PROBLEMS

Problem 5-1

A diffusion process in a medium bounded by the planes $x = 0$ and $x = 1$ which are maintained at constant concentrations $y_1 = 0.1$ and $y_2 = 0.3$, respectively is governed by the following BVP:

$$\ddot{y}(x) = 0$$

Provided that the diffusion coefficient is constant. Solve this BVP analytically and numerically. Show that the concentration $y(x)$ changes linearly from y_1 to y_2 through the medium .

Problem 5-2

Consider a catalyst particle surrounded by a stagnant film through which component A has to diffuse in order to arrive at the catalyst surface. At the catalyst surface we presume that the reaction $2A \rightarrow B$ occurs instantaneously and that the product B then diffuses back out through the gas film to the main bulk gas stream composed of A and B.

The local rate of conversion from A is expressed as follows:

$$\frac{d}{dz}\left(\frac{1}{1-0.5x}\frac{dx}{dz}\right) = 0$$

Solve this problem assuming that the effective gas film and the main gas stream mole fractions are given as follows:

$$x(0) = 0.3, \qquad x(0.1) = 0.$$

Problem 5-3

The equation for the steady state heat conduction through a large flat slab with a temperature-dependent heat conductivity ($\lambda = \lambda_0 + \alpha T$) is given below. The solution describes the temperature $T(x)$ as a function of x through the slab.

$$\frac{d}{dx}\left((\lambda_0 + \alpha T)\frac{dT}{dx}\right) = 0, \qquad T(0) = 5,\ \frac{dT}{dx}|_{x=0} = 1$$

It is more common to specify the temperature at two points on the slab and solve the differential equations to obtain $T(x)$ for the remaining x values. The conditions then might be

$$T(0) = 0,\ T(10) = 100$$

With these conditions the equation becomes a boundary value problem. Obtain the solution of this problem given that $\lambda_0 = 0.1$ and $\alpha = 0.01$.

Problem 5-4

Suppose that a gas diffuses into a liquid in a long, narrow pipe. Suppose that this process takes place for such a long period of time the concentration $y(x)$ of the gas in the pipe depends on the distance x from some initial point and is independent of time. The gas is assumed to be consumed by chemical reaction in the liquid phase and the the amount of gas that disappears by this reaction is of half-order kinetic form ($r = ky^{0.5}$). Then $y(x)$ satisfied the BVP

$$\ddot{y} - \frac{k}{D} y^{0.5} = 0$$

$$y(0) = 0.1, \qquad y(1) = 0$$

Solve this problem for the cases $k/D = 0.1, 1, 10$.

Problem 5-5

The stagnant film model for mass transfer with single chemical reaction is based on the following. The fluid can be divided into two zones; a stagnant film of thickness δ near the interface and a well-mixed bulk behind it in which no concentrations gradients occur. The mass transfer process is a stationary process (steady state). The homogenous chemical reaction takes place only in the film. The model equation which describes the change of the concentration C_A with the distant x is

$$D\frac{d^2C_A}{dx^2}+r_A=0$$

Obtain the concentration profiles for the following, taking $k/D = 0.1$:

(a) First order chemical reaction $r_A = k\,C_A$ with

$$C_A(0) = C_{A1},\ C_A(1) = C_{A2}$$

(b) First order chemical reaction $r_A = kC_A$ with

$$C_A(0)=C_{A1}, \qquad \frac{dC_A(\delta)}{dx}=0$$

(c) Second order chemical reaction $r_A = k\,CA^2$ with

$$C_A(0)=C_{A1}, \qquad \frac{dC_A(\delta)}{dx}=0$$

Problem 5-6

Suppose that component A in the gas phase reacts by a single reaction (order n) in the presence of a solid catalyst with an internal pore structure .The following differential equation describe the material balance on reactant A in the catalyst pore:

$$y''-\phi^2y^n=0$$

where y represents the ratio of the concentration of A inside the pore to that in the gas phase and ϕ is a physical parameter commonly called Thiele modulus. For the boundary conditions:

$$y(0)=1, \qquad \dot{y}(1)=0$$

Obtain the concentration profiles for the first order reaction and for the cases: ϕ = 0.5,1,2,5

Problem 5.7

Consider the annular flow of a fluid confined between two cylinders of finite lengths as shown by figure below. The motion of the fluid is induced by the motion of the internal cylinder. It is assumed that there is no leakage flow from the system. Over most of the length L, the flow is strictly laminar flow ($v = (v_z, 0)$). The momentum balance equation for this system is:

$$-\frac{1}{r}\frac{d(r\tau_{rz})}{dr} = \frac{\Delta P}{L}$$

In this problem we assume there is a finite pressure drop across the ends of the annular region. Newtonian behavior is assumed, so we write the shear stress as follows:

$$\tau = -\mu\frac{dv_z}{dr}$$

We take no-slip condition on v_z at the surfaces:

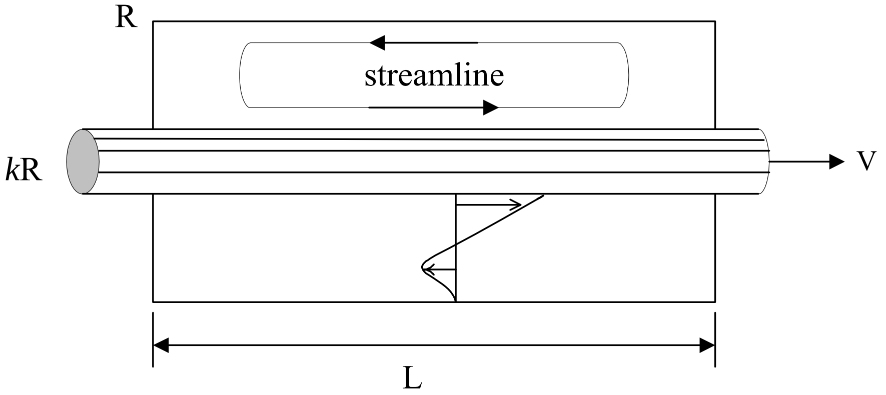
$$v_z = V \text{ at } r = \kappa R$$
$$v_z = 0 \text{ at } r = R$$

Solve this boundary value problem to obtain the velocity profile given that

$$\Delta P = -\frac{4\mu LV}{R^2}\frac{1-\kappa+2\kappa^2\ln(\kappa)}{(1-\kappa^4)\ln(\kappa)+(1-\kappa^2)^2}$$

and

$R = 0.01$ m, $L = 0.1$ m, $\kappa = 0.5$, $V = 3$ m/s, $\mu = 10$ Pa.s

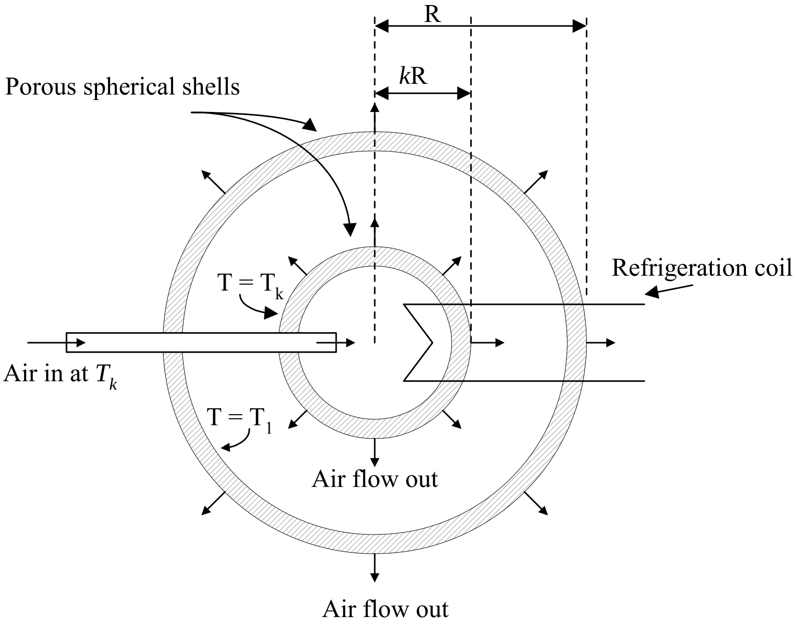


Problem 5.8

Consider two concentric porous spherical shells of radii κR and R as shown in Figure below. The inner surface of the outer one is at $T = T_1$ and the outer surface of the inner tube is to be maintained at a lower temperature T_κ. Dry air at temperature T_κ is blown outward radially from the inner shell into the intervening space and out through the outer shell. The differential equation that describes the temperature distribution $T(r)$ in the space between the two shells has the form:

$$\frac{dT}{dr} = \frac{1}{R_o}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)$$

where R_0 is $w_r c_p/4\pi k$, w_r is the radial mass flow rate of the air, c_p is the heat capacity of the air and k is the thermal conductivity. Solve this BVP using $T_1 = 60^o$C, $T_\kappa = 30^o$C, $R = 0.01$m, $\kappa = 0.5$, $R_0 = 1.0$.



Problem 5.9

In a heat exchanger shown in Figure (below), fluid *A* enters and leaves at the same end of the exchanger, whereas the shell fluid *B* always move in the same direction; there is thus both parallel flow and counterflow in the same apparatus. Under simplified assumptions the temperature distribution of the shell fluid can be shown to be

$$\frac{d^2\theta}{d\alpha^2}+R\frac{d\theta}{d\alpha}-\frac{\theta}{4}=0$$

where

$$\theta=\frac{T_B-T_{B2}}{T_{B1}-T_{B2}}$$

$$d\alpha=\frac{U}{w_A Cp_A}dA$$

$$R=\frac{w_A Cp_A}{w_B Cp_B}$$

Solve the equation with the boundary conditions:

$$\theta(0)=1, \qquad \theta(1)=0$$

using R = 1,2,10.