

# Optimal Operation of a Wastewater Treatment Unit Using Advanced Control Strategy

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**Abstract:** This paper considers investigating the closed-loop behavior of a wastewater treatment plant consisting of a bioreactor with recycle followed by a clarifier. A previously developed dynamic model for the process is used to conduct the closed-loop analysis utilizing conventional Proportional plus Integral (PI) and Non-linear Model Predictive Control (NLMPC) algorithms. The static version of the model was utilized first to determine the optimal operating conditions for the process. Afterward, the effectiveness of the two proposed control algorithms were tested for two control objectives. The first objective is a servo problem, i.e. to startup the plant from arbitrary operating condition to the optimal one. The second objective is the regulatory problem, i.e., rejecting the influence of common disturbance and load changes to the process. Both control algorithms performed very well for all cases, however, NLMPC was found superior to the PI in terms of faster and smoother feedback response and less tuning requirements.

## 1. Introduction

Since waste treatment processes generally suffer from frequent variations in their feed qualities such as concentration and flow rates, it is essential to incorporate process control systems for improved operation. Such a process used to be controlled by primitive control policies such as manual and/or on-off controllers. In the past, several efforts were reported that discuss the application of many classical control strategies to the control problem of the wastewater treatment plants (WWTP) [1,2,3,4]. It was pointed out also, that more sophisticated control system is needed for further improvement of the performance of such plants. With the advent of more reliable sensors, sophisticated process automation and fast computers, the implementation of advanced control strategies became tractable. Gudi *et. al.* [5] implemented a SISO non-linear control law to a Fed-Batch Fermentation process used in wastewater treatment. Zhao and Skogestad [6] demonstrated the effectiveness of their proposed controllability measure to a similar process. The controller is a SISO PID. Nielsen *et. al.* [7] and Youssef and Dahhou [8] studied the application of adaptive control laws to waste treatment plants. Several other authors have also reported their efforts of controlling WWTP using adaptive control [9-14]. Most of these efforts dealt with controlling WWTP using anaerobic digester processes. Only few dealt with WWTP using fed-fermentation processes. Application of stochastic control system to WWTP was also addressed by Tenno and Uronen [15]. Menzi and Steiner [16] reported their effort with controlling nitrogen-eliminating WWTP using a model based multivariable control based on model predictive control concept and  $H_\infty$  theory. An Internal Model Control (IMC) was also examined for the control of the effluent concentrations of a bioreactor used for ethanol production from glucose [17].

This paper will address the simulated implementation of a multi-variable constrained non-linear Model Predictive Control (MPC) to maintain the water quality effluent from WWPT, which utilizes fed-fermentation process, within desired trajectories when the process is under the influence of disturbances and load changes. The purpose of this research is, thus to highlight to control engineers the advantage gained by incorporating such advanced control system. The performance of the proposed control algorithm will be compared to that of the conventional PID. In addition, the control objective will be formulated as MIMO problem. Moreover, the desired trajectories for the process will be determined such that it provides the optimum operation of the plant, i.e., maximum conversion of the substrate.

## 2. The Process Model

### Reactor with no recycle

Here we consider the bioreactor shown in Fig. 1, but without recycle and clarifier. The dynamic model for this process is taken from Zhao and Skogestad [6] and given as follows:

$$\dot{X} = rX - DX \quad (1)$$

$$\dot{S} = D(S_i - S) - \frac{r}{Y} X \quad (2)$$

where the specific growth rate ( $r$ ) is defined as follows:

$$r = \frac{\mu S}{K + S} \quad (3)$$

where  $S$ ,  $X$  are the substrate and cell concentrations respectively,  $\mu$  is maximum specific growth rate,  $K$  is saturation constant,  $Y$  yield coefficient,  $D$  is dilution rate (ratio of fresh feed to reactor volume) and  $S_i$  is the inlet substrate concentration. During developing the dynamic model, it was assumed that no biomass is presented in the reactor influent, and the reactor condition is aerobic which means there is sufficient oxygen to carry out the reaction. In addition, kinetic model does not account for cell maintenance and cell death. The nominal plant steady state operating condition is given in table 1.

**Table 1. Steady state operating point**

$D$ (l/h)	$S_i$ (g/l)	$X$ (g/l)	$S$ (g/l)	$\mu$ (l/h)	$K$ (g/l)	$Y$ (g/g)
0.17	1.0	0.38	0.05	0.5	0.1	0.4

### Reactor with recycle

Here we consider the bioreactor with recycle as shown in Fig. 1. The dynamic model for the complete process is taken from Sundstrom *et. al.*[18] and given as follows:

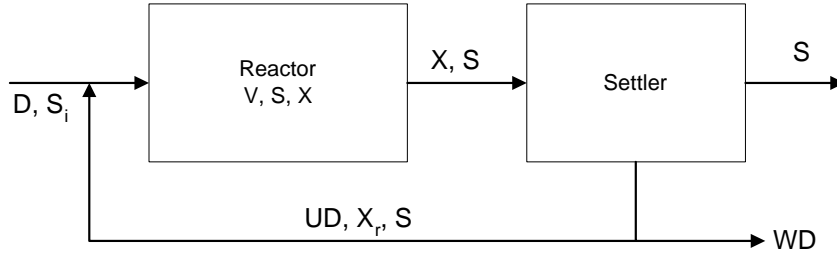
$$\dot{X} = DUX_r - D(1+U)X + rX - k_d X \quad (4)$$

$$\dot{S} = D(S_i - S) - r \frac{X}{Y} \quad (5)$$

$$X_r = X \frac{1+U}{U+W} \quad (6)$$

where  $r$  is the Mond reaction rate as defined before,  $U$  is recycle to fresh feed ratio,  $W$  is waste to fresh feed ratio,  $X_r$  is the cell concentration in recycle stream, and  $k_d$  is endogenous decay constant and equal to 0.005 l/hr in this paper. In addition to the assumption used for no recycle case, the following assumption were made during developing the dynamic model: no

reaction occurs in the settler such that the substrate concentration in the recycle flow is equal to that in the reactor effluent, the clarifier dynamic is neglected, and the surface area of the clarifier is so large such that the biomass concentration leaving the settler is zero. However, cell maintenance is corrected in this case. Although these assumptions leave the model incomplete, it is sufficient to provide unified approach for testing and comparing the performance of the proposed control algorithm with that of PID. Furthermore, in control practice, the model is never perfect and thus the inadequacy arising from the unmodeled parts are lumped together to formulate what is known as modeling errors. Consequently, robustness of a control system can be examined for various degrees of modeling errors.



**Figure 1. Schematic diagram of activated sludge process**

### 3. Optimal plant operating conditions

The given model exhibits specific phenomena related to the process conditions. When the feed flow becomes too low, the cell will die out of starvation because no enough nutrition is provided rapidly to maintain the cell metabolism. In the other hand, when the feed is too high, the residence time decreases to an extent that there will be no sufficient time for the cell (biomass) to grow resulting in no reaction, i.e., no conversion of the substrate. This is known as 'washout'. This occurs only when no cell recycle is used. In that case, avoiding washout impose an upper bound on the feed flow rate as follows [18]:

$$D < \mu \frac{1 + \bar{S}_i}{\bar{S}_i - \beta(1 + \bar{S}_i)} = D_c \quad (7)$$

where  $\beta = k_d/\mu$  and  $\bar{S}_i = S_i / K$ .

In the case of recycle, although recycle increases the cell concentration which in turn improve the conversion since the reaction is autocatalytic, it may deteriorate performance. This is because recycle dilute the substrate and lowers the residence time. According to Sundstrom *et al.* [18], if  $D > D_c$ , the fractional conversion of input substrate increases monotonically with both increasing recycle ratio ( $U$ ) and recycled cell concentration ( $X_r$ ). However, since  $U$  will be used as manipulated variable, washout may still occur if  $U$  becomes zero. For this case, the condition:  $D < D_c$  will be imposed in all simulations. For the latter case, Sundstrom *et al.* [18] illustrated that conversion increases with recycle ratio only if  $X_r$  exceeds certain critical value which is given as follows:

$$\bar{X}_r > \frac{\bar{S}_i}{1 + \beta\gamma} - \frac{1}{\gamma - 1 - \beta\gamma} = X_{rc} \quad (8)$$

where  $\bar{S}_i$  and  $\beta$  are defined as before,  $\gamma = \mu/D$  and  $\bar{X}_r = X_r / KY$ . For our case  $X_{rc}$  is found to be 0.2286 g/l and  $D_c$  to be 0.56 l/hr.

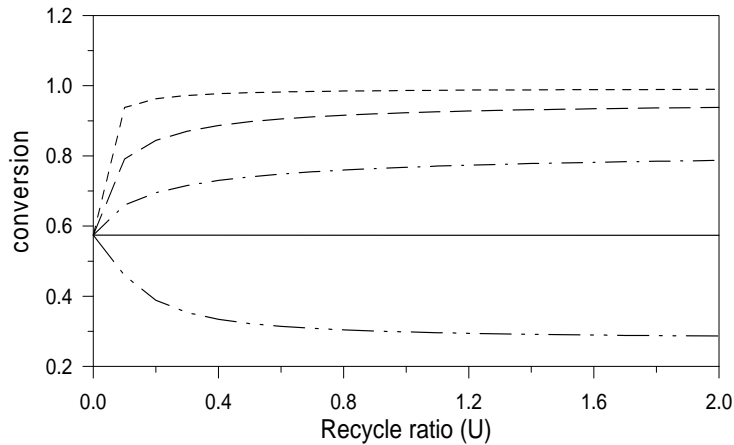
Thus, based on the above situations, one can obtain the optimum operating condition of the reactor that maximize the substrate conversion and avoid washout by solving the following optimization problem:

$$\max_{X,S,U,D,X_r} z = \frac{S_i - S}{S_i} + D \quad (9)$$

subject to:

$$\begin{aligned} \dot{X} &= 0 \\ \dot{S} &= 0 \\ 0 &\leq D < D_c \\ X_{rc} &< \bar{X} \end{aligned}$$

The second term in the objective function is added to ensure that the maximum conversion occurs at high throughput. The optimization problem is solved using MATLAB software and the results are listed in Table 2. This optimal operating point will be used as our desired set point for closed-loop simulations. It is more interesting to demonstrate the effect of varying  $U$  and  $X_r$  on the substrate conversion while fixing  $D$  at its optimal value listed in table 2. The result is shown in figure 3. The curves in the figure are obtained by finding the steady state values for equations 4 and 5 for fixed  $D$  and various values of  $U$  and  $X_r$ . It was observed that  $X_r$  must be kept above certain value of 0.2286 to ensure higher substrate conversion. It was also found that increasing  $U$  beyond 1.0 gives only marginal increase in the conversion. Thus, during closed-loop simulation  $U$  will be limited between 0 and 1.0. These observations were similar to those found by Sundstrom *et. al.*[18]. The above performance constraints on  $R$  and  $U$  present incentives for employing constrained MPC.



**Figure 2: Conversion of substrate; dotted curve:  $X_r = 10$ , dashed curve:  $X_r = 5$ , dash-and-dot curve  $X_r = 1$ , solid curve:  $X_r = 0.2286$ , dash-and-double dots curve:  $X_r = 0.1$**

#### 4. The Control Objective

In a typical bioreactor there is a limited control objectives. Traditionally, the control outputs for the process are dissolve oxygen concentration in the aerator, biomass and substrate concentrations in the reactor effluent and the liquid level in the settler. The common disturbances to the system are the feed flow rate and inlet substrate and cell concentrations. For simplicity we will assume that the oxygen concentration is perfectly controlled using the air flow rate which is maintained in proportion to the feed flow rate. The latter assumption

does not strongly affect the process efficiency [1] as long as the dissolved oxygen in the reactor remains above the minimum level. Moreover, we assume the sludge level in the settler is perfectly controlled by the sludge waste. This will leave us with two controlled outputs. Although frequent measurement of the outputs (cell and substrate concentrations) are not available, it will be considered that they can be inferred from other measurements such as oxygen consumption rate and carbon dioxide production rate. The possible manipulated variable are then the fresh-feed flow rate and the ratio of recycle to fresh-feed flow rate. With the utilization of the fresh-feed for regulatory action, the control objective of rejecting the changes in the feed flow rate is ruled out. Thus, in this paper, the control objective is to maintain  $X$  and  $S$  within their desired operating points despite the influence of plant disturbances. For this reason, traditional discrete PI controller tuned using Ziegler and Nichols method [19] and non-linear MPC are implemented and compared. In all the simulations hereafter, a sampling time of 0.5  $h$  is used. Formulation of NLMPC is given next.

**Table 2. Optimum operating point**

$X(g/l)$	$S(g/l)$	$D(l/hr)$	$X_r(g/l)$	$U$
3.4848	0.01	0.4	6.6172	1.0

### 4.3 Non-linear MPC algorithm

In this paper the structure of the MPC version developed by Ali and Zafiriou[20] that utilizes directly the nonlinear model for output prediction is used. A usual MPC formulation solves the following on-line optimization:

$$\min_{\Delta u(t_k), \dots, \Delta u(t_{k+M-1})} \sum_{i=1}^P \left\| \Gamma(y(t_{k+i}) - r(t_{k+i})) \right\|^2 + \sum_{i=1}^M \left\| \Lambda u(t_{k+i-1}) \right\|^2 \quad (10)$$

subject to

$$\hat{A}^T \Delta U(t_k) \leq b \quad (11)$$

For nonlinear MPC the predicted output  $y$  over the prediction horizon  $P$  is obtained by the numerical integration of:

$$\dot{x} = f(x, u, t) \quad (12)$$

$$y = g(x) \quad (13)$$

from  $t_k$  up to  $t_{k+P}$  where  $x$  and  $y$  represent the states and the output of the model respectively. The symbols  $\| \cdot \|$  denotes the Euclidean norm,  $k$  is the sampling instant,  $\Gamma$  and  $\Lambda$  are diagonal weight matrices.  $r$  is the vector of desired output trajectory.  $\Delta U(t_k) = [\Delta u(t_k) \dots \Delta u(t_{k+M-1})]^T$  is a vector of  $M$  future changes of the manipulated variable vector  $u$  that are to be determined by the on-line optimization. The control horizon ( $M$ ) and the prediction horizon ( $P$ ) are used to adjust the speed of the response and hence to stabilize the feedback behavior.  $\Gamma$  is usually used for tradeoff between different controlled outputs.  $\Lambda$ , in the other hand, is used to penalize different inputs and thus to stabilize the feedback response.

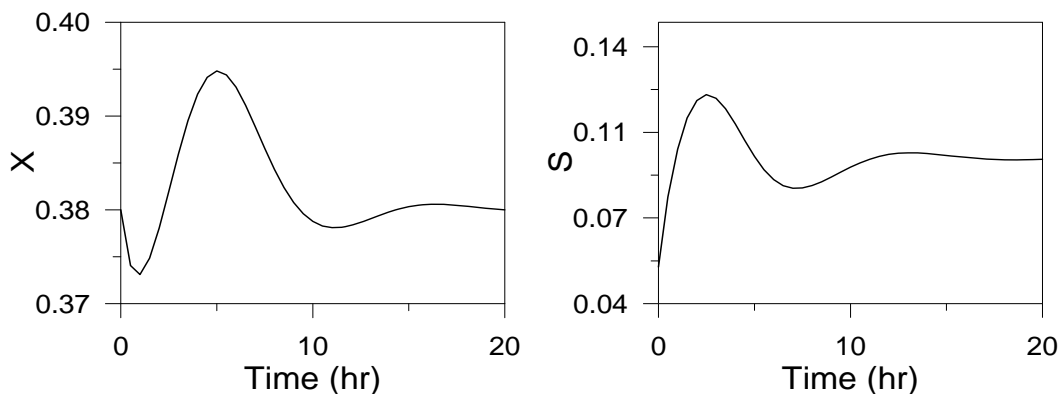
A disturbance estimate should also be added to  $y$  in equation (10) or alternatively it can be absorbed in  $r(t_{k+i})$ . In the standard MPC implementation, the disturbance is assumed constant over the prediction horizon, and set equal to the difference between plant and model outputs at present time  $k$ . The function of the ‘‘additive’’ constant disturbance in the model

prediction is to introduce integral action and thus removes steady state offset in the presence of model uncertainty or unmeasured disturbances. The usual implementation of MPC involve numerical integration of the model state equations over the prediction horizon  $P$  to obtain the future output behavior. Then, the objective function (Eq. 10) is solved on-line to determine the optimum value of  $\Delta U(k)$ . Only the current value of  $\Delta u$ , which is the first element of  $\Delta U(k)$ , is implemented on the plant. At the next sampling instant, the whole procedure is repeated.

## 5. Closed-loop simulation tests

### Reactor with no recycle

In this case, there are two controlled outputs ( $S$ ,  $X$ ) and one manipulated variable ( $D$ ). From controllability point of view, perfect control can not be achieved when the number of controlled outputs exceed the number of manipulated variables. Zhao and Skogestad[6] have demonstrated this fact where among the five SISO control configurations they tried, only one case had the best performance, i.e., when  $S_i$  is used as the manipulated variable and  $X$  as the controlled output. Usually  $S_i$  can not be used as a manipulated variable easily. Moreover, this control configuration can perform badly for -30% step disturbance in  $\mu$  as depicted by figure 3. It is obvious that  $S$  can not be kept within its acceptable upper bounds of 0.06 which is set by Zhao and Skogestad [6].



**Figure 3. Closed-loop response for -30% step disturbance in  $\mu$  using PI controller with  $S_i$  as the manipulated variable and  $X$  as the controlled output.**

Closed-loop response for 25% step disturbance in  $Y$  using PI controller with two SISO control structures, i.e., ( $D \rightarrow X$  or  $D \rightarrow S$ ) is shown in figure 4 (b,d). In either case, only the output under control can be perfectly regulated. By incorporating MPC which permits non-square control structure, due to its multi-variable nature, it provides a compromised performance as illustrated by figure 4(a,c). In this case,  $X$  and  $S$  are simultaneously controlled by  $D$ . When  $\Gamma = [1 \ 0]$  is used, which disactivate  $S$  as a controlled output, similar response to that of PI with  $D \rightarrow X$  structure is obtained. Similarly, when  $\Gamma = [0 \ 1]$  is used, similar response to that of PI with  $D \rightarrow S$  structure is obtained. Trade-off between the two outputs was achieved by using  $\Gamma = [1 \ 1]$ . Thus, a trade-off can be simply attained by adjusting the MPC tuning parameter  $\Gamma$  by software means, where in the case of PI can only be achieved through switching from one control loop to another by hardware means. Nevertheless, a compromised response like that of the dash-and dot curve in figure 4(a,c) can not be achieved by PI.

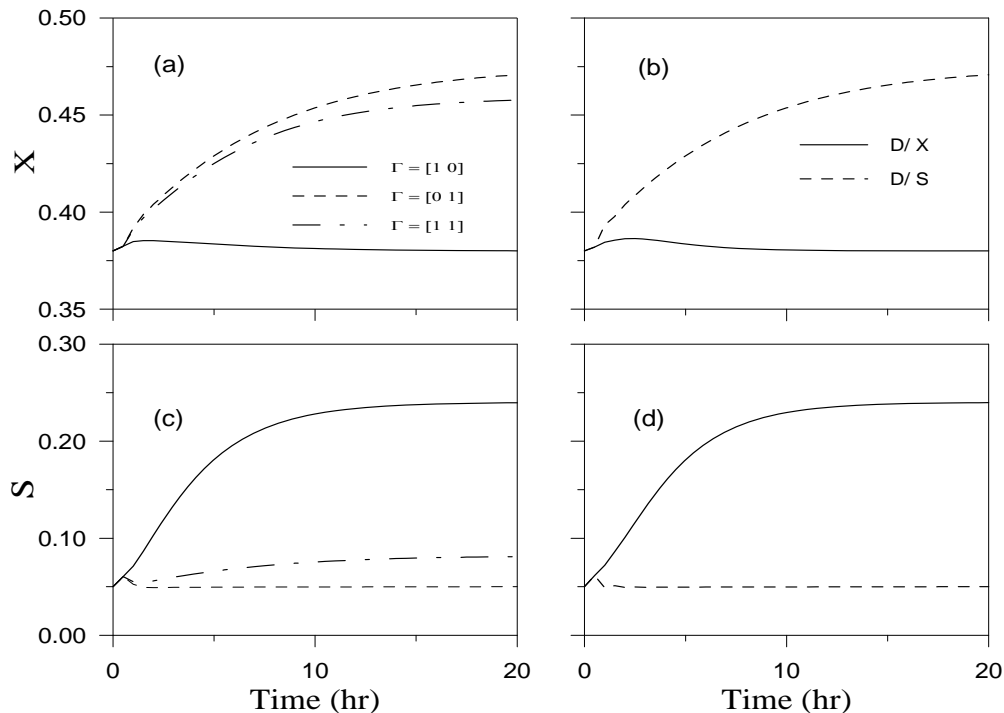
It is clear that exact control of both effluent concentrations can't be achieved. Thus, it is better to consider the realistic case where the process have a biomass recycle. In this case two manipulated variables are available namely,  $U$  and  $D$  where both of them are constrained

and two controlled variable namely,  $S$  and  $X$ . In the previous section, it was indicated that  $X_r$  has crucial effect on the overall performance of the process.  $X_r$  can be kept above its critical value by simply controlling  $X$ . This can be confirmed from equation 6 as  $X$  is fixed at its set point,  $X_r$  will always remain above its critical value for any value of  $U$  in the range  $[0,1]$ .

**Table 3. PI settings**

	$K_c$	$\tau_I$
D→S	120	0.5
U→X	100	2.0

First the closed-loop performance of PI and NLMPC for set point change from the steady state value of table 1 to the optimal one given in table 2 was examined. The result is shown by figure 5. Although both controllers managed to bring the process to the new optimal condition NLMPC had faster dynamics specially for the biomass concentration ( $X$ ). The PI settings are found by Ziegler and Nichols method and are listed in table 3. The NLMPC tuning parameters used were  $M = 1$ ,  $P = 4$ ,  $\Lambda = \text{diag}[0,0]$  and  $\Gamma = \text{diag}[1,1]$ .



**Figure 4: Closed-loop response for 25% step disturbance in  $Y$ . (a,c) using MPC; solid line:  $\Gamma=[1 \ 0]$ ; dashed line:  $\Gamma=[0 \ 1]$ ; dash-and dot line:  $\Gamma=[1 \ 1]$ . (b,d) using PI; solid line: D→X control configuration; dashed line: D→S control configuration**

Next the regulatory performance of PI and NLMPC is examined. Figure 6. illustrates the feedback performance for -20% step change in  $\mu$  which meant to simulate reaction deterioration due to disturbances in environment conditions. Both controllers showed excellent disturbance rejection as evident by the quick recovery to the desired optimal steady state. However, NLMPC outperformed the PI algorithm in terms of faster biomass response. This was also observed for the set point change case. Basically,  $X$  has slow dynamics in response to variation in  $U$ , which affects the closed-loop performance of the PI since  $X$  is

controlled directly by  $U$  only. While in the NLMPC case, both  $D$  and  $U$  are manipulated optimally and simultaneously to control  $S$  and  $X$ .

It is also interesting to examine the effect of modeling error on the controller performance. In the above cases,  $X_r$  was considered to be in proportion to  $X$  assuming no biomass leaves the clarifier. However, this may not always be true, thus, the controller performance is also examined for the case when the biomass concentration in the recycle ( $X_r$ ) is 10 % less due biomass leak with the clarifier effluent. The result is shown in figure 7. It is clear that both controller were able to maintain  $S$  and  $X$  at their corresponding optimal steady state and that they had almost similar performance. The NLMPC used the same tuning parameters values as before. However, the PI settings of table 3 generated unstable response which is not included in figure 7, thus, the PI controller gains had to be retuned to 30 and 50 respectively. The modified gains resulted in better closed-loop response as indicated by figure 7. The NLMPC tuning parameters is the same as before.

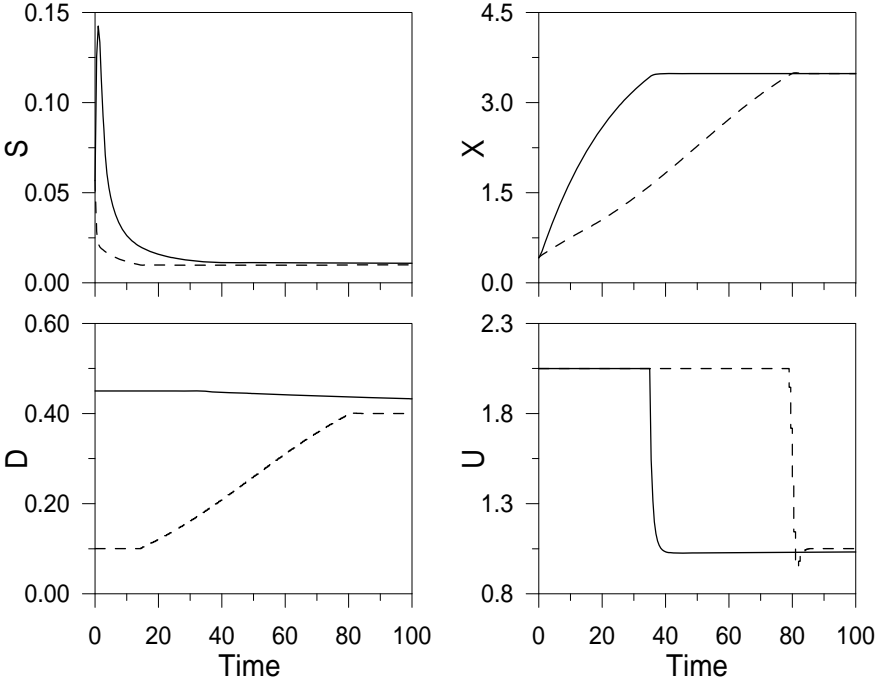
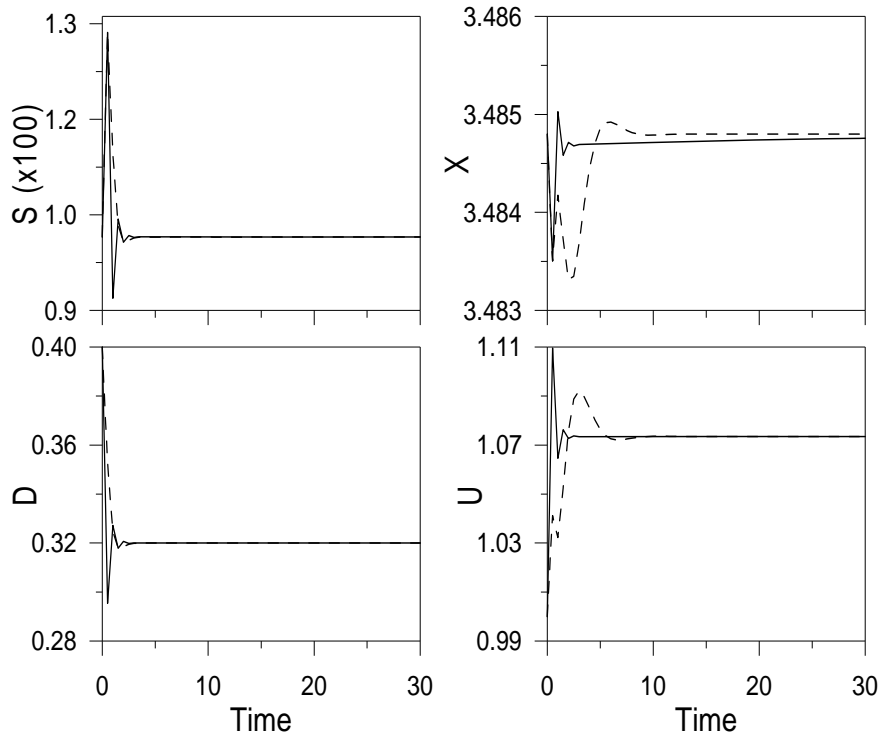
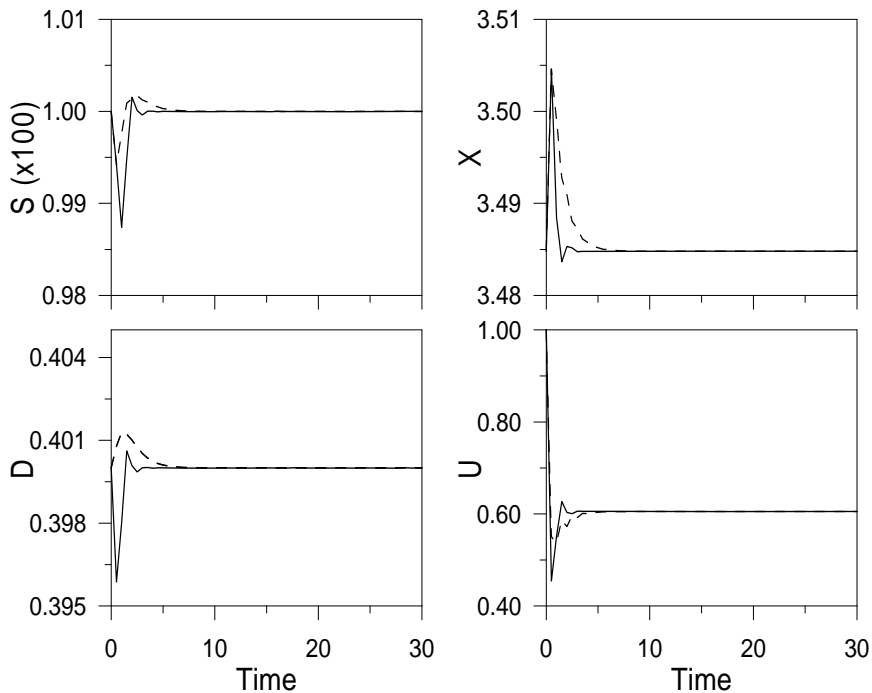


Figure 5. Set point change response; solid line: MPC; dashed line: PI



**Figure 6. Closed-loop response for -20% step change in  $\mu$ . solid line: MPC; dashed line: PI**



**Figure 7. Closed-loop response for 10% step change in  $X_r$ . Solid line: MPC, Dashed line: PI**

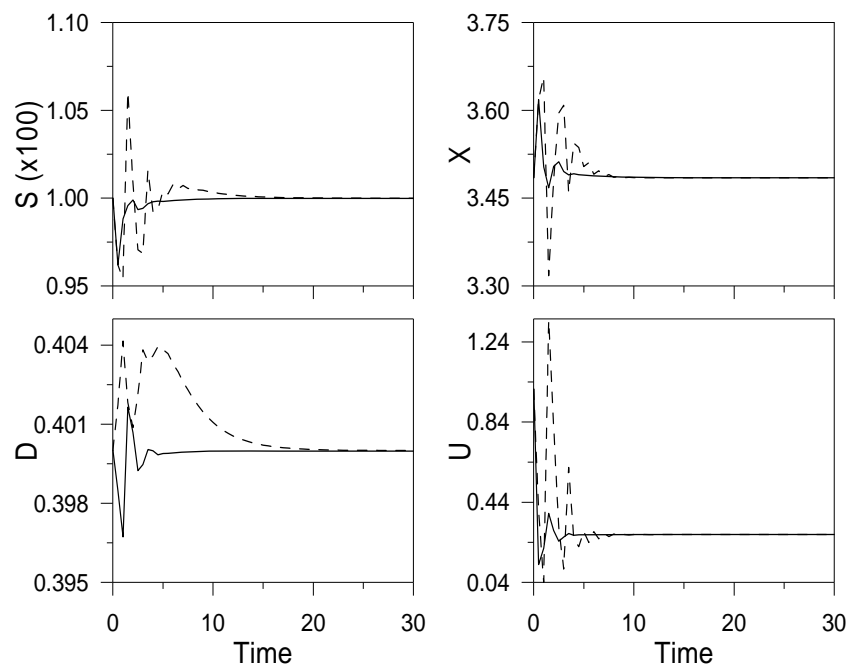
A common disturbance of the wastewater treatment plant is the sudden changes in the feed load conditions. The developed process model assumes no biomass enters with the fresh feed. For this reason, the process feedback behavior is investigated when the biomass concentration in the fresh feed steps to 0.1. In this case an extra term of  $DX_i$  is added to the

right hand side of equation (4) with  $X_i$  represents the biomass concentration in the fresh feed. The closed-loop simulation is depicted in figure 8. Although, both controller demonstrated good and quick rejection of the impact of disturbance in the fresh feed load condition, PI controller had inferior performance at high frequency (initially). Moreover, the PI controller gains were retuned further to 10 and 1.0 respectively for stability. The NLMPC tuning parameters is still the same as before.

It should be noted that, although not shown in the figures,  $X_r$  was perfectly kept much higher than the critical value. This was achieved, as mentioned before, by keeping  $X$  at its desired set point.

## 6. Conclusions

A previously developed model for wastewater treatment plant, consisting of a bioreactor with recycle and a clarifier, is used for steady state and dynamic analysis of the process. The steady state model is used to determine the optimal operating conditions of the plant that maximize the substrate conversion and avoid washout situations. The dynamic model is used to investigate the performance of standard PI and Non-linear Model Predictive Control algorithms for the two control objectives of the process. The first objective is to startup the plant from an arbitrary operating condition to the optimal one. While the second objective is to reject the effect of disturbance and load changes injected to the plant. In the two cases, both control algorithms presented excellent closed-loop performance. However, NLMPC outperformed PI in some case. In the case of controlling the bioreactor without recycle, NLMPC had the ability to provide compromised feedback responses for the two controlled outputs (substrate and biomass concentrations), while PI controller can only control one controlled output at a time due to its single loop structure. In the case of controlling the bioreactor with recycle, NLMPC provided faster and smoother feedback response than those provided by the PI controller. In addition, the PI controller gains had to be retuned for stability from case to case.



**Figure 8. Closed-loop response for 0.2 step change in  $X_i$  ; Solid line: MPC, Dashed line: PI**

**References**

- [1] Andrews, J. F. "Dynamic Models and Control Strategies for Wastewater Treatment Processes", *Water Res.*, 8, 1974, pp. 261-289.
- [2] H. Keli, D. Sundstrom, and A. Molvar, "Control of Well-mixed Biological Reactors Subject to Variations in Feed Concentration and Flow Rate", *J. Appl., Chem., Biotech.*, 25, 1975, pp. 535-548.
- [3] H. Keli, D. Sundstrom, "Automatic Control of an Activated Sludge Reactor", *J. Water. Poll. Cont. Fed.*, 46, 1974, pp. 993-998.
- [4] E. Williams, "Optimizing Waste Treatment Control systems", The Foxboro Co., Mass., 1970.
- [5] R. Gudi, S. Shah, M. Gray, and P. Yegneswaran, "Adaptive Nutrient Estimation and Control of Nutrition Levels in a Fed-Batch Fermentation Using Off-line and On-line Measurements", *The Can. J. of Chemical Eng.*, 75, 6, 1997, pp. 562-573
- [6] Y. Zhao, and S. Skogestad, "Comparison of Various Control Configurations for Continuous bioreactor", *Ind. Eng. Chem. Res.*, 36, 1997, pp. 697-705.
- [7] K. Nielsen, H. Madsen. And J. Carstensen, "Identification and Control of Nutrition removing Processes in Wastewater Treatment Plants", *Proceeding of the IEEE conference on Control Applications*, v2, 1994, pp. 1005-1010.
- [8] C. Youssef and B. Dahhou, "Multivariable adaptive Predictive Control of an Aerated lagoon for a Wastewater treatment process", *J. of Process Control*, 6, 5, 1996, pp. 265-275
- [9] M. Polihronakis, L. Petrou, and A. Deligiannis, "Parameter Adaptive Control Techniques for Anaerobic Digesters- Real Life Experiments", *Comp. Chem. Eng.*, 17, 12, 1993, pp. 1167-1179.
- [10] D. Dochain "Adaptive Control Algorithms for Non-minimum Phase Non-linear Bioreactors", *Comp. Chem. Eng.*, 16, 5, 1992, pp. 449-462.
- [11] O. Monroy, J. Ramirez, F. Cuervo, and R. Femat, "An Adaptive Strategy to control Anaerobic Digesters for Wastewater Treatment", *Ind. Eng. Chem. Res.*, 35, 1996, pp. 3442-3446.
- [12] S. Park, W. Ramirez, "Optimal Regulatory, Control of Bioreactor Nutrient Concentration Incorporating system Identification", *Chem. Eng. Sci.*, 45, 12, 1990, pp. 3467-3481.
- [13] D. Dochaion, N. Tali-Maamar, and J. Babary, "On Modeling, Monitoring and Control of Fixed Bed Reactors", *Comp. Chem. Eng.*, 21, 11, 1997, pp. 1255-1266.
- [14] C. Emmanouilides, and L. Petro, "Identification and Control of Anaerobic Digesters Using Adaptive on-line Trained Neural Networks", *Comp. Chem. Eng.* 21, 1, 1997, pp. 113-143.
- [15] R. Tenno, and P. Uronen, "Stochastic control for Wastewater treatment processes", *Control Engineering Practice*, 4, 5, 1996, pp. 601-614.
- [16] S. Menzi, and M. Steiner, "Model-Based Control for nitrogen-eliminating wastewater treatment plants", *IEEE conference on Control Applications*, 1995, pp. 842-847.
- [17] A. Aoyama, F. Doyle III, V. Subramnian, "Control-affine Neural Approach for Non-linear Minimum Phase Non-linear Process Control", *J. Process Control*, 6, 1, 1996, pp. 17-26
- [18] D. Sundstrom, H. Keli, A. Molvar, "The Use of Dimensionless Groups in the Design of Activated Sludge Reactors", *Water Research*, 7, 1973, pp. 1905-1913.
- [19] Ziegler, J. G. and N. B. Nichols, "Optimum Settings for Automatic Controllers," *Trans. ASME*, 64, 759 (1942).
- [20] Ali, E., and E. Zafiriou, "Optimization-Based tuning of Model Predictive Control with State Estimation" *J. Proc. Cont.*, 3, pp. 97-107, (1993)