Final Exam model solution

Question 1

$$T(s) = \frac{k_p(\tau s+1)}{(\tau_1 s+1)(\tau_2 s+1)} = \frac{a}{s} + \frac{b}{\tau_1 s+1} + \frac{c}{\tau_2 s+1}$$

$$a = \frac{k_p \tau}{\tau_1 \tau_2}$$

$$b = \frac{k_p(\tau - \tau_1)}{1-\tau_2/\tau_1}$$

$$c = \frac{k_p(\tau - \tau_2)}{1-\tau_1/\tau_2}$$

Inverting T(s) using the definition of *a*, *b* and *c* yields:

$$T(t) = k_p\left[\frac{\tau}{\tau_1\tau_2} + \frac{\tau-\tau_1}{\tau_1-\tau_2}e^{-t/\tau_1} + \frac{\tau-\tau_2}{\tau_2-\tau_1}e^{-t/\tau_2}\right]$$

$$T(t) = \left[\frac{20}{45} + \frac{17}{-12}e^{-t/3} + \frac{5}{12}e^{-t/15}\right]$$

$$\frac{\partial T}{\partial t} = \left[\frac{17}{12\times 3}e^{-t/3} - \frac{5}{12\times 15}e^{-t/15}\right] = 0$$

⇨ $t = 10.6$

T = 0.6

Question 2

(a) System is unstable because of positive root

(b)

$$y(s) = \frac{\dfrac{k_c}{(s-1)(s+2)}}{1+\dfrac{k_c}{(s-1)(s+2)}} r(s) = \frac{k_c}{(s-1)(s+2)+k_c} r(s)$$

$$y(s) = \frac{k_c}{s^2+s-2+k_c} r(s) = \frac{\dfrac{k_c}{k_c-2}}{\dfrac{1}{k_c-2}s^2+\dfrac{1}{k_c-2}s+1} r(s)$$

$k_c > 2$ makes the system stable

(c)

$$y(s) = \frac{\dfrac{k_c}{(s-1)(s+2)}}{1+\dfrac{k_c k_m}{(s-1)(s+2)(\tau_m s+1)}} r(s) = \frac{k_c(\tau_m s+1)}{(s-1)(s+2)(\tau_m s+1)+k_c k_m} r(s)$$

$$y(s) == \frac{2(\tau_m s+1)}{\tau_m s^3+(\tau_m+1)s^2+(1-2\tau_m)s+2} r(s)$$

The characteristic equation:

$$\tau_m(s)^3+(\tau_m+1)(s)^2+(1-2\tau_m)s+2=0$$

Setting $s=iw$ gives:

$$\tau_m (iw)^3 + (\tau_m + 1)(iw)^2 + (1 - 2\tau_m) iw + 2 = 0$$

Or

$$-\tau_m i w^3 - (\tau_m + 1) w^2 + (1 - 2\tau_m) iw + 2 = 0$$

Imaginary part gives:

$$-\tau_m w^3 + (1 - 2\tau_m) w = 0 \quad \Rightarrow \quad w = \sqrt{\frac{1 - 2\tau_m}{\tau_m}} \qquad (1)$$

The real part gives:

$$-(\tau_m + 1) w^2 + 2 = 0 \qquad (2)$$

From the last two equations we get $\tau_m = 0.28$

Question 3

The close loop characteristic equation is:

$$CE = 1 + G_c(s)G_v(s)G_p(s)G_m(s)$$

$$CE = 1 + \frac{k_c(4s+1)}{(2s+1)(s+1)(0.2s+1)(0.1s+1)}$$

$$CE = (2s+1)(s+1)(0.2s+1)(0.1s+1) + k_c(4s+1)$$

$$CE = 0.04s^4 + 0.66s^3 + 2.92s^2 + (3.3 + 4k_c)s + 1 + k_c$$

$s = iw$

$$CE = 0 = 0.04(w)^4 - 0.66(iw)^3 - 2.92(w)^2 + (3.3 + 4k_c)iw + 1 + k_c$$

Real Part gives:

⇨ $$\omega = \sqrt{\frac{3.3 + 4k_c}{0.66}} \qquad (1)$$

Imaginary part:

$$0 = 0.04(w)^4 - 2.92(w)^2 + 1 + k_c \qquad (2)$$

From (1) & (2)

$$0 = 16(k_c)^2 + 19.7k_c + 5.44$$

Solving for K_c yields:

$K_c = 0.8$

Question 4

The close loop characteristic equation is:

$$CE = 1 + G_c(s)G_v(s)G_p(s)G_m(s)$$

$$CE = 1 + \frac{k_c(\tau_I s + 1)}{\tau_I s}\frac{1}{s-1}$$

$$CE = \tau_I s(s-1) + k_c(\tau_I s + 1)$$

$$0 = \tau_I s^2 + \tau_I(k_c - 1)s + k_c$$

$$0 = \frac{\tau_I}{k_c}s^2 + \tau_I\frac{(k_c - 1)}{k_c}s + 1$$

For k_c =2 ➔ $0 = \frac{\tau_I}{2}s^2 + \tau_I\frac{1}{2}s + 1$

$$\zeta = \frac{\sqrt{\tau_I/2}}{2}$$

For Over-damped systems $\zeta > 1$

⇨ $\tau_I > 8$