IDENTIFICATION AND ESTIMATION (EMPIRICAL MODELS)

Introduction

When the process is mathematically too complex to model from the fundamental physical and chemical laws, empirical models can be obtained from experimental dynamic data. Characterizing and estimating the parameters of this model from its input/output experimental data is known as process identification.

1. Process Identification

Characterizing a process by an empirical model from its input/output experimental data is known as process identification. One may judge from input/output data if the system needs to be identified by a linear model or non-linear model. Mainly, if the output satisfies the superposition principle, that is, the response of the system to the sum of two inputs is the same as the sum of the response to the individual inputs, then a linear model will be adequate. Otherwise we need to identify the system by a non-linear model. Many books have been written on the subject of process identification, see for example, Bendat (1990), Box and Jenkins (1970), Box and Draper (1987), Eykhoff (1974), Graups (1972), Ljung (1980), Mehra and Lainiotis (1976), Ray and Lainiotis (1987), Sage and Melsa (1971), Seinfield and Lapidus (1974), Sinha and Kuszta (1988), Soderstrom and Stoica (1989) and Unbehauen and Rao (1987). In the next sections, we introduce linear and non-linear system identification.

2 Identification of linear Systems

The main task is to identify a suitable linear model to represent the data. Having suggested the model, methods of parameter estimation can then be used and statistical methods are then called for to test the adequacy of the proposed model. If not adequate, another model can be suggested. Models used can be of the form of:

2.1 The convolution integral

The general input/output relation is:

$$y(t) = \int_0^t g(\tau)u(t-\tau)d\tau \tag{1}$$

The above equation relates the output y(t) with the disturbance u(t) through the impulse response g(t). The discrete version of the relation takes the form:

$$y(k) = \sum_{i=0}^{k} g(i)u(k-i) \tag{2}$$

Instead of using impulse response function g(i), one can use the step response function β(i) such that:

$$y(k) = \sum_{i=0}^{k} \beta(i)\big(u(k-i) - u(k-i-1)\big) \tag{3}$$

where $\beta(i)=\sum_{j=1}^{i} g(j)$ and $g(i)=\beta(i)-\beta(i-1)$

The process of the determination of the impulse response function from input/output data is called "deconvolution" and involves the solution of a system of linear equations (2).

4.1.2 State Space Models

State space models equations (35) to (36) in the previous lecture can be used for process identification.

4.1.3 Laplace Transfer Function Models

In this case we use general linear transfer function with time delay of the form of equation (44). It is worth mentioning here that a second order system with time delay approximates the dynamics in many chemical processes.

From the output data of a process which is subjected to a certain type of disturbance, one can guess how the empirical model would look like. The input can be chosen to be an impulse, a step change or a sine wave. Next we present some possible output profiles from certain models subjected to step input change.

(1) *models from step change*:
According to the responses shown in Figure 1, we can classify the following:

(a) Output (*a*) is called Gain only process. It is characterized by the instantaneous response of the output and the model is given by:

$$\bar{y}=K\bar{u}$$

K is called the process gain.

(b) Output (*b*) is usually obtained for a first order system given by:

$$\bar{y}=\frac{K}{\tau s+1}\bar{u}$$

τ is called time constant of the process.

(c) When the output responds slowly to the change in the input (shown in *c*), the process can be modeled by a second order over-damped model. It takes the form;

$$\bar{y}=\frac{K}{(\tau_1 s+1)(\tau_2 s+1)}\bar{u}$$

(d) When the step response of a process has a decaying oscillation (shown in *d*), a second order under-damped model may be used. It takes the form;

$$\bar{y} = \frac{K}{T^2 s^2 + 2\zeta T s + 1} \bar{u}$$

where T is the characteristic time, and ζ is the damping ratio which is less than one for under-damped system.

In all above models, if the output is delayed for some time untill it feels the effect of the change in the input, all the above models can be multiplied by $e^{-\theta s}$ which is the transfer function of a dead time process and θ is the dead time.

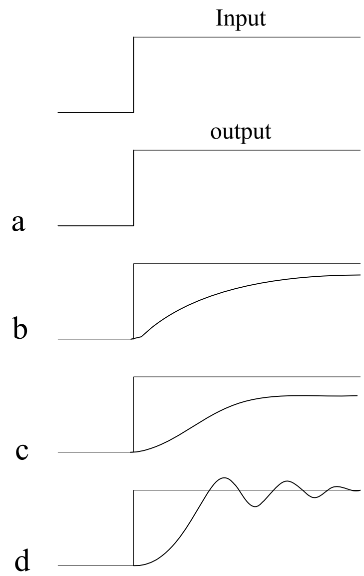


Figure 1: Step response of process

(2) *Models from sinusoidal response*

Here the input takes the form of a sine wave:

$$u(t) = A \sin \omega t$$

where A is the amplitude and ω is the frequency. After waiting for all transient to die out, the output could have different amplitude and the response is delayed by what is called phase lag.

The process is subjected to different sine waves with different frequencies. From the output, one can guess the suitable model which represents the process. The following rules are useful;

i) For an n-th order system, the phase lag is less than $\frac{n\pi}{2}$.

ii) The amplitude ratio between the output and the input is usually less than 1. If the amplitude ratio is greater than one for a range of frequency, this indicates an under-damped model (oscillating).
iii) There is no limit for the phase lag in case of dead time.

Parameter Estimation in Transfer Function Models

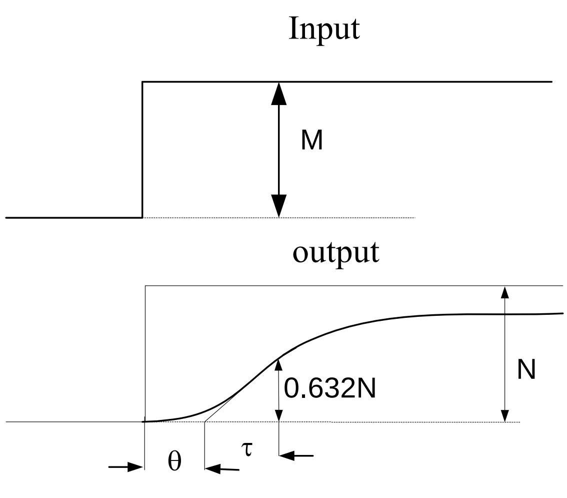
In most control packages, e.g. CONSYD, there are programs that can estimate optimum parameters using optimization methods such as Least Square method. Here we present some simple methods to obtain approximately estimates for the parameters.

(1) First order system with dead time

$$\bar{y} = \frac{Ke^{-\theta s}}{\tau s + 1}\bar{u}$$

where $K = \frac{N}{M}$

τ is the time constant, which is the time at which the change in the output is 63.2% of its ultimate value and θ is the dead time at the end of which the output variable starts to change.



(2) Second order over-damped system with dead time

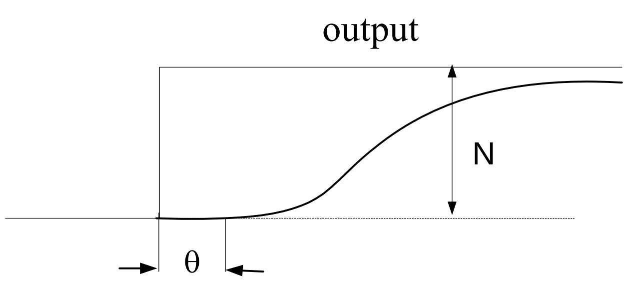
$$\bar{y} = \frac{Ke^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}\bar{u}$$

where $K = \frac{N}{M}$ and θ can be obtained easily from the diagram.

The step response of second order damped system in the time domain is:

$$y(t) = N\left(1 - \frac{\tau_2 e^{-(t-\theta)/\tau_2} - \tau_1 e^{-(t-\theta)/\tau_1}}{\tau_2 - \tau_1}\right)$$

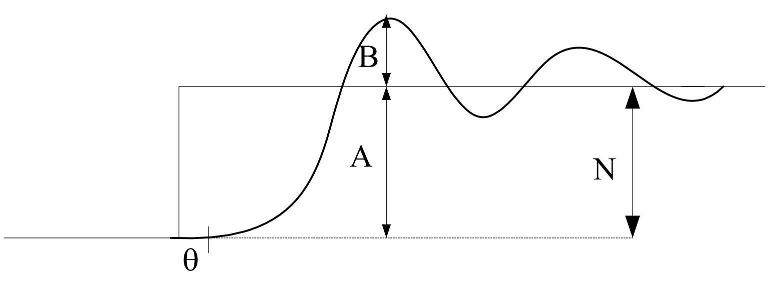
τ_1 and τ_2 can be obtained by solving non-linear equation in τ_1, τ_2 for two values of y(t), e.g. at y(t) = 1/3 N and y(t) = 2/3N.



(3) Second order under-damped system with time delay

$$\bar{y} = \frac{Ke^{-\theta s}}{T^2 s^2 + 2\zeta Ts + 1}\bar{u}$$

where K and θ can be estimated as before. ζ can be calculated from overshoot $= \frac{B}{A} = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$



4.2 Identification of Nonlinear System

We discuss here some methods for non-linear model identification. A recent review by Haber and Unbehauen (1990) discusses different methods for the identification of nonlinear systems.

4.2.1 Volterra Series Models

This is an extension of the convolution integral of a linear system which is:

$$y(t) = \int_0^t k(t-\tau)u(\tau)d\tau \tag{4}$$

The extension is given by:

$$y = k_0 + \int_{-\infty}^{\infty} k_1(\tau)u(t-\tau)d\tau + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k_2(\tau_1,\tau_2)u(\tau-\tau_1)(\tau-\tau_2)d\tau_1 d\tau_2$$
$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k_3(\tau_1,\tau_2,\tau_3)u(\tau-\tau_1)(\tau-\tau_2)(\tau-\tau_3)d\tau_1 d\tau_2 d\tau_3 + \cdots \tag{5}$$

where the integral kernels $k_1(\tau)$, $k_2(\tau_1,\tau_2)$, $k_3(\tau_1,\tau_2,\tau_3)$ are zero when any of their argument, is negative. Practical implementation of Voletrra series is discussed in Seinfeld and Lapidus (1974).

4.2.2 Neural Networks Model Identification:

Here the model is not in a form of explicit algebraic form with parameters to be estimated. Rather the model is in the form of a general structure consisting of a network of input models (neurons) hidden layers and output nodes. As shown in Figure (2), the input to the nodes of each hidden layer has adjustable weights that resemble the unknown parameters in algebraic form. The output y_i from a hidden layer is given by:

$$y_i = f\left(\sum_{j=1}^{n} w_{ij}u_j\right), \quad i = 1,2, \ldots \text{n} \tag{6}$$

Where u_j 's are the inputs, w_{ji} are the weight and f is a simple non-linear function, e.g.

$$f(x) = \frac{1}{1+e^{-\alpha x}} \tag{7}$$

Recently some researcher works have been presented for the application of neural networks for dynamic modeling and hence for control purposes. Bhat and Mcavoy (1990), Billing et al. (1992), Narendra and Parthasarathy (1990), Scott and Ray (1993a, 1993b), Su and Sheen (1992). El-Hewary (1992) gave an account for the prospects of the neural networks to desalination systems including control application.

6.2.3 NARMAX Models:

A general non-linear discrete time system of the form of:

$$y(t) = f(y(t-1),\cdots,y(t-n_g),u(t-1),\cdots,u(t-n_a),e(t-1),\cdots,e(t-n_e)+e(t)) \tag{8}$$

is called NARMAX model (Non-linear Autoregressive Moving Average with Exogenous input). Chen and Billing (1989a) describe a recursive prediction error parameter estimator. Chen and Billing (1989b) expand non-linear terms of the polynomials. Chen et al. (1990) compare the use of radial basis function with the use of output affine models, Polynomial models and rational models. Johansen and Foss (1993) used local ARAMAX models to construct a global NARMAX model.

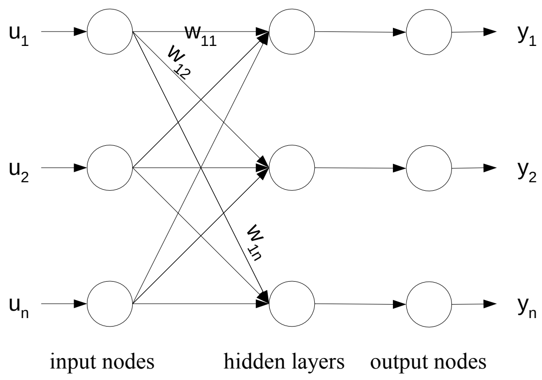


Figure (2): A Neural Network

Note: One of the workshop tutorials include an exercise on modeling input-output data of a multivariable process to an ARAMAX model (linear discrete time model).

Reference:

Bendat, J.S., *Nonlinear System Analysis and Identification from Random Data*, Wiely-Inetrscience, New York, 1990.

Box, G.E.P. and Jenkins. G.M., *Time Series Analysis Forecasting and Control*, Holden-Day, San Francisco, 1970.

Box, G.E.P., and Draper, N.R., *Empirical Model Building and Response Surface*, John Wiely, New York, 1987.

Bhat, N. and MCaAvoy, T. J., "Use of Neural Nets for Dynamic Modeling and Control of Chemical Process Systems", *Computer & Chemical Engineering*, 14, 573-582, 1990.

Billings, S.A., Jamaluddin, H.B. and Chen, S., "Properties Neural Networks with Application to Modeling Non-linear Dynamic Systems", *Int. J. Control*, 55, 193-224, 1992.

Chen, S., and Billings, S. A., "A recursive Prediction Error Estimator for Non-linear Models", *Int. J. Control*, 49, 1013-1032, 1989a.

Chen, S., and Billings, S. A., " Representation of Non-linear Systems: The NARMAX model", *Int. J. Control*, 49, 569-594, 1989b.

Chen, S., Billings, S. A., Cowan, C.E.N, and Grant, P., "Practical Identification of NARMAX Models Using Radial Basis Functions", *Int. J, Control*, 52, 1327-1350, 1990.

Eykhoff, P., *System Identification*, John Wiely, New York, 1974.

Graups, D., *Identification of Systems*, Van Nostrand Reinhold Co., New York, 1972.

Haber, R., and Unbehauen, H., "Structure Identification of Non-linear Dynamic Systems- A Survey of Input/Output Approaches", *Automatica*, 16, 651-667, 1990.

Johansen, T. A., and Foss, B. A., "Constructing NARMAX Models using ARMAX Models", *Int. J. Control*, 58, 1125-1153, 1993.

Ljung, L., *System Identification Theory for the User*, Prentice-Hall, Englewood Cliffs, New Jersey, 1980.

Mehra, R. K. and Lainiotis, D. G., *System Identification: Advances and Case Studies*, Academic Press, New York, 1976.

Narendra, S. and Parthasarathy, K., "Identification and Control of Dynamical Systems Using Neural Networks", *IEEE Trans. Neural Networks*, NN-1, 4-27, 1990.

Ray, W. H., and Lainiotis, D. G., *Distributed Parameter Systems, Identification, Estimation and Control*, Mercel Dekker, New York, 1987.

Sage, A. P., and Melsa, J.L., *System Identification*, Academic Press, New York, 1971.

Scott, G. M., and Ray, W. H., "Creating Efficient Non-linear Neural Network Process Models that Allow Model Interpretation", *J. Process Control*, 3, 163-178, 1993.

Sinha, N. K., and Kuszta, B., *Modeling and Identification of Dynamic Systems*, Van Nostrand Rheinhold Co., New York, 1983.

Soderstrom, T. and Stocia, P., *System Identification*, Prentice-Hall, Englewood Cliffs, New Jersey, 1989.

Su, Y. T. and Sheen, Y. T., "Neural Networks for System Identification", *Int. J. System Sci*., 23, 2171-2186, 1992.

Unbehauen, H., and Rao, G. P., *Identification of Continuous Systems*, North Holland, Amsterdam, 1987.

Seinfeld, J. H., and Lapidus, L., *Mathematical Methods in Chemical Engineering, vol. 3, Process Modeling, Estimation, and Identification*, Prentice-Hall, Englewood Cliffs, New Jersey, 1974.

Appendix

Computer Aided Design Packages

Much of the Programming burden required to apply the principles presented in this chapter is no longer needed since there are well developed computer packages that carry out simulation, identification, parameter estimation, model reduction and simplification, analysis and control system design. In the following table we give list of some famous package in the field.

Sources for Control Design Packages

Program	Source
CC	System Technology Inc. 13766, S. Hawthorne Blvd. Hawthorne, CA 90250, USA
CONSYD	Prof. H. Ray, Dept. of Chemical Engineering University of Wisconsin 1415 Johnson Drive Madison, WI 53708, USA
EASY5	Boeing Computer Services POBox 24346 Seattle, WA 98124, USA
KEDDC	Ingenieur buero Erble Jahnostr, 73 Grossbettilingen, Germany
MATLAB	Mathworks Inc. 24 Prince Park Way Natic, MA 01760, USA
SIMULINK	Mathworks Inc. 24 Prince Park Way Natic, MA 01760, USA
Control Station	Prof. Douglas Cooper, Chemical Engineering Dept. University of Connecticut 191 Auditorium Rd. Storrs, CT 06269
20sim	Controllab Products B.V. Drienerlolaan 5 EL-RT 7522 NB Enschede The Netherlands
Calerga	Calerga Sarl Yves Piguet Av. de la Chabliere 35 CH - 1004 Lausanne Switzerland
pIDtune	EngineSoft P.O. Box 877 Tempe, Arizona 85280-0877

SimApp	Buesser Engineering Bruno Buesser, dipl. El.Ing. ETH Wacht 28 CH-8630 Rueti ZH, Switzerland
Visim	Visual Solutions, Incorporated 487 Groton Road Westford, Massachusetts 01886