUD UNIVERSITY
College of Science
Department of Mathematics

M-106

First Semester (1430/1431)

Correction Second Mid-Exam

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 20

Time: 90 minutes

Marks:

Multiple Choice(1-10)	
Question # 11	
Question # 12	
Question # 13	
Question # 14	
Total	

Multiple Choice

Q.No.	1	2	3	4	5	6	7	8	9	10
$\{a,b,c,d\}$	c	b	b	b	a	a	c	a	a	c

Q. No: 1 The partial fraction decomposition of $\frac{1}{x^4-1}$ takes the form

- (a) $\frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x^2+1}$ (b) $\frac{A}{x^2-1} + \frac{Bx+C}{x^2+1}$ (c) $\frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$ (d) None of these

Q. No: 2 To evaluate the integral $\int \cos^3(x) dx$, we substitute

- (a) $u = \cos x$, (b) $u = \sin x$, (c) $u = \cos^2(x)$, (d) None of these

Q. No: 3 The integral $\int \frac{1}{\sqrt{x^2+2x+5}} dx$ is equal to

- (a) $\sinh^{-1}\left(\frac{x+1}{4}\right) + c$, (b) $\sinh^{-1}\left(\frac{x+1}{2}\right) + c$ (c) $\frac{1}{2} \sinh^{-1}\left(\frac{x+1}{2}\right) + c$, (d) None of these

Q. No: 4 $\lim_{x \rightarrow \infty} \frac{e^x + e^{2x}}{1 + e^{2x}}$ is equal to

- (a) ∞ , (b) 1, (c) 0, (d) None of these

Q. No: 5 If $f(x) = \cosh^{-1}(\sqrt{x})$, then $f'(x)$ is equal to

- (a) $\frac{1}{2\sqrt{x^2-x}}$, (b) $\frac{1}{2\sqrt{x-x^2}}$, (c) $\frac{1}{2\sqrt{x^2+x}}$, (d) None of these

Q. No: 6 The improper integral $\int_0^{\infty} \frac{1}{4+x^2} dx$

- (a) Converges to $\frac{\pi}{4}$, (b) Converges to $\frac{\pi}{2}$, (c) diverges, (d) None of these

Q. No: 7 The integral $\int x \sin(2x) dx$ is equal to

- (a) $\frac{x^2}{2} \cos(2x) + c$, (b) $\frac{x}{2} \cos(2x) - \frac{1}{2} \sin(2x) + c$, (c) $-\frac{x}{2} \cos(2x) + \frac{1}{4} \sin(2x) + c$,
 (d) None of these

Q. No: 8 The value of the integral $\int_3^8 \frac{\sqrt{1+x}}{x} dx$ is equal to

- (a) $2 + \ln\left(\frac{3}{2}\right)$, (b) $2 - \ln 2$, (c) $2 + \ln 2 - \ln 3$, (d) None of these

Q. No: 9 The integral $\int \sqrt{1+\sqrt{x}} dx$ is equal to

- (a) $\frac{4}{5}(1+\sqrt{x})^{\frac{5}{2}} - \frac{4}{3}(1+\sqrt{x})^{\frac{3}{2}} + c$, (b) $\frac{4}{5}(1+\sqrt{x})^{\frac{5}{2}} + c$, (c) $\frac{4}{3}(1+\sqrt{x})^{\frac{3}{2}} + c$, (d) None of these.

Q. No: 10 The integral $\int \frac{x-2}{x\sqrt{x^2-25}} dx$ is equal to

- (a) $\cosh^{-1}\left(\frac{x}{5}\right) - 2\sec^{-1}\left(\frac{x}{5}\right) + c$, (b) $\cosh^{-1}\left(\frac{x}{5}\right) - \frac{2}{5}\sec^{-1}(x) + c$, (c) $\cosh^{-1}\left(\frac{x}{5}\right) - \frac{2}{5}\sec^{-1}\left(\frac{x}{5}\right) + c$,
(d) None of these

Full Questions

Question No: 11 Evaluate $\int \sin^3(x)\cos^7(x)dx$

[2]

Let $u = \cos x \Rightarrow du = -\sin x dx$ (0.5)

So

$$\begin{aligned}\int \sin^3(x)\cos^7(x)dx &= \int u^7(u^2-1)du && (0.5) \\ &= \frac{1}{10}u^{10} - \frac{1}{8}u^8 + c \\ &= \frac{1}{10}(\cos x)^{10} - \frac{1}{8}(\cos x)^8 + c && (1)\end{aligned}$$

Question No: 12 Evaluate $\int \tan^3(x)\sec^6(x)dx$

[2]

Let $u = \sec x \Rightarrow du = \sec x \tan x dx$ (0.5)

So

$$\begin{aligned}\int \tan^3(x)\sec^6(x)dx &= \int u^5(u^2-1)du && (0.5) \\ &= \frac{1}{8}u^8 - \frac{1}{6}u^6 + c \\ &= \frac{1}{8}(\sec x)^8 - \frac{1}{6}(\sec x)^6 + c && (1)\end{aligned}$$

Question No: 13 Evaluate $\int \frac{1}{(x-1)(x^2+1)} dx$. [3]

$$\frac{1}{(x-1)(x^2+1)} = \frac{1}{2} \frac{1}{(x-1)} - \frac{1}{2} \frac{x+1}{(x^2+1)} \quad (1)$$

So

$$\int \frac{1}{(x-1)(x^2+1)} dx = \frac{1}{2} \int \left(\frac{1}{(x-1)} - \frac{2x}{2(x^2+1)} - \frac{1}{(x^2+1)} \right) dx \quad (0.5)$$

$$= \frac{1}{2} \ln|x-1| - \frac{1}{4} \ln(x^2+1) - \frac{1}{2} \tan^{-1} x + c \quad (0.5+0.5+0.5)$$

Question No: 14 Evaluate $\int (1-x^2)^{\frac{3}{2}} dx$. [3]

Let $x = \sin \theta \Rightarrow dx = \cos \theta d\theta$ (0.5)

So

$$\int (1-x^2)^{\frac{3}{2}} dx = \int \cos^4 \theta d\theta \quad (0.5)$$

$$= \int \left(\frac{1 + \cos(2\theta)}{2} \right)^2 d\theta \quad (0.5)$$

$$= \frac{1}{8} \int (3 + 4 \cos(2\theta) + \cos(4\theta)) d\theta$$

$$= \frac{1}{8} \left(3\theta + 2 \sin(2\theta) + \frac{1}{4} \sin(4\theta) \right) \quad (0.5)$$

$$= \frac{1}{8} \left(3 \sin^{-1}(x) + 2 \sin(2 \sin^{-1}(x)) + \frac{1}{4} \sin(4 \sin^{-1}(x)) \right) + c$$

$$= \frac{1}{8} \left(3 \sin^{-1}(x) + 4x\sqrt{1-x^2} + x\sqrt{1-x^2}(1-2x^2) \right) + c. \quad (1)$$