

CHE401 Exam 2 solution

Question 1

(a)

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SUBROUTINE MATRIX(A,B,N,X)
REAL, A(N,N),B(N),X(N),
INTEGER, N
X(1)=B(1)/A(1,1)
DO I=2,N
    SUM=0.0
    DO J=2,N
        SUM=SUM+A(I,J)*X(J)
    ENDDO
    X(I)=B(I)-SUM/A(I,I)
ENDDO
RETURN
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(B)

$$\frac{L^k}{L^0} = 1 \times 10^{-3} = 2^{-k}$$

$$\ln(1 \times 10^{-3}) = \ln(2^{-k})$$

$$-6.9 = -k \ln(2)$$

$$\frac{-6.9}{0.69} = -k$$

$$\mathbf{K = 100}$$

Question 2

$$\frac{dC_A}{dt} \equiv f = F(C_{Af} - C_A) - \frac{0.05C_A}{0.1 + C_A}$$

$$x_{i+1/2} = x_i + hf(t_i, x_i)$$

$$\bar{x}_{i+1/2} = f\left(t_i + \frac{h}{2}, x_{i+1/2}\right)$$

$$x_{i+1} = x_i + h\bar{x}_{i+1/2}$$

Since the independent variable t does not appear explicitly in the ODE, the algorithm reduces to:

$$x_{i+1/2} = x_i + hf(t_i, x_i)$$

$$x_{i+1} = x_i + hf(x_{i+1/2})$$

Applying the algorithm for the given parameter values gives:

i	T	C_A	$f(x)$
0	0	0.9550	0.0042
1	0.5000	0.9581	0.0008
2	1.0000	0.9587	0.00

Question 3

- (a) BVP because the dependent variable x is defined on both sides.
(b)

Let: $y_1 = dx/dz$

then

$$y_2 = y'_1 = y'$$

$$y'_2 = y''_1 = y''$$

Substituting in the original ODE gives:

$$\frac{dy_1}{dx} = y_2 \quad y_1(0) = y(0) = 3$$

$$\frac{dy_2}{dx} = -3y_2 + 10y_1 \quad y_2(0) = y'(0) = z$$

- (c) Applying the finite difference on the original ODE gives:

$$\frac{x^{i+2} - 2x^{i+1} + x^i}{h^2} + 3\frac{x^{i+1} - x^i}{h} - 10x^i = 0$$

Multiplying by h^2 and rearranging gives:

$$\frac{x^{i+2} - 2x^{i+1} + x^i}{h^2} + 3\frac{x^{i+1} - x^i}{h} - 10x^i = 0$$

$$x^{i+2} + (3h - 2)x^{i+1} + (1 - 3h - 10h^2)x^i = 0$$

Let $h = 2$

$$x^{i+2} + 4x^{i+1} - 45x^i = 0$$

$$i=0 \quad z=0 \quad x^2 + 4x^1 - 45x^0 = 0$$

$$i=1 \quad z=2 \quad x^3 + 4x^2 - 45x^1 = 0$$

knowing $x^0 = 3$ and $x^3 = 1$ the last two equations can be solved simultaneously yield:

$$x(z=2) = 88.6 \quad \text{and} \quad x(z=4) = 99.5$$