

Useful Finite Difference Approximations

Table A-1 First-Order Finite Difference Approximations

Difference	First Order Formula	
Forward Difference for First Derivative	$\frac{d}{dx}f(x_i) = \frac{f(x_{i+1}) - f(x_i)}{\Delta x}$	(A-1)
Backward Difference for First Derivative	$\frac{d}{dx}f(x_i) = \frac{f(x_i) - f(x_{i-1})}{\Delta x}$	(A-2)
Forward Difference for Second Derivative	$\frac{d^2}{dx^2}f(x_i) = \frac{f(x_i) - 2f(x_{i+1}) + f(x_{i+2}))}{\Delta x^2}$	(A-3)
Backward Difference for Second Derivative	$\frac{d^2}{dx^2}f(x_i) = \frac{f(x_i) - 2f(x_{i-1}) + f(x_{i-2}))}{\Delta x^2}$	(A-4)

Table A-2 Second-Order Finite Difference Approximations

Difference	Second Order Formula	
Forward Difference for First Derivative	$\frac{d}{dx}f(x_i) = \frac{-3f(x_i) + 4f(x_{i+1}) - f(x_{i+2}))}{2\Delta x}$	(A-5)
Central Difference for First Derivative	$\frac{d}{dx}f(x_i) = \frac{f(x_{i+1}) - f(x_{i-1}))}{2\Delta x}$	(A-6)
Backward Difference for First Derivative	$\frac{d}{dx}f(x_i) = \frac{3f(x_i) - 4f(x_{i-1}) + f(x_{i-2}))}{2\Delta x}$	(A-7)
Forward Difference for Second Derivative	$\frac{d^2}{dx^2}f(x_i) = \frac{2f(x_i) - 5f(x_{i+1}) + 4f(x_{i+2})) - f(x_{i+3}))}{\Delta x^2}$	(A-8)
Central Difference for Second Derivative	$\frac{d^2}{dx^2}f(x_i) = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1}))}{\Delta x^2}$	(A-9)
Backward Difference for Second Derivative	$\frac{d^2}{dx^2}f(x_i) = \frac{2f(x_i) - 5f(x_{i-1}) + 4f(x_{i-2})) - f(x_{i-3}))}{\Delta x^2}$	(A-10)