Example of using BVPFD subroutine

Example Statement

Consider the heat transfer problem shown in figure below, where a hot fluid flowing through the pipe at temperature T_h exchanging heat with a cold fluid at temperature T_c on the outside of the pipe. The energy balance equation around differential element inside the wall of the pipe gives:

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0$$

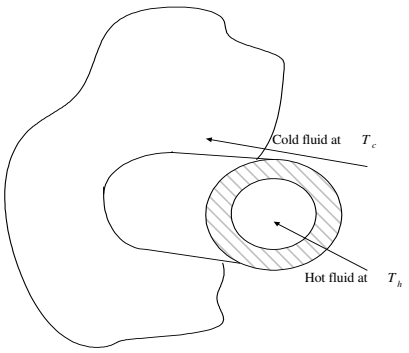
subject to boundary conditions,

$$r = r_i; \qquad T = T_h$$
$$r = r_o; \qquad T = T_c$$

The heat flux is given by the Fourrier's law: $q = -k\dfrac{dT}{dr}$

Using IMSL subroutine " " solve the temperature profile between $r = r_i$ *and* $r = r_o$ and compute also the flux for the following values:

$r_i = 0.05$, $r_o = 0.1$, $T_h = 200\ ^oC$, $T_c = 80\ ^oC$, $k = 41.8\ J/(s\ ^oC\ m)$



Fortran Program Listing

```
      INTEGER    LDYFIN, LDYINI, MXGRID, NEQNS, NINIT
      PARAMETER  (MXGRID=45, NEQNS=2, NINIT=10, LDYFIN=NEQNS,
     &           LDYINI=NEQNS)

      INTEGER    I, J, NCUPBC, NFINAL, NLEFT, NOUT
      REAL       ERREST(NEQNS), PISTEP, TFINAL(MXGRID), TINIT(NINIT),
     &           TLEFT, TOL, TRIGHT, YFINAL(LDYFIN,MXGRID),
     &           YINIT(LDYINI,NINIT)

      LOGICAL    LINEAR, PRINT
      INTRINSIC  FLOAT
      REAL       FLOAT

      EXTERNAL   BVPFD, SSET
      EXTERNAL   FCNBC, FCNEQN, FCNJAC
      REAL       FCNBC, FCNEQN, FCNJAC

      NOUT   = 6
      NLEFT  = 1
      NCUPBC = 0
      TOL    = 0.001
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```
  TLEFT  = 0.05
  TRIGHT = 0.1
  PISTEP = 0.0
  PRINT  = .FALSE.
  LINEAR = .TRUE.
  rk=   41.8

  DO 10  I=1, NINIT
     TINIT(I) = TLEFT + (I-1)*(TRIGHT-TLEFT)/FLOAT(NINIT-1)
10 CONTINUE

  DO 20  I=1, NINIT
     CALL SSET (NEQNS, 0.0, YINIT(1,I), 1)
20 CONTINUE

  CALL BVPFD (FCNEQN, FCNJAC, FCNBC, FCNEQN, FCNBC, NEQNS, NLEFT,
 &            NCUPBC, TLEFT, TRIGHT, PISTEP, TOL, NINIT, TINIT,
 &            YINIT, LDYINI, LINEAR, PRINT, MXGRID, NFINAL,
 &            TFINAL, YFINAL, LDYFIN, ERREST)

  WRITE (NOUT,97)
  WRITE (NOUT,98) (I,TFINAL(I),(YFINAL(J,I),J=1,NEQNS),
 &      rk*YFINAL(1,I),I=1,NFINAL)
  WRITE (NOUT,99) (ERREST(J),J=1,NEQNS)
97 FORMAT (4X, 'I', 7X, 'T', 14X, 'Y1', 13X, 'Y2', 13x, 'q')
98 FORMAT (I5, 4E15.6)
99 FORMAT (//,' Error estimates', 4X, 1P3E15.6)

  END

  SUBROUTINE FCNEQN (NEQNS, T, Y, P, DYDX)
  INTEGER    NEQNS
  REAL       T, P, Y(NEQNS), DYDX(NEQNS)

  DYDX(1) = Y(2)
  DYDX(2) = -Y(2)/T

  RETURN
  END

  SUBROUTINE FCNJAC (NEQNS, T, Y, P, DYPDY)
  INTEGER    NEQNS
  REAL       T, P, Y(NEQNS), DYPDY(NEQNS,NEQNS)

  DYPDY(1,1) = 0.0
  DYPDY(1,2) = 1.0
  DYPDY(2,1) = 0.0
  DYPDY(2,2) = -1.0/T

  RETURN
  END

  SUBROUTINE FCNBC (NEQNS, YLEFT, YRIGHT, P, F)
  INTEGER    NEQNS
  REAL       P, YLEFT(NEQNS), YRIGHT(NEQNS), F(NEQNS)

  F(1) = YLEFT(1) - 200.0
  F(2) = YRIGHT(1) - 80.0

  RETURN
  END
```