

CONTROL SYSTEM DESIGN AND STRUCTURE

Introduction

This lecture will describe a methodology for designing a multivariable control system that includes the elements of control strategy considerations and control structure. Control structure means the selection of controlled variables and manipulated variables from a set of many choices. In this part we focus attention on the selection of the controlled system configuration via the choice of controlled, manipulated and measured variables.

1. The influence of process design on process control

Traditionally, process design and control system design have been separate activities. This approach has serious limitations, since the plant design determines the process characteristics as well as the operability of the plant. In extreme situations, the plant may be uncontrollable even though the process design appears satisfactory from a steady-state point of view. In recent years there has been a growing recognition of the importance of considering dynamics and control issues early in plant design. This interplay between design and control has become especially important for modern plants, which tend to have a larger degree of material and energy integration and tighter performance specifications. Next we consider two examples of how process design affects dynamics and control

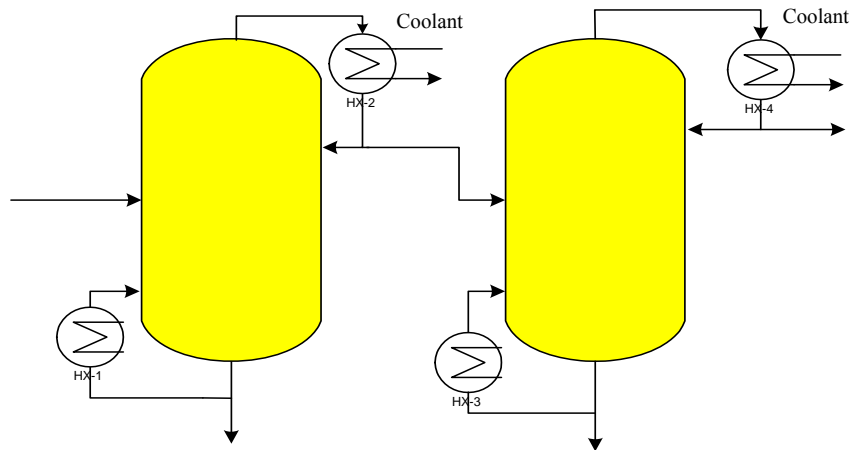
Example 1: Two-distillation column with and without heat integration

There has been considerable interest in reducing the energy costs of distillation trains by heat integration or thermal coupling of two or more columns. Figure 1 shows two distillation columns with and without heat integration. Heat integration reduces the energy cost by allowing the overhead stream from column 1 to be used as the heating medium in the reboiler for column 2. However this arrangement is more difficult to control for two reasons. First the process is more highly interacting; because the process upsets in one column affect the other column via the heat integration. Second the reboiler heat duty can no longer be independently manipulated as in the conventional scheme. In this case, the heat integration scheme has one less manipulated variable available for process control. These disadvantages can be overcome by utilizing more sophisticated control strategies.

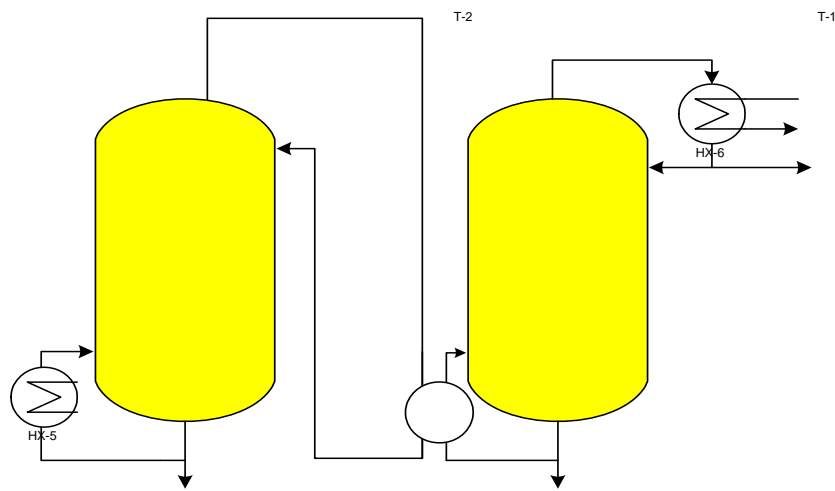
Example 2: Reactor feed heating schemes

A second type of heat integration is shown in Figure 2 for a fixed bed reactor. The conventional design uses a hot oil to heat the feed to the reactor. If the chemical reaction is exothermic, energy costs can be reduced by using the hot product stream to heat the cold feed in a heat exchanger. However, this reactor configuration has the same disadvantages as the heat-integrated distillation column. In particular, the feed-effluent heat exchanger introduces positive feedback and the possibility of a thermal runaway, since the temperature fluctuations in the effluent are transmitted to the feed stream.

Thus, they affect the rate of reaction and, consequently, the heat generation due to the reaction.



(a) Without heat integration



(b) With heat integration

Figure (1): Two distillation columns

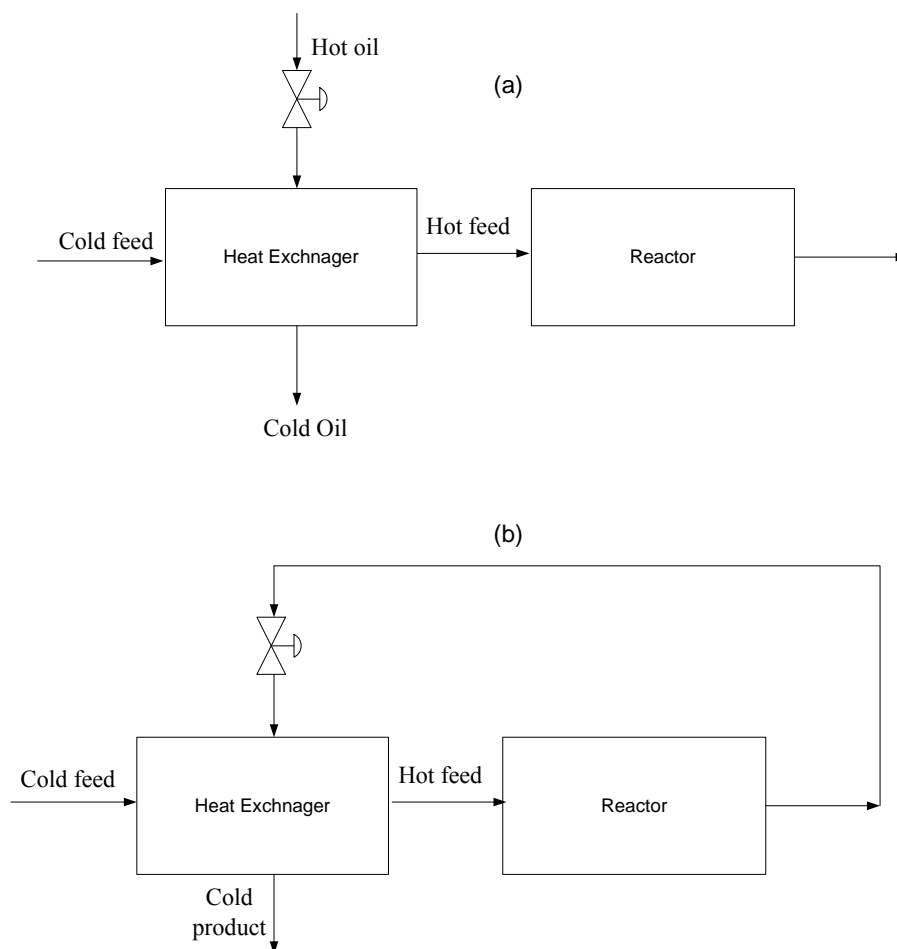


Figure (2) Alternative reactor feed heating schemes.

2. Classification of process variables

For control system design, it is convenient to classify process variables as either inputs or outputs, as is shown in Figure 3. The output variables (y_1, y_2, \dots, y_N) are process variables that ordinary associated with the exit streams of a process or conditions inside a process vessel (e.g., composition, temperatures, levels, and flow rates). A subset of the output variables is selected as variables to be controlled (i.e., controlled variables) in order to satisfy the plant and control objectives.

The process inputs (x_1, x_2, \dots, x_M) are physical variables that affect the process outputs. Typically the inputs are associated with the inlet streams (e.g., feed compositions, or feed flow rates) or environmental conditions (e.g., ambient temperature). However, an exit flow rate from a process can also be an “input” from control point of view if an exit flow rate is used as a manipulated variable. Some of the input variables are specified as manipulated variables, while the other inputs are considered to be disturbance variables, specified by the external environment.

In general it is not feasible to control all of the output variables for a number of reasons; two of them:

1. It may not be possible or economically to measure all of the outputs especially compositions.
2. There may not be possible manipulated variables.

In general controlled variables are measured on-line (direct or indirect) and the measurements are used for control. However, it is theoretically possible to control a process variable that is not measured by using a mathematical model of the process to calculate the value of the unmeasured controlled variable.

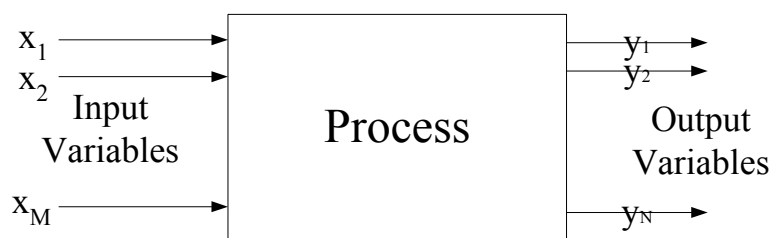


Figure (3) Process with multiple inputs and outputs

2.1 Selection of Controlled variables

The consideration of control objectives has led to a number of suggested guidelines for the selection of the controlled variables from the available output variables:

1. Select variables that are not self-regulating. A common example is liquid level in a storage vessel with a pump in the exit line.
2. Choose output variables that may exceed equipment and operating constraints (e.g. temperatures, pressures, and compositions)
3. Select output variables that are a direct measure of product quality (e.g., composition, refractive index) or that strongly affect it (e.g., temperature, pressure)
4. Choose output variables that seriously interact with other controlled variables. The steam header pressure for a plant boiler that supplies several downstream units is an example of this type of output variable.
5. Choose output variables that have favorable dynamic and static characteristics. Ideally, there should be at least one manipulated variables that has a significant direct, and rapid effect on each controlled variables.

2.2 Selection of Manipulated Variables

A number of guidelines have been proposed for the selection of manipulated variables from among of the input variables

1. Select inputs that have large effects on the controlled variables. Thus, for each control loop it is desirable that the steady-state gain matrix between the manipulated variable and the controlled variable be as large as possible. Also it is important that the manipulated variable have a large enough range. For example, if a distillation column operates at a steady-

state reflux ratio of five it will be much easier to control the level in the reflux drum by using the reflux flow rate rather than the distillate flow rate.

2. Choose inputs that rapidly affect the controlled variables. Any time delay that is associated with the manipulated variable should be small relative to the dominant time constant of the process.
3. The manipulated variable should affect the controlled variable directly rather than indirectly. For example, consider the problem of controlling the exit temperature of a process stream that is heated by steam in a shell and tube heat exchanger. It is preferable to use the steam flow to the heat exchanger as a manipulated variable rather than the condensate flow from the shell, since the steam flow rate has a more direct affect on the steam pressure and thus on the steam saturation temperature and the rate of heat transfer.
4. Avoid recycling of disturbances: It is preferable not to manipulate an inlet stream or a recycle flow, because disturbances tend to propagate forward or recycled back to the process. This problem can be avoided by manipulating a utility stream to absorb disturbances or an exit stream that allows the disturbances to be passed downstream, provided that the exit stream changes do not upset downstream process units.

2.3 Selection of Measured Variables

Safe, efficient operation of processing plants is made possible by the on-line measurement of key process variables. Clearly, output variables that are used as controlled variables should be measured. Other output variables are measured to provide additional information to plant operators or for use in model-based control schemes. It is also desirable to measure selected input variables as well as output variables, since recorded measurements of manipulated inputs provide useful information for tuning controllers. Also, measurements of disturbances can be used in feedforward control schemes. In choosing which outputs to measure and in locating measurement points, both static and dynamic considerations are important.

1. Reliable, accurate measurements are essential for good control.
2. Select measurement points that have an adequate degree of sensitivity.
3. Select measurement points that minimize time delays and time constant.

3. Degree of freedom for process control

To design a control system, it is necessary to select the appropriate number of manipulated variables. The number of process variables that can be manipulated cannot exceed the degree of freedom of the process. The degree of freedom of a process is the number of process variables that must be specified to determine the remaining process variables. It is defined as follows:

$$N_F = N_V - N_E$$

where N_F is the degree of freedom, N_V is the number of process variables, and N_E is the number of independent equations.

In general adding a single control loop introduces an additional equation, the control law, and thus uses up one degree of freedom, the manipulated variable. It can be argued that the control law also introduces a new process variable, the set point. However the value of the set point is usually specified. Thus, the net result of controlling a process variable is to reduce the degrees of freedom by one.

For under specified process ($N_F > 0$), the degree of freedom are utilized in two ways:

1. Choice of manipulated variables (N_M)
2. Identification of process variables that are fixed by the process environment (N_S).

Thus we can write

$$N_F = N_M + N_S$$

This suggests the very important result that the number of manipulated variables is always less than or equal to the available degree of freedom.

Now we consider an example that illustrates how the number of manipulated variables can be determined.

Example: Two tank blending system (Figure 4)

Assume that primary control objective is to control exit composition c_5 of a key component and that all streams have the same density. The makeup stream has a constant composition c_4 , but its flow rate q_4 can be adjusted. Streams 1 and 2 have variable flow rates and compositions.

Assuming that each tank is perfectly mixed one can write the dynamic overall mass and component balances as follows:

Tank 1

$$A_1 \frac{dh_1}{dt} = q_1 + q_2 - q_3$$

$$A_1 \frac{d(h_1 c_3)}{dt} = q_1 c_1 + q_2 c_2 - q_3 c_3$$

Tank 2

$$A_2 \frac{dh_2}{dt} = q_3 + q_4 - q_5$$

$$A_2 \frac{d(h_2 c_5)}{dt} = q_3 c_3 + q_4 c_4 - q_5 c_5$$

There are four equations and twelve process variables:

$$h_1, h_2, c_1, c_2, c_3, c_4, c_5, q_1, q_2, q_3, q_4, q_5$$

It follows that there are $12-4 = 8$ degrees of freedom. There are four external disturbance variables (c_1, q_1, c_2, q_2); also c_4 is assumed to be constant. Thus, there are three remaining degrees of freedom that can be used to specify no more than three manipulated variables. For example we could choose q_3, q_4, q_5 as the manipulated variables. For controlled variables we should select the exit concentration c_5 , because it is the primary controlled variable, and liquid levels to maintain adequate liquid inventories without having the two tanks drain completely or overflow. One reasonable multiloop control configuration consists of the following pairing of controlled and manipulated variables:

$$c_5 \rightarrow q_4, h_1 \rightarrow q_3, h_2 \rightarrow q_5$$

However further analysis is necessary to determine whether this proposed pairing is the best one from the $3! = 6$ alternatives.

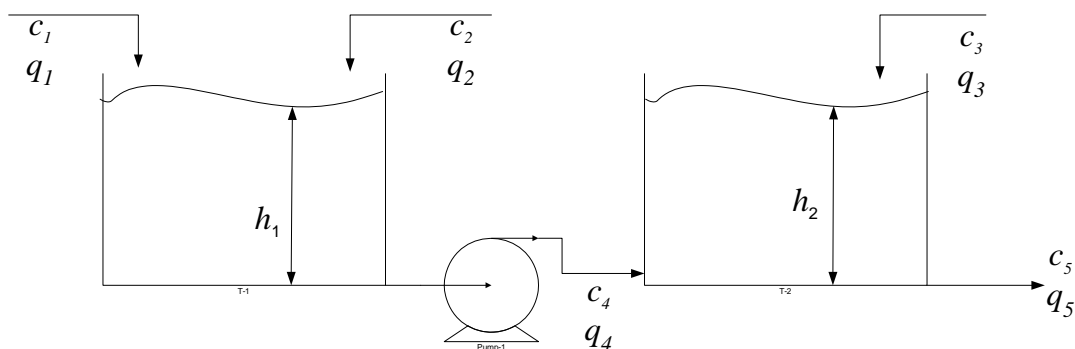


Figure (4): Two tank blending system

An important question then arises as to the number of manipulated variables necessary to control certain number of controlled variables N_c . If offsets cannot be tolerated, then there must be at least as many manipulated variables as controlled variables.

$$N_M \geq N_c$$

For example, if a process has five controlled variables, then offsets can be avoided by using five manipulated variables (five controllers). By contrast, if only four manipulated variables are available, then offset can be eliminated in only four of the five controlled variables.

Occasionally, situations occur where there are more manipulated variables than controlled variables.

Next we consider an evaporator control problem that will be used to illustrate the guidelines maintained above.

Example

An evaporator is used to concentrate a dilute solution of a single solute in a volatile solvent as shown in Figure 5. We would like to specify a control configuration for the evaporator for two situations:

- (a) The product composition is measured on-line
- (b) The product composition is not measured on-line

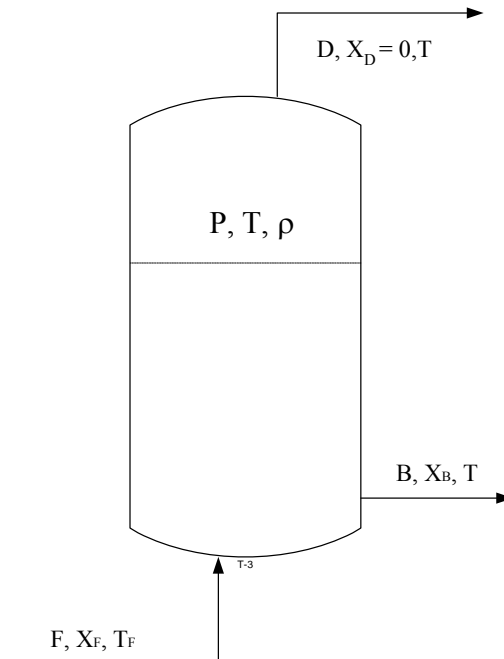


Figure (5) Schematic diagram of an evaporator

A dynamic model of the evaporator can be developed based on the following assumptions:

1. Liquid is perfectly mixed
2. The thermal capacitance of the vapor is negligible compared with that of the liquid.
3. The dynamics of the steam is negligible
4. The feed stream and bottoms streams have a constant molar density c and a constant heat capacity c_p .
5. The vapor and liquid are in thermal equilibrium
6. Heat loss and heat of solution is negligible

The model equations can be written as follows:

Total material balance

$$Ac \frac{dh}{dt} = F - B - D$$

Where F , B , and D are molar flow rates, A is the cross-sectional area, and c is the molar density (mol/m^3). The average solution molecular weight is assumed to be constant, since changes in composition are small.

Solute balance

$$Ac \frac{dx_B}{dt} = Fx_F - Bx_B$$

where x_F and x_B denote mole fractions.

Energy Balance

$$AC_{pl} \frac{d(hT)}{dt} = C_{pl}FT_F - C_{pl}BT - C_{pv}DT + UA_s(T_s - T) - \Delta H_v E$$

where

C_{pl} : specific heat of the liquid

C_{pv} : specific heat of the vapor

A_s : heat transfer area

U : overall heat transfer coefficient

ΔH_v : latent heat of evaporation

E : rate of evaporation

T_s : steam temperature

For normal operation, the sensible heat changes in the liquid are small compared with the latent heats associated with the condensing steam and the evaporating solvent. Thus, the derivative of the energy balance equation is small, and this equation can be approximated by:

$$UA_s(T_s - T) = \Delta H_v E$$

Material Balance on Vapor

$$\frac{1}{M} \frac{d\rho V}{dt} = E - D$$

where ρ is the vapor density, V is the volume of vapor, and M is the molecular weight of the solvent.

Equation of state

The density of the vapor ρ can be related to the pressure and the temperature T by an equation of state: $\rho = \varphi_1(P, T)$

Vapor Pressure Relation

The pressure in the evaporator P is equal to the vapor pressure of the liquid solution P_{vap} , which depends on the temperature T as $P_{vap} = \varphi_2(T)$. Since $P = P_{vap}$ we can write $P = \varphi_2(T)$

Volume Relation

Since the liquid level can change, the vapor space volume also can change. However, these two variables are related by:

$$V_o = V + Ah$$

where V_o is the fixed volume of the evaporator.

Thermodynamic Relation

If the steam is saturated, the relation between the steam pressure and temperature can be obtained from the steam table data:

$$P_s = \phi_3(T)$$

Degree of freedom

Thus, the simplified dynamic model of the evaporator consists of 8 equations: (Total material balance, solute balance, energy balance, vapor material balance, equation of state, vapor pressure relation, volume relation, thermodynamic relation), and 14 variables:

$$h, F, B, D, x_F, x_B, T, T_F, T_s, E, \rho, V, P, P_s$$

Thus there is six degree of freedom. However, the feed conditions (F, x_F, T_F) are normally fixed by operations in an upstream unit. Consequently, the maximum number of variables that can be manipulated is $6-3=3$.

Case (a): Product composition is measured on-line

Controlled Variables:

Because three degrees of freedom are available, three process variables can be controlled without offset by adjusting three manipulated variables. Since the primary objective is to obtain a product stream with a specific composition, mole fraction x_B is the primary controlled variable. Liquid level h should also be controlled since it is not self-regulating. The evaporator pressure P should also be controlled since it has a major influence on the evaporator operation.

Manipulated Variables:

The obvious manipulated variables are B, D , and P_s . Liquid flow rate B has a significant effect on h but a relatively small effect on P and x_B . Therefore, it is reasonable to control h by manipulating B . Vapor flow rate D has a direct and rapid effect on P while having less direct effect on h and x_B . Thus P should be paired with D . This leaves P_s as a manipulated variable to control the product composition x_B , which is physically reasonable, since the most direct way of regulating x_B is by adjusting the amount of solvent that is evaporated via the steam pressure.

Measured Variables

Clearly, the three controlled variables should be measured. It is also desirable to measure the three manipulated variables, since this information is useful for controller tuning and troubleshooting. If large and frequent disturbances in the feed stream occur,

then measurements of load variables F and x_F could be used in a feedforward control strategy. Figure (6) shows a schematic diagram of the controlled evaporator.

Case (b): Product composition cannot be measured on line

The controlled variables are the same as in Case (a), but since the third controlled variable x_B is not measured, standard feedback control is not possible. However, a simple feedforward control strategy can be developed based on a steady state version

$$0 = \bar{F}\bar{x}_F - \bar{B}\bar{x}_B, \quad \text{OR} \quad \bar{B} = \bar{F} \frac{\bar{x}_F}{\bar{x}_B}$$

This equation provides the basis for the feedforward control law. Replacing \bar{B} and \bar{F} by the actual flow rates B and F , and replacing the nominal product composition x_B by the set point, \bar{x}_{Bsp} , gives:

$$B = F \frac{\bar{x}_F}{x_{Bsp}}$$

Thus, the manipulated variable B is adjusted based on the measured load variable F , the set point x_{Bsp} , and the nominal value of the feed composition \bar{x}_F . The manipulated variables are the same for the Case (a): D , B , and P_s . B has already been used in the feedforward method of controlling x_B . Clearly the P-D pairing is still desirable for reasons given in the first case. This leaves h to be controlled by adjusting the rate of evaporation via P_s .

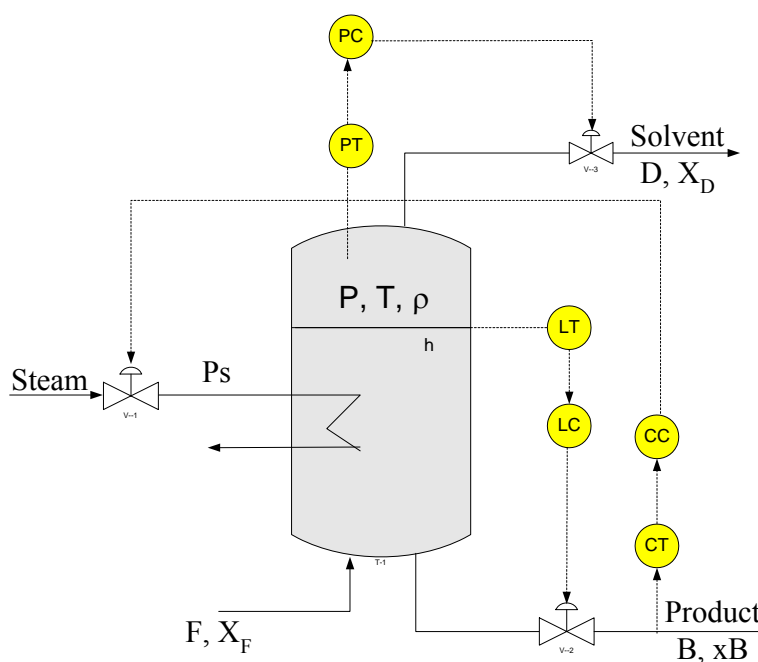


Figure (6): Evaporator control strategy for Case (a)

A schematic diagram for the controlled evaporator is shown in Figure (7). This control strategy has two disadvantages. First, it is based on the assumption that the unmeasured feed composition is constant at a known steady-state value. Second, the feedforward control was based on steady state considerations. Thus, it may not perform well during transient conditions unless dynamic compensation is added. Nevertheless, this scheme provides a simple, indirect method for controlling the product composition when it cannot be measured.

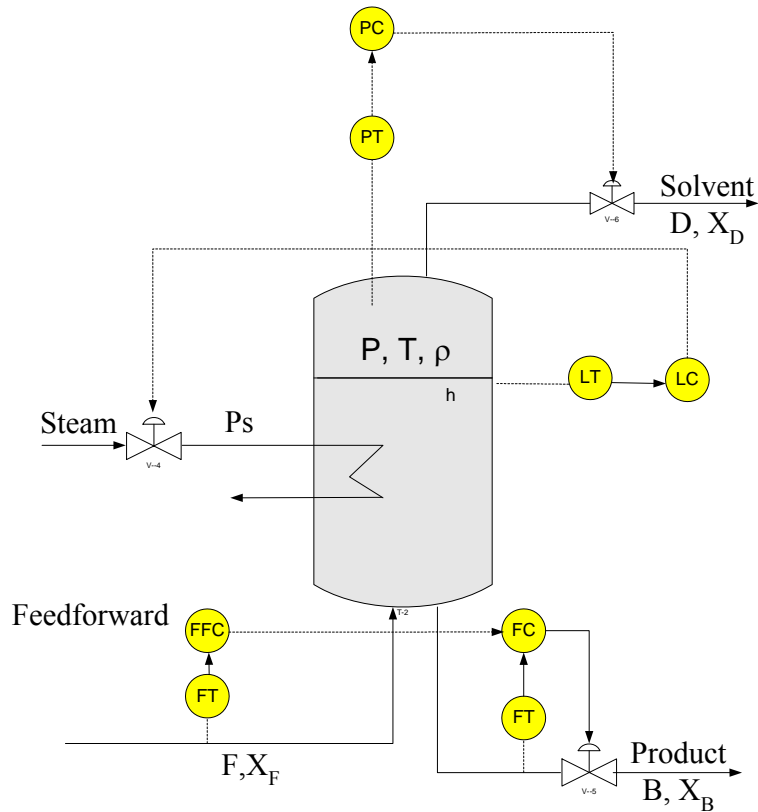


Figure (7): Evaporator control strategy for Case (b)

4. Process interactions and control loop interactions

When designing controllers for multi-input/multi-output (MIMO) processes, it seems natural to start out by considering the possibility of pairing the input (manipulated) variables and output (controlled) variables and assigning a feedback controller to each selected pair. With this approach, the first issue to be resolved is how to pair the input and output variables.

Consider, as an example, the 2×2 system in block diagram form in Figure (8). The effect of the input m_i on the output y_i can be represented by the input-output functions or transfer functions:

$$y_1 = g_{11}m_1 + g_{12}m_2$$

$$y_2 = g_{21}m_1 + g_{22}m_2$$

Where g_{ij} is the input-output relationship between y_i and m_j .

If m_1 has been assigned to y_1 and by default m_2 to y_2 , each of these control loops will experience interactions from the other loop. For example, loop 1 ($m_1 - y_1$) will experience interactions coming from loop 2 via the g_{12} element.

There is a second control configuration that can be considered. If the input/output pairing were switched, each loop would still be subject to interactions from the other loop (this time, the interactions would come via g_{11}, g_{22}).

Since there are two possible input/output pairing configurations for this system, the control engineer should choose the configuration that result in the minimum interaction. A means of discriminating between the $n!$ different configuration possible with a $n \times n$ system is required to determine which input/output pairing configuration to be used.

4.1 Quantifying control loop interactions

Consider the 2×2 system with two outputs and two inputs. When both loops are open, m_1 and m_2 can be manipulated independently and the effects of the inputs on the outputs are represented by the input-output function:

$$y_1 = g_{11}m_1 + g_{12}m_2$$

$$y_2 = g_{21}m_1 + g_{22}m_2$$

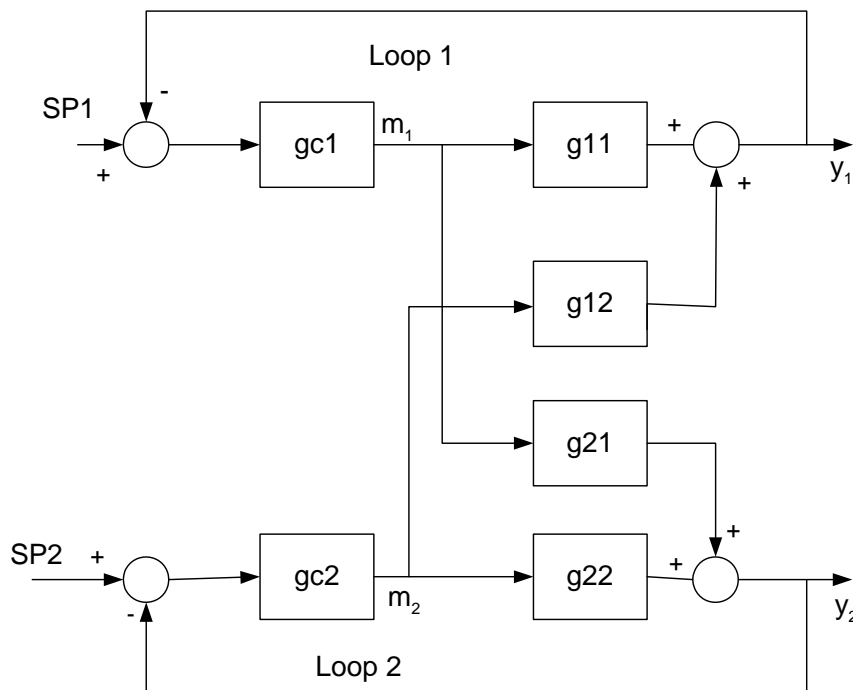


Figure (8) Loop interactions for a 2 X 2 system.

Let us now consider m_1 as a candidate input to pair with y_1 . To evaluate this choice against the alternative of using m_2 , it is necessary to perform two experiments on the system

Experiment 1: Unit step change in m_1 , with both loops open. With this change, the outputs y_1, y_2 will change but for now the primary interest is in the response of y_1 . The change in the steady-state value of y_1 is equivalent to the steady state gain between y_1 and m_1 :

$$\Delta y_{1m} = k_1$$

Experiment 2: Unit step change in m_1 with Loop 2 closed. This time, Loop 2 is closed and the controller g_{c2} manipulate m_2 to reject upsets occurring in y_2 due to change in m_1 . The following happens to the process due to the change in m_1 :

1. y_1 changes because of g_{11} , but so does y_2 and this is because of interactions via the g_{12} element.
2. Under feedback control, Loop 2 wards off this interaction effect on y_2 by manipulating m_2 until y_2 is restored to its initial state.
3. The change in m_2 affects y_1 .

Thus changes observed in y_1 are from two different sources:

- (1) The direct influence of m_1 on y_1 (Δy_{1m})
- (2) The indirect influence by the retaliatory action of controller in Loop 2 while it is rejecting the effect of m_1 on y_2 , which is Δy_{1r} .

After steady-state has been achieved, there will be a net change observed in y_1 given by:

$$\Delta y_1^* = \Delta y_{1m} + \Delta y_{1r}$$

It can be shown that

$$\Delta y_1^* = k_{11} \left(\frac{k_{12}k_{21}}{k_{11}k_{22}} \right) = k_{11}^*$$

where k_{11}^* is the steady state gain between m_1 and y_1 obtained in the second experiment, and k_{ij} is the open-loop steady-state gain between the input i and output j . The ratio of the two gains can be used as a good measure of how well the process can be controlled if m_1 is used to control y_1 :

$$\lambda_{11} = \frac{\Delta y_1 m}{\Delta y_1^*}$$

The quantity λ_{11} is known as the relative gain between the output y_1 and the input m_1 . The relative gain provides a measure of the influence of process interaction when m_1 is used to control y_1 . Similar experiments can be performed to determine the relative gain quantities between the controlled outputs and the possible inputs. Although the example introduced this quantity λ_{ij} , is for a 2×2 system, it can be generalized to any other multivariable system of arbitrary dimension. The relative gain, λ_{ij} , between output y_i and input m_j can be defined in general as:

$$\lambda_{ij} = \frac{\left(\frac{\partial y_i}{\partial m_j} \right)_{\text{all loops open}}}{\left(\frac{\partial y_i}{\partial m_j} \right)_{\text{all loops closed except the } m_j \text{ loop}}} = \left(\frac{\text{open loop gain}}{\text{closed loop gain}} \right)_{\text{for loop } i \text{ under control of } m_j}$$

Calculation of the relative gains for all combinations of input/output pairs in a multivariable system leads to the following matrix known as the relative gain array (RGA) or the Bristol Array:

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{1n} \\ \lambda_{21} & \lambda_{22} & \lambda_{2n} \\ \lambda_{n1} & \lambda_{n2} & \lambda_{nn} \end{bmatrix}$$

The most important properties of the RGA are as follows:

1. The elements of the RGA across any row, or down any column sum up to 1.
2. λ_{ij} is dimensionless.
3. the value of λ_{ij} is a measure of the steady-state interaction expected in the i th loop of the multivariable system if its output y_i is paired with m_j . In particular, $\lambda_{ij} = 1$ implies that m_j affects y_i without interacting with, and/or eliciting interactions from the other control loops. While $\lambda_{ij} = 0$ means that m_j has absolutely no effect on y_i .
4. When λ_{ij} is negative, it indicates that changing m_j in the closed loop situation has the opposite effect on y_i compared to its effect on y_i when other loops are open. Such input/output pairing are potentially unstable and should be avoided.

4.2 Calculating the RGA

Depending on the availability of the process model, RGA can be calculated using the following methods:

1. Process model is available:

The transfer function matrix G , of the process can be derived based on the process model and is defined as follows:

$$Y = GM$$

where Y is a vector of output variables, M is a vector of the input variables. Let the steady-state gain matrix of the process be defined as follows

$$K = G_{ss}$$

where “ss” indicates steady-state. Then, let R be the transpose of the inverse of the steady state gain matrix:

$$R = (K^{-1})^T$$

The elements of the RGA can be obtained as follows:

$$\lambda_{ij} = k_{ij}r_{ij}$$

where k_{ij} and r_{ij} are the elements of the K and R matrices respectively.

2. Process model is not available:

When a process model is not available, it is still possible to obtain the RGA from experimental process data. One may adopt either of the following two approaches:

- a. Determine the steady state gain matrix K , by implementing step change in the process outputs one at a time and observing the ultimate change in each of the process outputs. The steady state gain between the i th output and the j th input can be found using:

$$k_{ij} = \frac{\Delta y_i}{\Delta m_j}$$

Once the gain matrix is known we can easily obtain the RGA.

- b. It is possible to determine each element of the RGA directly from experiment. Each RGA element can be determined by performing two experiments. The first experiment determines the open-loop steady-state gains by measuring the response of y_i to input m_j when all the other loops are opened. In the second experiment, all other loops are closed using PI controllers to ensure that there

will be no steady state offset and the response of y_i to a change in input m_j is predetermined. By definition the ratio of these gains gives us the desired relative gain elements.

4.3 Loop pairing using the RGA

Now we will consider how the RGA may be used as a guide for selection of input/output pairs that lead to minimum interaction among control loops. The interpretation of the values of the RGA can be classified according to the following categories:

1. $\lambda_{ij} = 1$, indicates that open loop gain between y_i and m_j is identical to the closed-loop gain. Loop i will not be subject to retaliatory actions from other control loops when they are closed. Thus, m_j can control y_i without interference from other control loop. **Pairing recommendation:** Pairing y_i and m_j will therefore be ideal.
2. $\lambda_{ij} = 0$, indicates that open-loop gain between y_i and m_j is zero. This means m_j has no direct influence on y_i . **Pairing recommendation:** Do not pair y_i with m_j . Pairing m_j with some other outputs will however be advantageous, since at least y_i will be immune to interaction from this loop.
3. $0 < \lambda_{ij} < 1$, indicating the open-loop gain between y_i and m_j is smaller than the closed-loop gain. Since the closed-loop gain is the sum of the open-loop gain and the retaliatory effect from the other loops, the loops are definitely interacting; and they do so in such way that the retaliatory effect from the other loops is in the same direction as the main effect of m_j on y_i . Thus, the loop interactions assist m_j in controlling y_i . The extent of the assistance from the other loops is indicated by how close λ_{ij} is to 0.5. When $\lambda_{ij} = 0.5$, the main effect of m_j on y_i is identical to the retaliatory effect it provokes from other loops but complementary. When $\lambda_{ij} > 0.5$ and < 1 , this retaliatory effect from other interacting loops is lower than the main effect of m_j on y_i . While, if $0 < \lambda_{ij} < 0.5$, then the retaliatory effect is more substantial than the main effect. **Pairing recommendation:** if possible avoid pairing y_i with m_j whenever $\lambda_{ij} < 0.5$.
4. $\lambda_{ij} > 1$, indicating that the open-loop gain between y_i and m_j is larger than the closed-loop gain. The loops interact, and the retaliatory effect from the other loops acts in opposition to the main effect of m_j on y_i , thus reducing the loop gain when the other loops are closed. However, the main effect is still dominant,

otherwise λ_{ij} will be negative. For large values of λ_{ij} , the controller gain for loop i will have to be chosen much larger than when all the other loops are open. This could cause loop i to become unstable when the other loops are open. **Pairing recommendation:** The higher the value of λ_{ij} , the greater the opposition m_j experience from the other control loops in trying to control y_i . Thus, where possible, do not pair m_j with y_i if λ_{ij} takes a very high value.

5. $\lambda_{ij} < 0$, indicating that open-loop and closed-loop gains between y_i and m_j have opposite signs. The loops interact, and the retaliatory effect from the other loops is not only in opposition to the main effect of m_j on y_i , it is also the more dominant of the two effects. This is a potentially dangerous situation because opening the other loops may cause loop i to become unstable. **Pairing recommendation :** Because the retaliatory effect opposes to the main effect, and it is dominant, avoid pairing m_j with y_i .

The foregoing discussion leads to the following rule:

RGA RULE A: *pair input and output variables that have positive RGA elements and closets to one.*

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Even though pairing Rule A is usually sufficient in most cases; it does not consider the stability of the resulting control structure. Therefore, it is necessary to check the stability of the resulted control structure. This can be according to the Niederlinsky theorem.

Consider the $n \times n$ multivariable system whose input and output variables have been paired as follows: $y_1 \rightarrow m_1, y_2 \rightarrow m_2, \dots, y_n \rightarrow m_n$, resulting in a transfer function model of the form:

$$Y = GM$$

In this model, each element of G , g_{ij} , is rational and open loop stable. Furthermore, assume there are no individual feedback controllers with integral action and each controller is stable when the other $n-1$ loops are open. When all loops are closed, the system will be unstable for all possible values of controller parameters (Structurally monatomic unstable), if the Niederlinsky index N defined in the following equation is negative.

$$N = \left[\frac{|G|}{\prod_{i=1}^n g_{ii}} \right]_{SS}$$

this leads to the second rule for input/output pairing:

RULE B: *any pairing is unacceptable if it leads to a control system for which the Niederlinsky index is negative.*

5. RGA example: conventional distillation column

In this section, RGA analysis is used to find the appropriate pairing for a conventional distillation column (Figure 9). There are typically two control schemes for distillation columns: single and dual composition control. The single composition control scheme maintains the composition of one of the products at a desired value, whereas in dual control the compositions of both products are regulated.

If column pressure is being controlled (using coolant flow rate in the condenser), then the following variables can be used as manipulating variables:

1. Reboiler duty (Q_R)
2. Reflux flow (L)
3. Distillate flow (D)
4. Bottom product flow (B)

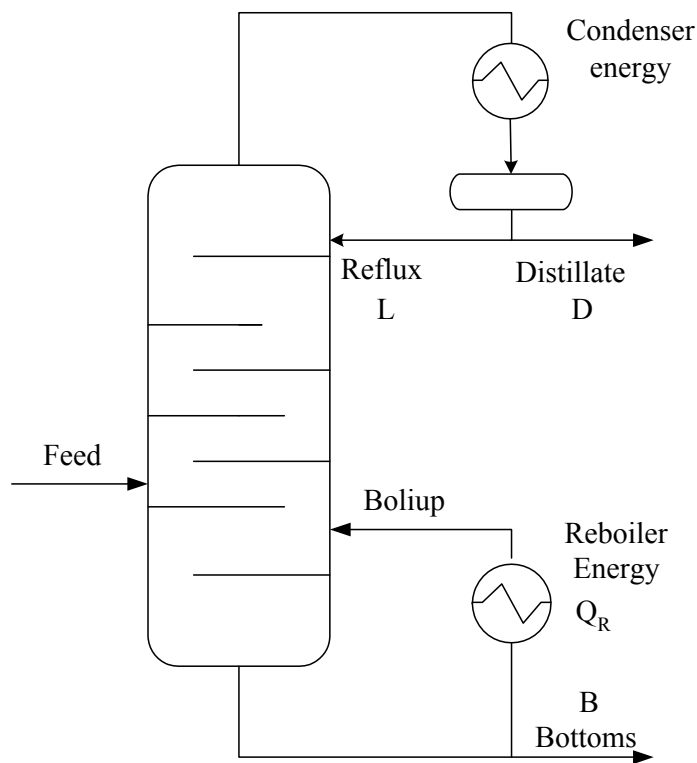


Figure 9: Distillation column example

The reasons why feed flow rate and reflux ratio are not considered as manipulating variables are as follows:

- The feed stream of the column is usually downstream of other units. Thus, its characteristics are usually set based on the operating condition of upstream units.

- Using reflux ratio as one of the manipulating variables results in a shock to the column whenever the distillate flow rate changes.

The variables usually considered as the process outputs for a distillation column are liquid levels at the base of the column and reflux drum and product compositions in dual composition control. Since there are four inputs that can be used to control four outputs, there are four different combinations. These combinations are shown in Table (1).

A preliminary screening of these 24 alternatives based on the dynamic response of the manipulated variable to the measured variable results in the three alternatives; 4, 10 and 18, which are shown in bold face in Table (1). The reasons for discarding other pairings are as follows:

- Schemes 1, 3, 5, 7, 9, 11, 13, 15, 19, 20, 23 and 24 are discarded since they involve control of base level by reflux flow or distillate flow.
- Schemes 6, 8, 14 and 19 are discarded since they involve manipulating flow rate of bottom product or reboiler heat to control the liquid level in the reflux drum.
- Schemes 21 and 22 are discarded since they do not regulate the material balance.

Table 1 Pairings in dual composition control

Case	Reflux drum	Column base	Top comp	Bottom comp.
1	D	L	B	Q _R
2	D	Q _R	B	L
3	L	D	B	Q _R
4	L	B	D	Q_R
5	B	L	D	OR
6	B	Q _R	D	L
7	Q _R	D	B	L
8	Q _R	B	D	L
9	D	L	Q _R	B
10	D	Q_R	L	B
11	L	D	Q _R	B
12	L	B	Q _R	D
13	B	L	Q _R	D
14	B	Q _R	L	D
15	Q _R	D	L	B
16	Q _R	B	L	D
17	D	B	Q _R	L
18	D	B	L	Q_R
19	B	D	L	Q _R
20	B	D	Q _R	L
21	L	Q _R	D	B
22	L	Q _R	B	D
23	Q _R	L	B	D
24	Q _R	L	D	B

- Schemes 2, 12 and 17 are discarded since each involves the control of one (or both)

product composition(s) at the end of the column using a manipulated variable at the other end of the column.

6. Base case steady state solution

The best pairing among these three alternatives, 4, 10, and 18, can be found through RGA analysis for a water-ethanol distillation column. Consider a conventional distillation column used in the separation of a water-ethanol mixture. A process simulation package such as HYSIS can be used to determine the necessary gains from a steady-state solution for RGA analysis and the Niederlinski Index to determine the appropriate pairings for dual composition control. The condenser and reboiler levels will be assumed to be under perfect control. For this type of system we will use the NRTL activity model with the ideal gas vapor model. The column is fed by the stream shown in Table 2.

Table 2 Characteristics of the column feed

Conditions and composition	
Temperature, °C	20.0
Pressure, kPa	101.3
Comp Molar Flow of Water, kmol/h	60.00
Comp Molar Flow of Ethanol, kmol/h	40.00

The distillation column has 20 stages and a total condenser. A steady-state solution for the distillation column can be found using the information in Table 3.

Table 3 Distillation column data

Column characteristics		Column pressure	
No. of stages	20	Condenser pressure, kPa	95
Feed stage	10	Condenser ΔP	0
Condenser type	Total	Reboiler pressure, kPa	105

Table 4 Distillation column specifications for base case steady-state

Specification	Value
Reflux ratio	2.0
Distillate rate, kmol/h	30

The steady-state solutions for the column yield the following results:

Table 5 Base case steady-state solution

Mole fraction of ethanol in distillate	0.8165
Mole fraction of ethanol in bottoms	0.2214
Reboiler duty, kJ/h	4.2×10^6

5.1 RGA Calculation

For this exercise, the steady-state values for the compositions will be considered to be the desired set points for the controllers. The best control pairings must be determined for maintaining the distillate and bottoms product compositions. The RGA will be calculated for each pairing of the three possible pairings. Cases 4, 10, and 18 from Table 1.

5.2 Pairing comparison for cases 4, 10, and 18

At this point, the steady-state gain between the distillate flow rate and the distillate composition, k_{11} , will be calculated using the steady-state distillation column model. Perform a step input in the distillate flow rate from 30 to 40 kmol/h. Change one of the column specifications from reflux ratio to reboiler duty, specifying a reboiler duty equal to the base case steady-state solution of 4.2×10^6 kJ/h. The new specifications for the column should be the same as those given in Table 6.

Table 6 Specifications for case 4 open loop

Specification	Value
Reboiler duty, kJ/h	4.2×10^6
Distillate rate, kmol/h	30

Run the column and determine the new mole fractions for ethanol in the distillate and bottoms. The results should be very close to those shown in Table 7.

Table 7 Steady-state solution for case 4 open loop

Mole fraction of ethanol in distillate	0.7890
Mole fraction of ethanol in bottoms	0.1407
Reboiler duty, kJ/h	4.2×10^6

The open loop gain is then calculated as follows:

$$k_{11} = \frac{\Delta x_D}{\Delta D} = \left(\frac{0.7890 - 0.8165}{40 - 30} \right) = -2.75 \times 10^{-3}$$

The closed loop gain, k_{11}^* can also be calculated from the steady-state solution. One can close the bottoms composition control loop by making the desired bottoms composition a steady-state specification. The closed loop specifications are shown in Table 8.

Table 8 Specifications for case 4 closed loop

Specification	value
Mole fraction of ethanol in bottoms	0.2214
Distillate rate, kmol/h	30

Run the column and determine the new mole fractions for ethanol in the distillate and bottoms. The results should be very close to those shown in Table 9.

Table 9 Steady-state solution for case 4 closed loop

Mole fraction of ethanol in distillate	0.6680
Mole fraction of ethanol in bottoms	0.2214
Reboiler duty, kJ/h	2.6×10^6

Now, the closed loop gain is calculated as follows:

$$k_{11}^* = \frac{\Delta x_D}{\Delta D} = \left(\frac{0.6680 - 0.8165}{40 - 30} \right) = -1.48 \times 10^{-3}$$

At this point, the RGA matrix can be calculated because this is a 2 x 2 system.

$$\lambda_1 = \frac{k_{11}}{k_{11}^*} = \left(\frac{-2.75 \times 10^{-3}}{-1.48 \times 10^{-3}} \right) = 0.185$$

The RGA matrix is then:

$$\begin{bmatrix} \lambda_{11} & 1 - \lambda_{11} \\ 1 - \lambda_{11} & \lambda_{11} \end{bmatrix} = \begin{bmatrix} 0.185 & 0.815 \\ 0.815 & 0.185 \end{bmatrix}$$

The step changes to calculate λ_{11} for Cases 10 and 18 are shown in Table 10. The resulting RGA matrix for Case 10 is:

$$\begin{bmatrix} \lambda_{11} & 1 - \lambda_{11} \\ 1 - \lambda_{11} & \lambda_{11} \end{bmatrix} = \begin{bmatrix} 0.985 & 0.015 \\ 0.015 & 0.985 \end{bmatrix}$$

Table 10 Open and closed loop results with the corresponding relative gain

Case	Steady state specifications	New value	New distillate X_{EtOH}	New distillate x_{EtOH}	Steady state gain	Relative gain λ_{11}
10 Open loop	L = 60	L = 70				0.9487
	B = 70	L = 70	0.8239	0.2180	$k_{11} = 7.4 \times 10^{-4}$	
Closed loop	$Bx_{EtOH} = 0.2214$	L = 70	0.8243	0.2214	$k_{11}^* = 7.4 \times 10^{-4}$	
	18 Open loop	L = 60	L = 70			2.8590
	Reboiler duty = 4.2×10^6		0.8388	0.2880	$k_{11} = 2.3 \times 10^{-3}$	
Closed loop	$Bx_{EtOH} = 0.2214$	L = 70	0.8243	0.2214	$k_{11}^* = 7.8 \times 10^{-4}$	

And for Case 18 is:

$$\begin{bmatrix} \lambda_{11} & 1-\lambda_{11} \\ 1-\lambda_{11} & \lambda_{11} \end{bmatrix} = \begin{bmatrix} 2.859 & -1.859 \\ -1.859 & 2.859 \end{bmatrix}$$

Using the three RGA matrices calculated from the steady-state model of distillation column, it can be concluded that Case 10 would be the best pair of measured variables to manipulated variables. The RGA matrix associated with Case 10 has elements that approach unity, indicating very little interacts.

The Niederlinski Index can be calculated from the full steady-state gain matrix. Using the steady-state model of the distillation column, the remaining elements for the steady-state gain matrix can be calculated for Case 10. The resulting matrix is:

$$K = \begin{bmatrix} 7.4 \times 10^{-4} & -3.4 \times 10^{-4} \\ 1.7 \times 10^{-3} & 7.0 \times 10^{-3} \end{bmatrix}$$

The Niederlinski Index can then be calculated from:

$$N = \frac{|K|}{\prod_{i=1}^n k_{ii}} = 1.11$$

Because the Niederlinski Index is not negative, the Niederlinski Index indicates that the system is stable. However, as mentioned earlier, a positive index value for higher order systems would indicate only that the system is not definitely unstable. The positive index value does not indicate stability for higher order systems: the system may or may not be unstable. Therefore one should also test the selected control scheme extensively via dynamic simulation before adoption.

Note that the workshop tutorial "Tut_poly4" provides an exercise on the calculation of the RGA.

References

- Shinsky, F., *Process Control Systems*, McGraw Hill, New York, 1988.
- Luyben, M. and Luyben, W., *Essentials of Process Control*, McGraw Hill, New York, 1997.
- Ogunnaike, B. and Ray, W., *Process Dynamics, Modeling and Control*, Oxford University Press, UK, 1994.
- Marlin, T., *Process Control*, McGraw Hill, New York, 1995.
- Clair, D., *Controller Tuning and Control Loop Performance*, Straight-line Control Co., USA, 1993.
- Svreck, W., Mahoney, D. and Young, B., *A Real Time Approach to Process Control*, Wiley & sons, New York, 2000.
- Astrom, K. and Hagglund, T., *PID Controllers: Theory, Design and Tuning*, ISA, NC, USA, 1995.
- Friedmann, P., and Stoltenberg, T., *Continuous Process Control*, ISA, NC, USA, 1996.
- Murrill, P., *Application Concept of Process Control*, ISA, NC, USA, 1988.
- Murrill, P., *Fundamentals of Process Control Theory*, ISA, NC, USA, 1991.
- Luyben, W., *Process Modeling, Simulation and Control for Chemical Engineers*, McGraw Hill, New York, 1990.
- Smith, C. and Corripio, A., *Principles and Practice of Automatic Process Control*, Wiley & sons, New York, 1997.
- Seborg, D., Edgar, T., and Mellichamp, D., *Process Dynamics and Control*, Wiley & sons, New York, 1989.
- Stephanopoulos, G., *Chemical Process Control: An Introduction to Theory and Practice*, Prentice Hall, 1984.
- Deshpande, P., *Multivariable Process Control*, ISA, NC, USA, 1989.

Skogestad, S. and Postlethwaite, I., *Multivariable Process Control, Analysis and Design*, Wiley & sons, New York, 1996.

Luyben, M., Luyben, W., and Tyreus, B., *Plant Wide Process Control*, McGraw Hill, NEW York, 1999.

Erickson, K. and Hedrick, J., *Plant Wide Process Control*, Wiley & sons, New York, 1999.