

PROBLEMS

Problem 1.1

Find the condition number of the following functions:

$$\ln(x), \cos^{-1}(x), \quad x^{-2}e^x, (x-a)^b$$

Problem 1.2

Consider the following sequence:

$$x_0 = 1, x_1 = \frac{1}{3}$$
$$x_{n+1} = \frac{13}{3}x_n - \frac{4}{3}x_{n-1} \quad n \geq 1$$

1. Show that the algorithm converge to: $x_n = \frac{1}{3^n}$
2. Compute the first 20 numbers of these iterations using the algorithm. Compare with the true values.
3. Is the algorithm stable?. Explain.

Problem 1.3

Consider the problem of finding the roots of a function f , i.e.

$$f(x) = 0 \tag{P1.1}$$

Our objective is to examine the conditioning of this problem. Suppose that r is a simple root of f , i.e.

$$f(r) = 0 \text{ and } f'(r) \neq 0 \tag{P1.2}$$

Let a small perturbation ε be introduced, so that f becomes \tilde{f} such that,

$$\tilde{f}(x) = f(x) + \varepsilon g(x) \quad (\text{P1.3})$$

As a result the root r changes by h :

$$\tilde{r} = r + h \quad \text{and} \quad \tilde{f}(r + h) = 0 \quad (\text{P1.4})$$

- a. By using a second order Taylor expansion of Equation (P1.4) around h , show that:

$$h \approx -\varepsilon \frac{g(r)}{f'(r) + \varepsilon g'(r)} \approx -\varepsilon \frac{g(r)}{f'(r)} \quad (\text{P1.5})$$

- b. As an application consider the problem of finding the roots of the polynomial:

$$f(x) = (x-1)(x-2)(x-3)\cdots(x-20)$$

Consider a perturbation of order ε in the first coefficient (x^{20}) of the polynomial.

- Identify $g(x)$ in Eq (P1.3)
- Using the relation (P1.5) how is the root $r = 20$ affected by a change of order 10^{-5} in the coefficient?
- State your conclusions concerning the conditioning of the problem.

