Exam 1 model solution

Question 1

(a)

Mass balance for constant density gives:

$$A\frac{dh}{dt} = F_1 - F_2$$

Component molar balance gives:

$$V\frac{dC_A}{dt} = F_1 C_{Ai} - F_2 C_A - VkC_A$$

$$V\frac{dC_B}{dt} = F_1 C_{Bi} - F_2 C_B - VkC_A$$

$$V\frac{dC_c}{dt} = -F_2 C_c + VkC_A$$

(b)

Knowing $VC_i = n\, y_i$

Letting $F=F_i n/V$

The component balance can be written as

$$n\frac{dy_A}{dt} = Fy_{Ai} - Fy_A - kny_A$$

$$n\frac{dy_B}{dt} = Fy_{Bi} - Fy_B - kny_B$$

$$n\frac{dy_c}{dt} = Fy_{ci} - Fy_c - kny_c$$

Question 2

$$n\dot{y} = Fy_i - Fy - kny$$

$$n\dot{y} + (F + kn)y = Fy_i$$

$$\frac{n}{F + kn}\dot{y} + y = \frac{F}{F + kn}y_i$$

$$\tau\dot{y} + y = k_p y_i$$

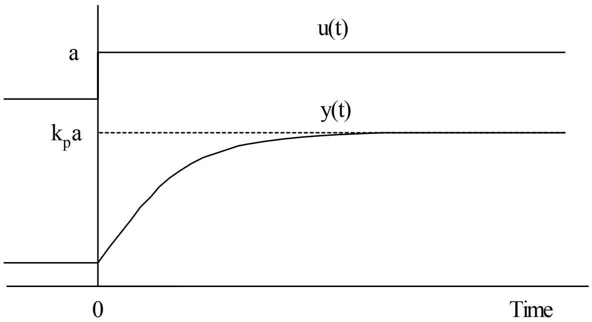
Applying Laplace gives:

$$y(s) = \frac{k_p}{(\tau s + 1)} y_i(s)$$

For a unit step change in y_i

$$y(s) = \frac{k_p}{(\tau s + 1)} \frac{1}{s}$$

Inverting gives: $y(t) = k_p(1 - e^{-t/\tau})$



Question 3

$$\Im[e^{-at}\sin(wt)] = \int_0^\infty e^{-at}\sin(wt)e^{-st}dt = \int_0^\infty e^{-at}\frac{e^{iwt}-e^{-iwt}}{2i}e^{-st}dt = \frac{1}{2i}\int_0^\infty e^{-(a+si-w)t} - e^{-(a+s+iw)t}dt$$

$$= \frac{1}{2i}\left(\frac{-1}{a+s-iw} e^{-(a+s-iw)t}\Big|_0^\infty - \frac{-1}{a+s+iw} e^{-(a+s+iw)t}\Big|_0^\infty \right)$$

$$\frac{1}{2i}\left[\frac{-(0-1)}{a+s-iw} - \frac{-(0-1)}{a+s+iw}\right]$$

$$= \frac{1}{2i}\left[\ \frac{a+s+iw-(a+s-iw)}{(a+s-iw)(a+s+iw)}\ \right]$$

$$= \frac{w}{(s+a)^2 + w^2}$$