CHE401 Final Exam solution

Question 1

$$F(f) = \sqrt{\frac{1}{f}} + 0.86 \ln\left(\frac{\varepsilon / D}{3.7} + \frac{2.51}{N_r \sqrt{f}}\right)$$

$$c = \frac{1}{2}(a_o + b_o)$$

Applying Bisection method for the given parameter values yields:

i	*a*	*b*	*c*	f_a	f_c	|*b*-*a*|
1	0.0100	0.0300	0.0200	2.9585	-0.2345	0.0200
2	0.0100	0.0200	0.0150	2.9585	0.9677	0.0100
3	0.0150	0.0200	0.0175	0.9677	0.3037	0.0050
4	0.0175	0.0200	0.0188	0.3037	0.0215	0.0025

Question 2

$$f \equiv \frac{dX}{dV} = \frac{k}{v}\frac{1-X}{1+X}\frac{T_0}{T}$$

$$k = 3.58\exp\left[34222\left(\frac{1}{T_0}-\frac{1}{T}\right)\right]$$

Applying Explicit Euler method;

$X^{i+1} = X^i + hf$

Using h =0.1 and the given parameter values yield:

V	x	f
0	0	28.2973
0.1000	2.8297	-13.5196
0.2000	1.4778	-5.4563
0.3000	**0.9321**	0.9939
0.4000	1.0315	-0.4391
0.5000	**0.9876**	0.1764
0.6000	1.0052	-0.0741
0.7000	0.9978	0.0306
0.8000	1.0009	-0.0127
0.9000	0.9996	0.0053
1.0000	1.0002	-0.0022

We can observe that Euler method with large h=0.1 gives unrealistic results, i.e. conversion is higher than 1.

Question 3

$$\min_x f(x) = \frac{x^3}{10} - 1400x \; ; \qquad x \in [63,255]$$

Applying the Golden Search method for three iterations using the initial values of *x* gives:

$L = b - a$
$x_1^i = a + 0.382L$
$x_2^i = b - 0.382L$

a	b	x_1	x_2	$f(x_1)$	$f(x_2)$	$\|\|x^{i+1}-x^i\|\|_2$
6.3000e+001	2.5500e+002	1.3634e+002	1.8165e+002	6.2577e+004	3.4512e+005	1.0372e+002
1.3634e+002	2.5500e+002	1.8167e+002	2.0967e+002	3.4525e+005	6.2824e+005	5.3286e+001
1.8167e+002	2.5500e+002	2.0968e+002	2.2698e+002	6.2834e+005	8.5174e+005	3.2931e+001

Question 4

Taking the first derivative of the objective function gives:

$$\frac{\partial f}{\partial P_2} = \frac{RT}{P_2}\left[\left(\frac{P_2}{P_1}\right)^n - \left(\frac{P_2}{P_3}\right)^n\right]$$

$$\frac{\partial f}{\partial P_3} = \frac{RT}{P_3}\left[\left(\frac{P_3}{P_2}\right)^n - \left(\frac{P_4}{P_3}\right)^n\right]$$

Setting each equation to zero and solving for P_2 and P_3 yields:

$$P_2 = (P_1\ P_3)^{1/2}, \qquad P_3 = (P_4\ P_2)^{1/2}$$

Or

$$P_2 = (P_1^2 P_4)^{1/3}, \qquad P_3 = (P_4^2 P_1)^{1/3}$$

Taking the second derivative gives:

$$\frac{\partial^2 f}{\partial P_2^2} = \frac{(n-1)}{P_2^2}\left(\frac{P_2}{P_1}\right)^n + \frac{(n+1)}{P_2^2}\left(\frac{P_3}{P_2}\right)^n$$

$$\frac{\partial^2 f}{\partial P_3^2} = \frac{(n-1)}{P_3^2}\left(\frac{P_3}{P_2}\right)^n + \frac{(n+1)^2}{P_3}\left(\frac{P_4}{P_3}\right)^n$$

$$\frac{\partial^2 f}{\partial P_2 \partial P_3} = \frac{\partial^2 f}{\partial P_3 \partial P_2} = -n\left(\frac{P_3}{P_2}\right)^n\left(\frac{P_2}{P_3}\right)$$

For Positive semi-definite it is enough to show that the diagonal elements are positive because the matrix is simply 2x2.

This can hold if $n > 1$