A NEW APPROACH FOR THE ANALYSIS OF THREE PHASE SELF-EXCITED INDUCTION GENERATORS

K.F. Ali A.I. Alolah

EE Dept., College of Eng., King Saud University, P.O. Box 800, Riyadh 11421, Saudi Arabia

***Abstract*-- This paper presents a new approach for the steady state analysis of three phase self-excited induction generator. The problem is formulated to obtain the performance of the generator by a simple iteration scheme with the help of the machine equivalent circuit. The proposed method is essentially suitable for education as it avoids the long and tedious analytical derivation of several equations, as the case of the most of the reported methods of analysis.**

List of Symbol

F, u	p.u frequency and speed, respectively
C, X_c	per phase value of excitation capacitance(μ F) and its p.u reactance (at base frequency), respectively
R_s, R_r, R_L	p.u stator, rotor and load resistances, respectively
X_s, X_r, X_L	p.u stator leakage, rotor leakage and load reactances at base frequency, respectively
X_m, X_o	p.u saturated and unsaturated magnetizing reactances at base frequency, respectively
E_g, V	Air gap and terminal voltages, respectively
I_c, I_l, I_s, I_r	p.u. per phase excitation capacitance, load, stator and rotor currents, respectively
V_b, I_b, Z_b	Base voltage, current and impedance, respectively
f_b, N_b	Base frequency and speed in *Hz* and *rpm*, respectively

1. Introduction

When the rotor of an induction machine is driven at a suitable speed, the rotor residual magnetism induces a small emf in the stator windings. This emf can be made to build up if a stator current at a leading power factor is supplied by a suitable means. This can be achieved by connecting a capacitor bank of sufficient value, across the stator terminals. In this case the armature reaction flux assists the original flux causing the emf to continue to build up until an equilibrium is reached due to the magnetic saturation of the machine. Under these conditions the machine is a self-excited generator which has several advantages such as low unit and running cost, free from current collecting problems, ruggedness and self protection against large overloads and short circuit faults. The frequency and value of the voltage generated by these generators are highly dependent on speed, excitation capacitance and load [1, 2]. This paper deals with a new approach for the analysis of a three phases self excited induction generator. This approach can be summarized as given below.

2. Method of Analysis

The per phase equivalent circuit of the induction generator under inductive load is shown in Fig.1. From this circuit and with an assumed value for X_m:

$$Z_{eq} - jX_c/F^2 = 0 \qquad (1)$$

where $Z_{eq} = Z_L // [Z_s+(jX_m//Z_r)]$.
Eqn.1 implies that:

$$Real\,(Z_{eq}) = 0 \qquad (2)$$

$$Im(Z_{eq}) - jX_c/F^2 = 0 \qquad (3)$$

Eqn.2 is a 4^{ht} order polynomial in F which has two real roots and another two complex roots. The realistic root is the real root, which is very close to u. In this paper a value of X_c can be found from eqn.3 by substituting for the solution of F from eqn.2 and the assumed value of X_m. E_g is obtained from the representation of the magnetization curve of the machine. Eqn.2 can be written as:

$$a_4F^4 + a_3F^3 + a_2F^2 + a_1F + a_0 = 0 \qquad (4)$$

The proposed approach to obtain the real roots of eqn.4 is as follows:
(i) Eqn.4 is rewritten as:

$$A_2F^2 + A_1F + A_0 = 0 \qquad (5)$$

where
$A_2=a_2+a_4F^2,\ A_1=a_1+a_3F^2$ *and* $A_0=a_0$

(ii) The value of u is assumed as an initial value for F, the values of the coefficients of eqn.5 are computed then eqn.5 is solved for F.

(iii) The positive real root, which is closer to u, is chosen. Update the value of F. Step (ii) is repeated until a convergent solution is reached.

(iv) The value of X_c is found from eqn.3.

Application of this approach to practical cases showed that the two roots are real and positive. Few number of iterations is usually required to achieve a convergent solution.
The open circuit load condition can be easily manipulated by taking the limit as $R_L \rightarrow \infty$ in eqn.4. Doing so eqn.4 reduces to a quadratic equation in F:

$$D_2F^2+D_1F+D_0=0 \qquad (6)$$

0-7803-5293-9/99 $10.00

Expressions for the different coefficients of eqns. 4 and 6 are given in the Appendix.

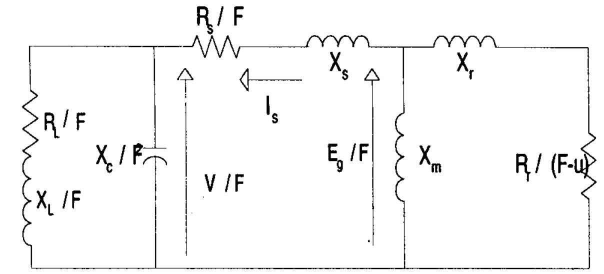

Fig.1 Equivalent circuit of self-excited induction generator under R-L load

The nonlinearly encountered due to the magnetic saturation of the machine is efficiently treated by the proposed new approach as follows:

The magnetization curve of an induction machine can be represented as

$$E_g / F = K_0 - K_1 X_m - K_2 X_m^2 \qquad (7)$$

The proposed approach described above, is modified to include the representation of eqn.7 as follows:

1. Initial values of F and X_c are taken from the results of the open circuit load condition.
2. Using the equivalent circuit of Fig.1, the air gap voltage E_g, is determined from an assumed value of the terminal voltage.
3. An estimate value for X_m is determined from eqn.7.
4. The steps described before are followed to obtain a new values for F and X_c.
5. The steps 2-4 are repeated to achieve a convergent values for F, X_c and X_m.

3. Results

To test the validity of the proposed approach it is applied to a laboratory machine of the following parameters:

R_s=25.6 Ω, R_r=14.72 Ω, X_s=X_r=22.38 Ω, X_o = 440.0 Ω
R=220.0 Ω, X=0.0
V_{rated}=220.0 V $\qquad$ I_{rated}=1.0 A
$E_g/F = -59.77 + 2.723\, X_m - 0.00559\, X_m^2$

For a per unit speed u = 1 with X_m = 300 Ω, the roots of 4th order polynomial as calculated with the help of MATLAB are:

$F_1 = 0.9236 \qquad F_2 = 0.7187$
$F_3, F_4 = 0.1789 \pm j\,2.2478$

The simple iterative approach described above is applied and the two roots of the quadratic equation, obtained after 18 iterations for a tolerance of 10^{-5}, are as follows:

$F_1 = 0.923611 \qquad F_2 = 0.508300$

The root F_1 is identical with that calculated above by MATLAB.

The capacitor value, as calculated from the imaginary part equals 15.995 µF.

When the magnetization curve is introduced in the solution, convergent values for F, X_m and C are obtained after few number of iterations as shown in the table. It is worthwhile to mention here that the two roots of the quadratic equation are obtained in this case after 10 iterations only.

Iterations	F, p.u.	X_m, Ω	C, μF
1	0.924625	314.5014	15.3242
2	0.923305	296.6362	16.1612
3	0.923402	297.6854	16.1088
4	0.923396	297.6220	16.1121
5	0.923397	297.6257	16.1118
6	0.923396	297.6255	16.1119
7	0.923396	297.6255	16.1119

For the machine used by the authors of reference [3], the present approach is applied for the condition of constant terminal voltage at a value equal to 1.0 p.u. under different resistive load conditions.

Variation of C, F and the efficiency η versus output power is shown in Fig. 2.

Variation of I_s, I_r and I_c against output power is shown in Fig. 3. The results of these two figures are in a good agreement with the results shown in Fig. 7a and Fig. 7b of reference [3]. It is worthwhile to mention here that the approach applied in reference [3] is based on Newton Raphson technique that needs, in addition to the iterative solution, several differentiations to develop the Jacobian matrix.

4. Conclusions

A new approach for the steady-state analysis of self-excited induction generators is introduced. The proposed method is simple and avoids the long and tedious analytical derivation of several equations as compared to the most of the reported methods of analysis. The results obtained ensure the validity and accuracy of the proposed approach.

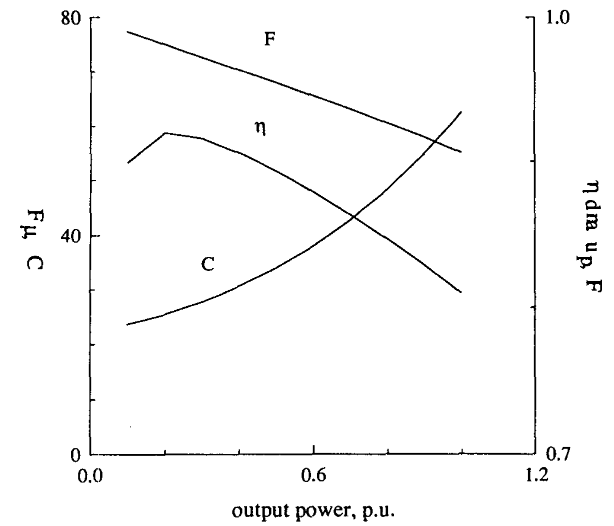

Fig. 2 Variation of C, F, and η versus output power for the Machine of Ref. [3]

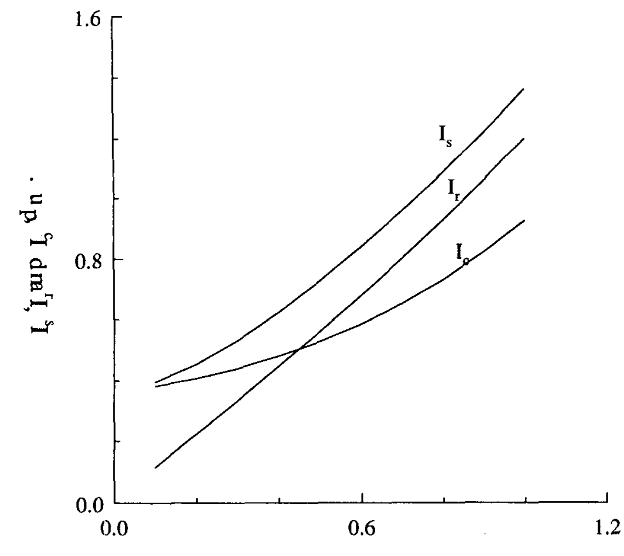

Fig. 3 Variation of I_s, I_r and I_c versus output power for the Machine of Ref. [3]

References

1. Malik, N. and Haque, S., "Steady Steady Analysis And Performance Of An Isolated Self-Excited Induction Generator", IEEE Trans. Vol-EC 1(3), 1986, pp.134-139.
2. Al-Jabri, A. and Alolah, A., "Limits On The Performance Of Three Phase Self-Excited Induction Generator", IEEE Trans. Vol-EC 5(2), 1990, pp.155-159.
3. Malik, N. and Al-Bahrani, A., "Infleunce of the terminal capacitor on the Performance Characteristics of a Self-Excited Induction Generator", IEE Proc., Pt.C, Vol. 137(2), 1990, pp.168-173.

Appendix

$$D_0 = R_s \left(R_r^2 + X_{22}^2 u^2\right)$$
$$D_1 = -u D_2 - u R_s X_{22}^2$$
$$D_2 = \left(R_s X_{22}^2 + R_r X_m^2\right)$$

Resistive Load Case

$$a_{0_R} = R_s R_T \left[R_r^2 + u^2 X_{22}^2\right]$$
$$a_{1_R} = -uR_T \left(R_r X_m^2 + R_s X_{22}^2\right) - uR_s \left(R_r X_m^2 + R_T X_{22}^2\right)$$
$$a_{2_R} = R_T \left(R_r X_m^2 + R_s X_{22}^2\right) + R_r \left(R_s X_m^2 + R_r X_{11}^2\right) + u^2 \left(X_s X_{22} + X_r X_m\right)^2$$
$$a_{3_R} = -2u\left(X_s X_{22} + X_r X_m\right)^2$$
$$a_{4_R} = \left(X_s X_{22} + X_r X_m\right)^2$$

Inductive Load Case

$$a_{0_X} = R_L a_{0_R}$$
$$a_{1_X} = R_L a_{1_R}$$
$$a_{2_X} = R_L a_{2_R} + X_L^2 R_s \left(R_r^2 - X_{22}^2 u\right)$$
$$a_{3_X} = R_L a_{3_R} - X_L^2 u R_r X_m^2$$
$$a_{4_X} = R_L a_{4_R} + X_L^2 R_r X_m^2$$

where

$X_{11} = X_s + X_m$

$X_{22} = X_r + X_m$

$R_T = R_s + R_L$