Modelling of Single Evaporators

Anders Gränfors and Bernt Nilsson
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Abstract: This paper discusses a lumped model of a single evaporator for steady-state and dynamic simulations. The paper includes a description of a selection of MATLAB M-files for calculations of physical properties

Introduction

This paper describes in detail a lumped model of an evaporator. The model is used both in steady-state and dynamic simulations. The paper also present some MATLAB M-files for calculations of some physical properties in black liquor evaporation.

Lumped Model Balance Equations

The modelling of an evaporator includes the formulation of total mass and component balances together with an energy balance. The balances are derived with the whole evaporator as the control volume. The resulting balances are listed below:

Total material balance:

$$\frac{d}{dt}(\rho V) = \rho_{IN} F_{IN} - \rho_{OUT} F_{OUT} - W \tag{1}$$

Component balance with respect to **water**:

$$\frac{d}{dt}(X_w \rho V) = X_{w,IN} \rho_{IN} F_{IN} - X_{w,OUT} \rho_{OUT} F_{OUT} - W \tag{2}$$

Component balance with respect to **black liquor**:

$$\frac{d}{dt}(X_s \rho V) = X_{s,IN} \rho_{IN} F_{IN} - X_{s,OUT} \rho_{OUT} F_{OUT} \tag{3}$$

Energy balance:

$$\frac{d}{dt}(\rho V H) = \rho_{IN} F_{IN} H_{IN} + Q - \rho_{OUT} F_{OUT} H_{OUT} - W H_{OUT}^{V} \tag{4}$$

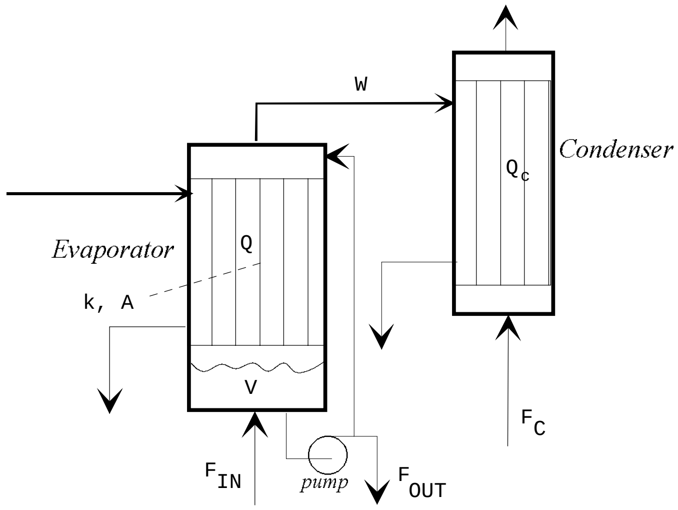

Figure 1- Single evaporator with a condenser

The equations (1)-(4) are the basic four balance equations that can be derived under the assumption that the liquid is well stirred. However it is convenient to do the following rearrangements of the equations.

The L.H.S. of equation (2) can be written as:

$$\frac{d}{dt}\left(X_w \rho V\right) = X_w \frac{d}{dt}\left(\rho V\right) + \left(\rho V\right)\frac{d}{dt}\left(X_w\right) \tag{5}$$

Inserting equation (1) and (2) to (5) gives:

$$\frac{dX_w}{dt} = \frac{1}{\left(\rho V\right)}\left[\rho_{IN} F_{IN}\left(X_{w,IN} - X_{w,OUT}\right) - W\left(1 - X_{w,OUT}\right)\right] \tag{6}$$

The same procedure can be used for black liquor. The L.H.S. of equation (2) can be written as:

$$\frac{d}{dt}\left(X_s \rho V\right) = X_s \frac{d}{dt}\left(\rho V\right) + \left(\rho V\right)\frac{d}{dt}\left(X_s\right) \tag{7}$$

Inserting equation (1) and (3) to (7) gives:

$$\frac{dX_s}{dt} = \frac{\rho_{IN} F_{IN}}{\left(\rho V\right)}\left(X_{s,IN} - X_{s,OUT}\right) \tag{8}$$

Similarly, the L.H.S. of equation (2) can be written as:

$$\frac{d}{dt}\left(\rho V H\right) = H \frac{d}{dt}\left(\rho V\right) + \left(\rho V\right)\frac{d}{dt}\left(H\right) \tag{9}$$

Inserting equation (1) and (4) to (9) gives:

$$\frac{dH}{dt} = \frac{1}{(\rho V)}\left[\rho_{IN} F_{IN}\left(H_{IN} - H_{OUT}\right) - W\left(H_{OUT}^{V} - H_{OUT}\right) + Q\right] \tag{10}$$

The equations (1) and (10) together with one of (6) or (8) are a lumped model that describes the unsteady-state behaviour of an evaporator. One example of a lumped model is seen below (using equation (8)). The model results in a set of three ordinary differential equations, ODEs.

$$\frac{d}{dt}(\rho V) = \rho_{IN} F_{IN} - \rho_{OUT} F_{OUT} - W$$
$$\frac{dX_s}{dt} = \frac{\rho_{IN} F_{IN}}{(\rho V)}\left(X_{s,IN} - X_{s,OUT}\right)$$
$$\frac{dH}{dt} = \frac{1}{(\rho V)}\left[\rho_{IN} F_{IN}\left(H_{IN} - H_{OUT}\right) - W\left(H_{OUT}^{V} - H_{OUT}\right) + Q\right]$$

The steady-state behaviour can be calculated if the derivatives on the L.H.S., i.e. the accumulation terms, are set to zero, resulting in a nonlinear equation system.

Additional Physical Relations

The physical model needs description of the heat transfer from condensing steam on the "hot" side to the boiling liquid on the "cold" side. In our application we can assume that the "hot" side is condensing steam at equilibrium.

$$Q = kA(T_{hot} - T_{cold}) \tag{11}$$

The mass flow of evaporated steam, W, is the same as the amount of condensing steam in the condenser, the first relation in Equation set (12). The other two relations below are heat balances over cooling water and heat transfer in the condenser. The cooling temperature, $T_{c,out}$, the steam flow, W, and the heat flow, Q, can not be calculated directly and the equation set must be manipulated.

$$\begin{aligned} Q_c &= W \Delta H_{steam} \\ Q_c &= {}_c\rho_c C_{p_c}\left(T_{c,IN} - T_{c,OUT}\right) \\ Q_c &= k_c A_c\left(T - T_{c,OUT}\right) \end{aligned} \tag{12}$$

Physical Data Calculations

To solve the evaporator model above one needs a number of physical data descriptions for black liquor and water/steam.

Physical properties routines for **black liquor** (in MATLAB):

`dnslblq(T ,X)` Density as a function of temperature and concentration of black liquor. Density ρ in kg m^{-3}**:**

$$\rho = 1007.4 - 0.495T + 600X \ \left(\mathrm{kg/m^3}\right)$$

`entlblq(T ,X)` Enthalpy as a function of temperature and concentration of black liquor. Enthalpy H in kJ kg^{-1}**:**

$$H = C_P T \quad \left(\mathrm{kJ/kg}\right)$$

where $C_P = 4.1868 - 2.261X \qquad \left(\mathrm{kJ/kg^\circ C}\right)$

`tlblq(X ,H)` Temperature as a function of concentration of black liquor and enthalpy. The function includes the effect of boiling point increase β in °C:

$$\beta = y \cdot 0.016203(273 + T - y)^2 / (2513 - 2.5833T)$$

where $y = 7.3X/(1 - X)$

`eqpblq(T ,X)` Equilibrium pressure as a function of temperature and concentration of black liquor.

Physical properties routines for **water** (in MATLAB):

`eqth2o(P)` Equilibrium temperature as a function of pressure

`eqph2o(T)` Equilibrium pressure as a function of temperature

`epvh2o(P ,T)` Enthalpy for pure vapour as a function of pressure and temperature

`eplh2o(P ,T)` Enthalpy for pure liquid as a function of pressure and temperature

Nomenclature

Roman symbols

A	heat transfer area	(m^2)
C_P	specific heat capacity (kJ/kg°C)	
e	control error	

F	volume flow	(m^3/s)
H	enthalpy	(kJ/kg)
k	heat transfer coefficient	(kJ/m^2°C)
K_c	controller gain	
m	mass flow cooling water	(kg/s)
P	pressure	(kPa)
Q	energy flux	(kJ/s)
T	temperature	(°C)
X	concentration	(kgH_2O/kg)
u	control signal	(m^3/s)
V	volume	(m^3)
W	mass flow steam out of evaporator	(kg/s)

Greek symbols

Δ	difference	
ρ	density	(kg/m^3)

Subscript

OUT	out from the evaporator
IN	in from the evaporator
C	condenser
steam	heat of evaporation
LN	average logarithmic temperature
L	liquid
V	vapour

Superscript

V	vapour