

Problem Illustrating The Transport of A Pollutant in A Stream

One Curie of $^{134}\text{Cesium-134}$ (^{134}C) is accidentally released into a small stream. The stream channel has an average width of 40 m and an average depth of 2 m. The average water flow in the stream is $40 \text{ m}^3/\text{s}$ and the stream channel drops 1 meter in elevation over a distance of 10 km. Assuming that the ^{134}Cs is evenly distributed across the stream channel, estimate the distribution of ^{134}Cs as a function of distance downstream (using a maximum distance of 30 km) at 1, 3, 6, and 12 hours. Also estimate the ^{134}C activity (concentration) at a distance of 10 km at 6 hours after the release. (^{134}Cs has a half-life of 2.07 years.)

Solution:

- (1) Calculate the average stream velocity in m/s.

cross-sectional area of stream channel = width * depth = $(40\text{m})(2\text{m}) = 80 \text{ m}^2$
 average velocity = $(40 \text{ m}^3/\text{s})/(80 \text{ m}^2) = 0.50 \text{ m/s}$

- (2) Calculate the rate constant, k , for ^{134}Cs .

$$\ln \frac{C}{C_0} = -kt$$

For a first-order reaction:

where C = the concentration (or activity of ^{134}C) at time t
 C_0 = the initial concentration (or activity) of ^{134}C
 k = the decay rate constant, and
 t = time.

At the half-life($t_{1/2}$), one-half of the original concentration remains.

Substitution of this into the equation above yields:

$$\ln \left(\frac{\frac{1}{2}C_0}{C_0} \right) = -kt_{1/2} \quad \text{or}$$

$$-\frac{\ln 0.5}{t_{1/2}} = -\frac{\ln 0.5}{2.05 \text{ yr}} = k = 0.338 \text{ yr}^{-1}$$

$$(0.338 \text{ yr}^{-1}) \left(\frac{\text{yr}}{365 \text{ d}} \right) \left(\frac{\text{d}}{24 \text{ hr}} \right) \left(\frac{\text{hr}}{60 \text{ min}} \right) \left(\frac{\text{min}}{60 \text{ s}} \right) = 1.07 \times 10^{-8} \text{ s}^{-1}$$

Thus, the decay rate constant for ^{134}Cs is $1.07 \times 10^{-8} \text{ s}^{-1}$.

(3) Calculate the longitudinal dispersion coefficient, E (also referred to as the coefficient of eddy diffusion).

$$\text{slope} = \frac{1 \text{ m}}{10000 \text{ m}} = 10^{-4}$$

$$u = \sqrt{gds} = \sqrt{(9.81 \text{ m/s}^2)(2 \text{ m})(10^{-4})} = 0.044 \text{ m/s}$$

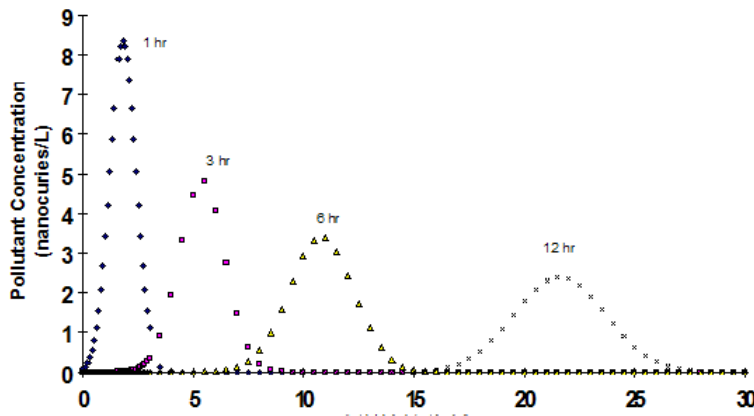
$$E = 0.011 \frac{v^2 w^2}{du} = 0.011 \frac{(0.50 \text{ m/s})^2 (40 \text{ m})^2}{(2 \text{ m})(0.044 \text{ m/s})} = 50 \text{ m}^2/\text{s}$$

(4) Arrange data in the proper units:

$M_0 = 1 \text{ curie} = 1 \times 10^6 \text{ mCi}$ (In the program this is entered as 1 Ci)
 $w = 40 \text{ m}$
 $d = 2 \text{ m}$
 $E = 50 \text{ m}^2/\text{s}$
 $t = \text{variable in seconds (s)}$
 $x = \text{variable in meters (m)}$
 $v = 0.50 \text{ m/s}$
 $k = 1.07 \times 10^{-8} \text{ s}^{-1}$

(5) Input data to program and obtain graph

**Example Problem: Longitudinal Concentration Profiles
as A Function of Time**



(6) Calculate $C(x,t)$ at 10 km and at 6 hr. ($x = 10,000 \text{ m}$ and $t = 6 \text{ hr} = 21,600 \text{ s}$)

$$\begin{aligned}
 C(x,t) &= \frac{M_0}{wd\sqrt{4\pi Et}} \exp\left(-\frac{(x-vt)^2}{4Et} - kt\right) \\
 &= \frac{1 \times 10^6 \text{ } \mu\text{Ci}}{(40 \text{ m})(2 \text{ m})\sqrt{4\pi(50 \text{ m}^2/\text{s})(21600 \text{ s})}} \exp\left(-\frac{(10000 \text{ m} - (0.50 \text{ m/s})(21600 \text{ s}))^2}{4(50 \text{ m}^2/\text{s})(21600 \text{ s})} - (1.07 \times 10^{-8} \text{ s}^{-1})(21600 \text{ s})\right) \\
 &= 3.39 \times 10^{-5} e^{-0.148} = 3.39 \times 0.862 \\
 &= 2.92 \times 10^{-5} \text{ Ci/m}^3 \\
 &= 2.92 \times 10^{-9} \text{ Ci/L} = 2.92 \text{ nCi/L}
 \end{aligned}$$

Options Helpful Info Navigation

Instantaneous Release Into A Stream

This section of EnviroLand allows the user to predict the concentration of a pollutant in a stream. The input of the pollutant is treated as an instantaneous input; for example, the immediate release and mixing of 50 gallons of acetone in a stream. The model used here is a one-dimensional advection-dispersion equation that can also account for the first-order degradation or removal.

Step 1:
Manually convert input data to metric units of:
Meters, Kilograms, or Curies

Step 2:
Enter or calculate the longitudinal dispersion coefficient, E:

☒ Enter the known Eddy dispersion coefficient:
E = 49.67 (in m^2/s)

☐ Calculate the Eddy dispersion coefficient:

Enter data:

☒ channel slope: $s = 0.0001$ (unitless) ☒ stream depth: $d = 2$ m

☒ water velocity: $v = 0.5$ m/s ☒ stream width: $w = 40$ m

☒ gravity constant: $g = 9.81$ m/s^2

[Click here to calculate Eddy Dispersion Coefficient \("E"\)](#)

E = 49.67 m^2/s Eq

Options

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Options Helpful Info Navigation

Step 3:
Calculate first-order rate constant:

☒ Enter half-life: (choose a unit) 2.06 years = 3.36E-1 /year [Click here to reset time units](#)

Step 4:
Enter data and create plot:

☒ Total mass: $M_0 = 1$ Ci (choose a unit) [Click here to reset unit choice](#)

☒ Width: $w = 40$ m

☒ Depth: $d = 2$ m

☒ Velocity: $v = 0.5$ m/s

Dispersion: $E = 49.67$ m^2/s

Rate constant: $k = 6.4\text{E-}7$ /minutes

Plot Concentration as a Function of:

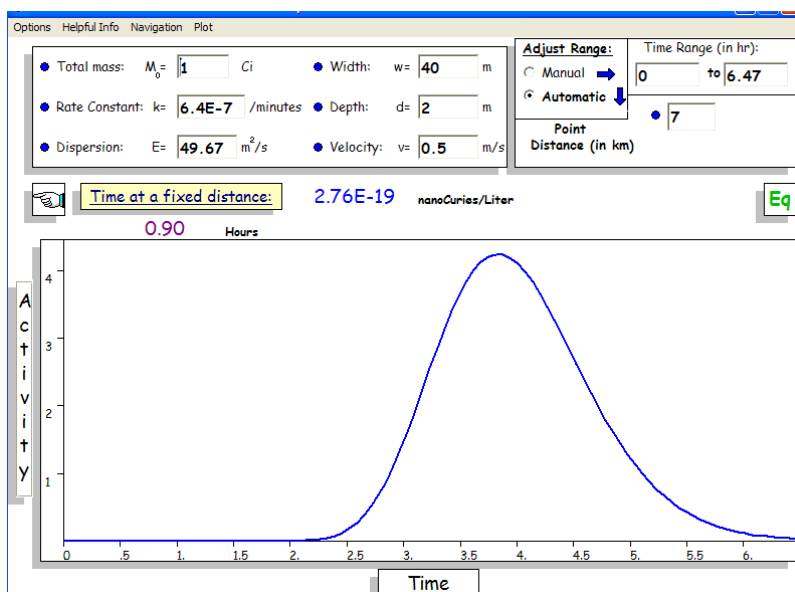
- ☐ Time, but at a fixed distance
- ☐ Distance, but at a fixed time
- ☐ Varying distance and time

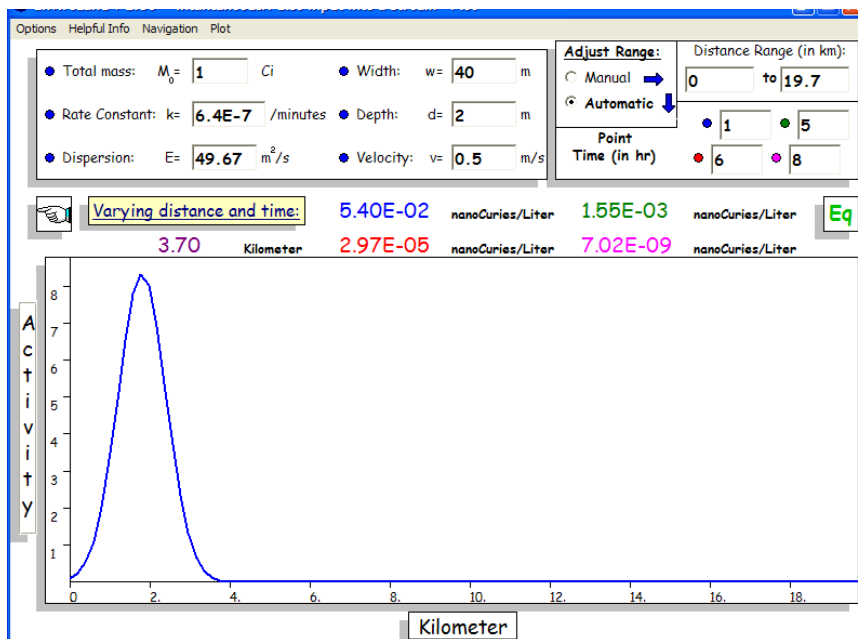
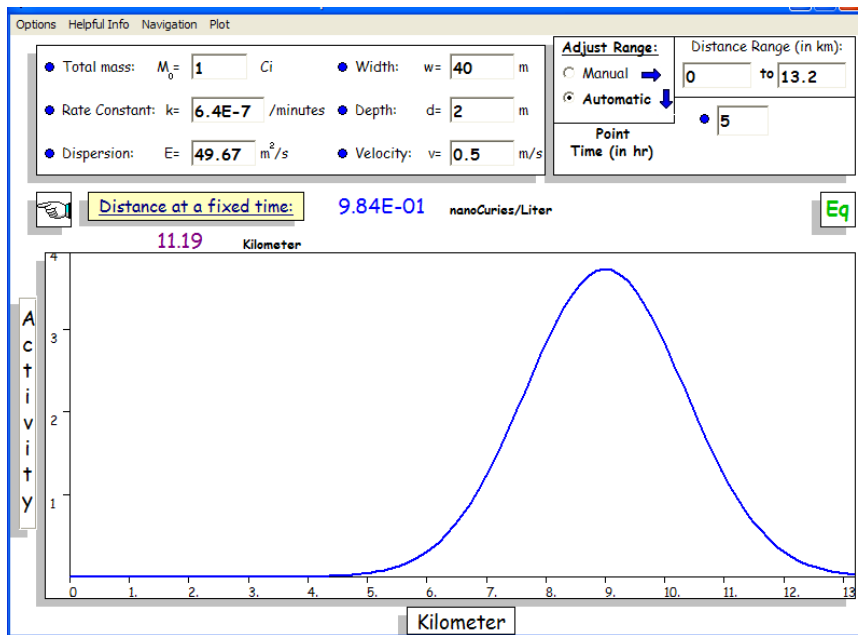
Step 5: (optional)
Calculate pollutant concentration at a specific distance and time:

☒ Enter distance: $x = 0$ (in meters) ☒ Enter time: $t = 0$ hours days (choose a unit)

[Click for Concentration =](#) Curies/Liter

[Eq](#)





Transport of Pollutants in Rivers and Streams

The close proximity of chemical factories, railways, and highways to natural waterways frequently leads to unintentional releases of hazardous chemicals to these systems. Once hazardous chemicals are in the aquatic system, they can result in a number of detrimental affects for considerable distances downstream from their source. This program allows the user to predict the concentration of a pollutant downstream of an instantaneous release of the pollutant. Examples of instantaneous releases can be as simple as small discrete releases such as dropping a liter of antifreeze off a bridge; or they can be more complex such as a transportation accident that results in the release of acetone from a tanker-car. Once released to the system, the model assumes that the pollutant and stream water are completely mixed (i.e. there is no cross-sectional concentration gradient in the stream channel). This is a reasonably good assumption for most systems. The model used here accounts for longitudinal dispersion (spreading in the direction of stream flow), advection (transport in the direction of stream flow at the flow rate of the water), and a first-order removal term (biodegradation or radioactive decay).

The governing equation is obtained by initially setting up a mass balance on a cross-section of the stream channel as described by Metcalf and Eddy (1972). When the dispersion term given above is included in a cross-sectional mass balance of the stream channel, each term can be described as

$$\text{Inflow: } QC\Delta t - EA \frac{\partial C}{\partial x} \Delta t$$

$$\text{Outflow: } Q \left(C + \frac{\partial C}{\partial x} \Delta x \right) \Delta t - EA \left(\frac{\partial C}{\partial x} + \frac{\partial^2 C}{\partial x^2} \Delta x \right) \Delta t$$

$$\text{Sinks: } vkC\Delta t$$

where Q = volumetric flow rate (m^3/s)
 C = concentration (mg/m^3)
 E = longitudinal dispersion coefficient (m^2/s)
 x = distance downstream from point source (m)
 v = average water velocity (m/s)

The two longitudinal dispersion terms in these equations:

$$EA \frac{\partial C}{\partial x} D\Delta t \quad \text{and} \quad EA \left(\frac{\partial C}{\partial x} + \frac{\partial^2 C}{\partial x^2} D\Delta x \right) D\Delta t$$

were derived from the following equation:

$$\frac{\partial M}{\partial t} = -EA \frac{\partial C}{\partial x}$$

where $\partial M / \partial t$ = mass flow
 $\partial C / \partial x$ = concentration gradient
 A = cross-sectional area
 E = coefficient of turbulent mixing

From this equation, it can be seen that whenever a concentration gradient exists in the direction of flow ($\partial C/\partial x$), a flow of mass ($\partial M/\partial t$) occurs in a manner to reduce the concentration gradient. For this equation, it is assumed that the flow rate is proportional to the concentration gradient and the cross-sectional area over which this gradient occurs. The proportionality constant, E, is commonly called the coefficient of eddy diffusion or turbulent mixing. Thus, the driving force behind this reduction in concentration is the turbulent mixing in the system, characterized by E and the concentration gradient.

The inflow, outflow, and sink equation given earlier can be combined to yield the pollutant concentration at a given cross section as a function of time. This combination of terms is generally referred to as the general transport equation and can be expressed as

$$\text{Accumulation} = \text{inputs} - \text{outputs} + \text{sources} - \text{removal}$$

Combining the inflow, outflow, and sink terms into the mass balance expression and integrating for the equilibrium case where $\partial C/\partial t = 0$, results in the following governing equation for the transport of a pollutant in a stream system:

$$C(x,t) = \frac{M_0}{wd\sqrt{4\pi Et}} \exp\left(-\frac{(x-vt)^2}{4Et} - kt\right)$$

where $C(x,t)$ = the pollutant concentration (in mg/L or mCi/L for radioactive compounds) at distance x and time t

- M_0 = mass of pollutant released (in mg or mCi)
- w = average width of the stream
- d = average depth of the stream
- E = longitudinal dispersion coefficient (m^2/s)
- t = time (s)
- x = distance downstream from input (m)
- v = average water velocity (m/s)
- k = first order decay or degradation rate constant (1/s)

Note that EXP represents "e" (the base of the natural logarithm).

When there is no (or negligible) degradation of the pollutant, k is set to zero. The longitudinal dispersion coefficient, E, is characteristic of the stream or more specifically the section of the stream that is being modeled. Values of E can be determined experimentally by adding a known mass of tracer to the stream and measuring the tracer concentration at various points as a function of time. The equation given above is then fitted to the data at each sampling point and a value for E is estimated. Unfortunately, this experimental approach is very time and cost intensive, and is rarely used. One common approach for estimating E values is given by Fischer et al. (1979)

$$E = 0.011 \frac{v^2 w^2}{du}$$

and

$$u = \sqrt{gds}$$

where v = average water velocity (m/s)
 w = average stream width (m)
 d = average stream depth (m)
 $g = 9.81 \text{ m/s}^2$ (acceleration due to gravity)
 s = the slope of the stream bed (unitless)

From these equations it can be seen that the downstream concentration of a pollutant (in the absence of degradation) is largely a function of the longitudinal dispersion which, in turn, is determined by the mixing in the system and the slope of the stream bed.

References:

Fischer, H.B., List, E.J., Koh, R.C.Y., Imberger, I., and Brooks, N.H. 1979. *Mixing in Inland and Coastal Waters*, Academic Press, New York.

Metcalf and Eddy, Inc., 1972. *Wastewater Engineering: Collection, Treatment, Disposal*. McGraw Hill Series in Water Resources and Environmental Engineering, McGraw Hill, New York.

Problem Illustrating The Step Input of A Pollutant into A Stream

An abandoned landfill leaches water into an adjacent stream at a rate of 1500 L/min. The concentration of 2-chlorophenol in the water is 500 mg/L. The stream has dimensions of 20 m wide and 2.3 m deep, and has a water velocity of 0.85 m/s. The regional slope of the stream channel is 1 ft per 1500 ft distance and the first-order half-life of 2-chlorophenol is 2.5 d. What is the concentration profile of 2-chlorophenol in the stream and what is the concentration 25 km downstream from the point source?

Solution:

- (1) Calculate the mass input to the stream in kg/s.

$$\text{mass input} = \left(\frac{1500 \text{ L}}{\text{min}} \right) \left(\frac{\text{min}}{60 \text{ s}} \right) \left(\frac{500 \text{ mg}}{\text{L}} \right) \left(\frac{\text{g}}{1000 \text{ mg}} \right) \left(\frac{\text{kg}}{1000 \text{ g}} \right)$$

$$\text{mass input} = 0.0125 \text{ kg/s}$$

- (2) Calculate the flow rate of the stream in m³/s.

$$\text{flow rate} = \text{width} * \text{depth} * \text{water velocity} = (20 \text{ m}) (2.3 \text{ m}) (0.85 \text{ m/s}) = 39.1 \text{ m}^3/\text{s}$$

- (3) Calculate the rate constant, k, for the first-order decay of 2-chlorophenol.

$$\ln \frac{C}{C_0} = -kt$$

For a first-order reaction:

| | | |
|-------|----------------|---|
| where | C | = the concentration at time t |
| | C ₀ | = the initial concentration of 2-chlorophenol |
| | k | = the first order decay constant, and |
| | t | = time in seconds |

At the half-life (t_{1/2}), one-half of the original concentration remains. Substitution of this into the equation above yields:

$$\ln \left(\frac{\frac{1}{2} C_0}{C_0} \right) = -kt_{1/2} \quad \text{or}$$

$$-\frac{\ln 0.5}{t_{1/2}} = -\frac{\ln 0.5}{2.5 \text{ d}} = k = 0.277 \text{ d}^{-1}$$

$$(0.277 \text{ d}^{-1}) \left(\frac{\text{d}}{24 \text{ hr}} \right) \left(\frac{\text{hr}}{60 \text{ min}} \right) \left(\frac{\text{min}}{60 \text{ s}} \right) = 3.21 \times 10^{-6} \text{ s}^{-1}$$

Thus, the decay rate for 2-chlorophenol is 3.21 × 10⁻⁶ s⁻¹.

- (4) Calculate the longitudinal dispersion coefficient, E (also referred to as the coefficient of eddy diffusion).

$$\text{slope} = \frac{1 \text{ m}}{1500 \text{ m}} = 6.67 \times 10^{-4}$$

$$u = \sqrt{gds} = \sqrt{(9.81 \text{ m/s}^2)(2.3 \text{ m})(6.67 \times 10^{-4})} = 0.123 \text{ m/s}$$

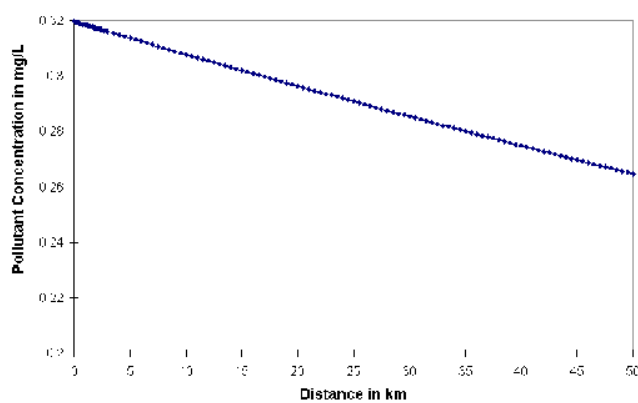
$$E = 0.011 \frac{v^2 w^2}{du} = 0.011 \frac{(0.85 \text{ m/s})^2 (20 \text{ m})^2}{(2.3 \text{ m})(0.123 \text{ m/s})} = 11.24 \text{ m}^2/\text{s}$$

(5) Arrange data into proper units:

$$\begin{aligned} Q &= 39.1 \text{ m}^3/\text{s} \\ W &= 0.0125 \text{ kg/s} \\ w &= 20 \text{ m} \\ d &= 2.3 \text{ m} \\ v &= 0.85 \text{ m/s} \\ E &= 11.24 \text{ m}^2/\text{s} \\ x &= \text{variable in meters (m)} \\ k &= 3.21 \times 10^{-6} \text{ s}^{-1} \end{aligned}$$

(6) Input data into program and obtain graph

Example Problem: Longitudinal Concentration Profile as a Function of Distance from The Point Source



(7) Calculate $C(x)$ at 25 km.

$$\begin{aligned} C &= \frac{W}{Q \sqrt{1 + \frac{4kE}{v^2}}} \exp \left[\frac{vx}{2E} \left(1 - \sqrt{1 + \frac{4kE}{v^2}} \right) \right] \\ C &= \frac{0.0125 \text{ kg/s}}{39.1 \text{ m}^3/\text{s} \sqrt{1 + \frac{4(3.21 \times 10^{-6} \text{ s}^{-1})(11.24 \text{ m}^2/\text{s})}{(0.85)^2}}} \times \\ &\quad \exp \left[\frac{(0.85 \text{ m/s})(25000 \text{ m})}{2(11.24 \text{ m}^2/\text{s})} \left(1 - \sqrt{1 + \frac{4(3.21 \times 10^{-6} \text{ s}^{-1})(11.24 \text{ m}^2/\text{s})}{(0.85 \text{ m/s})^2}} \right) \right] \\ C &= 3.20 \times 10^{-4} \text{ EXP}(-0.0944) \\ C &= 2.91 \times 10^{-4} \text{ kg/m}^3 \\ C &= 0.291 \text{ mg/L} \end{aligned}$$

Problem Illustrating the Use of The Streeter-Phelps Equation

A city discharges 25 million gallons per day (mgd) of domestic sewage into a stream whose typical rate of flow is 250 cubic feet per second (cfs). The velocity of the stream is approximate 3 miles per hour. The temperature of the sewage is 21 °C, while that of the stream is 15 °C. The 20 °C BOD₅ of the sewage is 180 mg/L, while that of the stream is 1.0 mg/L. The sewage contains no dissolved oxygen, but the stream is 90 percent saturated upstream of the discharge. At 20 °C, k_1 is estimated to be 0.34 per day while k_2 is 0.65 per day.

- (1) Determine the critical-oxygen deficit and its location.
- (2) Also estimate the 20 °C BOD₅ of a sample taken at the critical point. Use temperature coefficients of 1.135 for k_1 and 1.024 for k_2 .
- (3) Plot the dissolved-oxygen-sag curve.
- (4) Determine the dissolved oxygen concentration at 1000 km from the point source.

SOLUTION:

- (1) Determine the dissolved oxygen in the stream before discharge.

Saturation concentration at 15 °C (from table on worksheet) = 10.2 mg/L

Dissolved oxygen in stream = 0.90 (10.2 mg/L) = 9.2 mg/L

- (2) Determine the temperature, dissolved oxygen, and BOD of the mixture using the mass balance approach. Note that units should be compatible.

Flow rate of stream (conversion to liters):

$$\left(\frac{250 \text{ cubic feet}}{\text{s}} \right) \left(\frac{7.48 \text{ gal}}{\text{cubic foot}} \right) \left(\frac{60 \text{ s}}{\text{min}} \right) \left(\frac{60 \text{ min}}{\text{hr}} \right) \left(\frac{24 \text{ hr}}{\text{d}} \right) = 161.6 \times 10^6 \text{ gallons / d}$$

$$\left(161.6 \times 10^6 \text{ gallons} \right) \left(\frac{3.79 \text{ L}}{\text{gal}} \right) = 612 \times 10^6 \text{ L / d} = 612 \text{ million liters / d}$$

Flow rate of sewage effluent:

$$\left(\frac{25 \times 10^6 \text{ gallons}}{\text{d}} \right) \left(\frac{3.79 \text{ L}}{\text{gal}} \right) = 94.8 \times 10^6 \text{ L / d} = 94.8 \text{ million liters / d}$$

NOTE: Visual Basic limits the magnitude of some numbers, therefore flowrates should be expressed in million gallons/day, million liters/day, or million cubic feet/day.

Temperature of mixture:

Net Change in Temperature (DT) = Stream Input + Sewage Input - Output
 $0 = (\text{stream flow rate})(\text{stream temp.}) + (\text{sewage flow rate})(\text{sewage temp.}) - (\text{mixture flow rate})(\text{mixture temp.})$
 $0 = (612 \times 10^6 \text{ L / d})(15 \text{ C}) + (94.8 \times 10^6 \text{ L / d})(20 \text{ C}) - (612 \times 10^6 \text{ L / d} + 94.8 \times 10^6 \text{ L / d})T_{\text{mixture}}$
 upon rearrangement yields:

$$T_{\text{mixture}} = \frac{(612 \times 10^6 \text{ L / d})(15 \text{ C}) + (94.8 \times 10^6 \text{ L / d})(20 \text{ C})}{612 \times 10^6 \text{ L / d} + 94.8 \times 10^6 \text{ L / d}} = 15.67 \text{ C}$$

Dissolved oxygen of mixture:

Net Change in D.O. = Stream Input + Sewage Input - Output
 $0 = (\text{stream flow rate})(\text{stream D.O.}) + (\text{sewage flow rate})(\text{sewage D.O.}) - (\text{mixture flow rate})(\text{mixture D.O.})$
 $0 = (612 \times 10^6 \text{ L/d})(9.2 \text{ mg/L}) + (94.8 \times 10^6 \text{ L/d})(0.0) - (612 \times 10^6 \text{ L/d} + 94.8 \times 10^6 \text{ L/d})(\text{D.O.}_{\text{mixture}})$
 upon rearrangement yields:

$$\text{D.O.}_{\text{mixture}} = \frac{(612 \times 10^6 \text{ L/d})(9.2 \text{ mg/L}) + (94.8 \times 10^6 \text{ L/d})(0.0 \text{ mg/L})}{612 \times 10^6 \text{ L/d} + 94.8 \times 10^6 \text{ L/d}} = 7.91 \text{ mg/L}$$

BOD₅ of mixture:

Net Change in BOD₅ (DBOD₅) = Stream Input + Sewage Input - Output
 $0 = (\text{stream flow rate})(\text{stream BOD}_5) + (\text{sewage flow rate})(\text{sewage BOD}_5) - (\text{mixture flow rate})(\text{mixture BOD}_5)$
 $0 = (612 \times 10^6 \text{ L/d})(1.0) + (94.8 \times 10^6 \text{ L/d})(180) - (612 \times 10^6 \text{ L/d} + 94.8 \times 10^6 \text{ L/d})(\text{BOD}_{5\text{mixture}})$
 upon rearrangement yields:

$$\text{BOD}_{5\text{mixture}} = \frac{(612 \times 10^6 \text{ L/d})(1.0) + (94.8 \times 10^6 \text{ L/d})(180)}{612 \times 10^6 \text{ L/d} + 94.8 \times 10^6 \text{ L/d}} = 25.01 \text{ mg/L}$$

BOD_L of mixture (at 20 °C):

$$\text{BOD}_L = \frac{\text{BOD}_5}{1 - e^{-k'(x/v)}} = \frac{25.0 \text{ mg/L}}{1 - e^{-(0.34/\text{d})(5\text{d})}} = 30.68 \text{ mg/L}$$

(3) Correct the rate constants to 15.7 °C:

Rate constants are not linearly related to changes in temperature, therefore we must correct them using an exponential relationship. Typically these can be corrected using the two constants and equations given below. Note that 20 °C is used as the reference point since this is where the original data for the k's were collected.

$$k' = 0.34(1.135)^{15.7-20} = 0.197 \text{ day}^{-1}$$

$$k'_2 = 0.65(1.024)^{15.7-20} = 0.587 \text{ day}^{-1}$$

(4) Determine the critical time (t_c) and critical distance (x_c). In the table note that the saturation value for O_2 at 15.7 °C = 10.1 mg/L, however the stream is at 90% of the saturation value (9.2 mg/L). Thus, the initial oxygen deficit,

$$\begin{aligned} D_o &= (\text{the initial stream } O_2 \text{ value} - \text{the } O_2 \text{ of the mixture}) \\ &= (10.15 - 7.91) = 2.24 \text{ mg } O_2/\text{L} \end{aligned}$$

$$\begin{aligned} t_c &= \frac{1}{k'_2 - k'} \ln \frac{k'_2}{k'} \left[1 - \frac{D_o(k'_2 - k')}{k' \text{BOD}_L} \right] \\ t_c &= \frac{1}{0.587/\text{d} - 0.197/\text{d}} \ln \frac{0.587/\text{d}}{0.197/\text{d}} \left[1 - \frac{2.24 \text{ mg/L}(0.587/\text{d} - 0.197/\text{d})}{0.197/\text{d}(30.6 \text{ mg/L})} \right] \\ t_c &= 2.4 \text{ d} \end{aligned}$$

$$\begin{aligned} x_c &= vt_c \\ &= \left(\frac{3 \text{ miles}}{\text{h}} \right) \left(\frac{24 \text{ h}}{\text{d}} \right) (2.4 \text{ d}) = 172.8 \text{ miles} \\ \text{or} \\ &= \left(\frac{3 \text{ miles}}{\text{h}} \right) \left(\frac{1.61 \text{ km}}{\text{mile}} \right) \left(\frac{24 \text{ h}}{\text{d}} \right) (2.4 \text{ d}) = 278.2 \text{ km} \end{aligned}$$

(5) Determine D_c .

To calculate the critical oxygen deficit, D_c , we must first convert the water velocity into units of miles/d or km/d. Therefore, 3 miles/hr = 72 miles/d.

$$D_c = \frac{k'}{k'_2} BOD_L e^{-k'(x_c/v)}$$

$$= \left(\frac{0.197/d}{0.587/d} \right) (30.68) e^{-(0.197/d)(172.8 \text{ miles})/(72 \text{ mile/d})} = 6.41 \text{ mg/L}$$

Thus, the D.O. will be depressed 6.41 mg/L from its saturation value. The initial O_2 concentration of the stream will be the saturation value minus the D_c ,
or $10.15 - 6.41 = 3.74 \text{ mg } O_2/L$.

(6) Determine the BOD_5 of a sample taken at distance x_c .

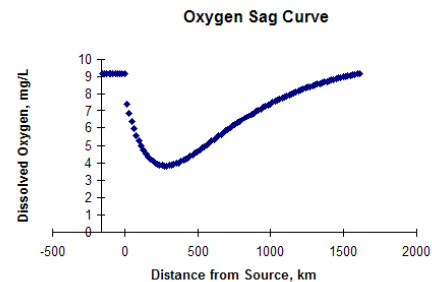
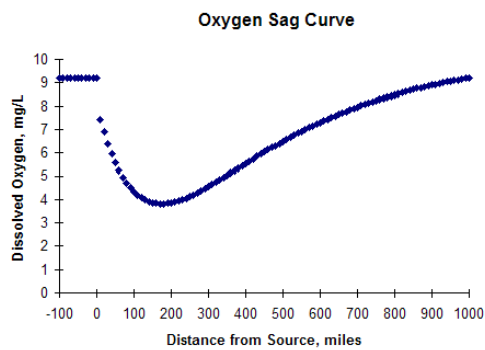
$$BOD_{L5} = BOD_L e^{-k'(x/v)}$$

$$= (30.6 \text{ mg/L}) e^{-0.197/d(174.2 \text{ miles}/72 \text{ miles/d})} = 19.0 \text{ mg/L}$$

$$20^\circ C \text{ } BOD_5 = BOD_L [1 - e^{-(k')(5)}]$$

$$= 19.0 \text{ mg/L} [1 - e^{-(0.34/d)(5d)}] = 15.5 \text{ mg/L}$$

(7) Draw the oxygen sag curves for both miles and km from the point source.



Note, that in this example all O_2 concentration values are above those normally encountered in the Zone of Active Decomposition. Thus, we basically have a Zone of Degradation and a Recovery Zone, delineated by the low point in the data plot.

(8) Determine the dissolved concentration at 1000 km from the point source.

$$D = \frac{k' BOD_L}{k'_2 - k'} \left(e^{-k'(x/v)} - e^{-k'_2(x/v)} \right) + D_c e^{-k'_2(x/v)}$$

$$= \frac{(0.197/d)(30.6 \text{ mg/L})}{(0.587/d) - (0.197/d)} \left[\text{EXP} \left(-\frac{(0.197/d) \cdot 1 \times 10^6 \text{ m}}{1.16 \times 10^5 \text{ m/d}} \right) - \text{EXP} \left(-\frac{(0.587/d) \cdot 1 \times 10^6 \text{ m}}{1.16 \times 10^5 \text{ m/d}} \right) \right]$$

$$+ 2.13 \text{EXP} \left(-\frac{(0.587) \cdot 1 \times 10^6 \text{ m}}{1.16 \times 10^5 \text{ m/d}} \right)$$

$$= 15.46 [\text{EXP}(-1.70) - \text{EXP}(-5.06)] + 2.13 \text{EXP}(-5.06)$$

$$= 15.46(0.18) + 2.13(0.01)$$

$$= 2.80 \text{ mg/L}$$

The calculated oxygen deficit created by the BOD is 2.80 mg/L. The initial oxygen concentration in the natural stream is 9.2 mg/L. Thus, the D.O. at 1000 km will be $10.15 - 2.8 = 7.35 \text{ mg/L}$ which agrees with the data plot shown above.

Explanation of Example Problem for a Step Input to a Lake

A lake in a rural community has an average surface area of 5 km² and a mean depth of 50 m. A stream exits the lake with an average annual flow rate of 45,000 m³/yr. Aerial application of an insecticide in the area introduces the compound into the lake. The average annual loading to the lake from the atmospheric and from agricultural runoff is estimated at 50 kg/day. Assuming a first-order removal of the insecticide (half-life = 45 days) from the lake and the initial background concentration of insecticide in the lake are negligible:

- (1) Estimate the detention time of water in the lake.

The volume of the lake is equal to the average surface area times the mean depth,

$$\text{Volume} = (5000 \text{ m}^2) (50 \text{ m}) = 250,000 \text{ m}^3$$

$$\text{Detention time} = \frac{250,000 \text{ m}^3}{45,000 \text{ m}^3 / \text{yr}} = 5.56 \text{ yr}$$

- (2) Calculate the equilibrium concentration of insecticide in the lake.

In order to solve this portion of the problem, we must first convert the first-order half-life to a rate constant, k , expressed in units of reciprocal years. The half-life of 45 days is equal to a half-life of 0.12 years.

$$\begin{aligned} \ln \frac{C}{C_0} &= -kt \\ \ln (0.5) &= -k (0.12 \text{ yr}) \\ k &= 5.78 / \text{yr} \end{aligned}$$

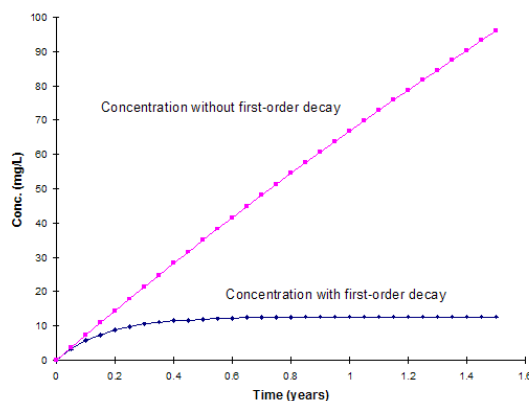
Now,

$$C = \frac{W}{\beta V}$$

$$\beta = \frac{1}{t_o} + k = \frac{1}{5.56} + 5.78 = 5.96$$

$$\begin{aligned} C &= \frac{W}{\beta V} = \frac{(50 \text{ kg / day})(365 \text{ day / yr})}{5.96 (250,000 \text{ m}^3)} \\ &= (1.26 \times 10^{-2} \text{ kg / m}^3)(1000 \text{ g / kg})(10^3 \text{ mg / g})(1 \text{ m}^3 / 1000 \text{ L}) \\ &= 12.5 \text{ mg / L} \end{aligned}$$

- (3) Determine if the first-order decay is an important removal process by constructing a plot using the first-order decay rate and another plot without the first-order decay



It is evident from these two plots, that the decay rate is important in reducing the concentration of pollutant.

- (4) Calculate the concentration after 1 year.

$$C = \frac{W}{\beta V} (1 - e^{-\beta t})$$

$$\text{where } \beta = \frac{1}{t_o} + k$$

$$\beta = \frac{1}{t_o} + k = \frac{1}{5.56} + 5.78 = 5.96$$

$$C = \frac{W}{\beta V} (1 - e^{-\beta t})$$

$$C = \frac{(18250 \text{ kg / yr})(1000 \text{ g / kg})(10^3 \text{ mg / g})}{5.96(250,000 \text{ m}^3)(1000 \text{ L / m}^3)} (1 - e^{-5.96 \times 1})$$

$$C = 12.25 (1 - e^{-5.96 \times 1})$$

$$C = 12.25 - 12.25(2.58 \times 10^{-3})$$

$$C = 12.2 \text{ mg / L}$$

Explanation of Example Problem for an Instantaneous Input to a Lake

Here, we will use the same problem used in the step input example problem, but we will monitor the removal of the insecticide if the input is ceased after 1 year. We found in the previous problem that after 1 year the average concentration in the lake was 12.2 mg/L. The previous problem statement was: "A lake in a rural community has an average surface area of 5 km² and a mean depth of 50 m. A stream exits the lake with an average annual flow rate of 45,000 m³/yr. Aerial application of an insecticide in the area introduces the compound into the lake. The average annual loading to the lake from the atmospheric and from agricultural runoff is estimated at 50 kg/day. Assuming a first-order removal of the insecticide (half-life = 45 days) from the lake and the initial background concentration of insecticide in the lake are negligible:"

From the previous problem, the volume of the lake is equal to the average surface area times the mean depth,

$$\begin{aligned}\text{Volume} &= (5000 \text{ m}^2) (50 \text{ m}) = 250,000 \text{ m}^3 \\ \text{Detention time} &= \frac{250,000 \text{ m}^3}{45,000 \text{ m}^3 / \text{yr}} = 5.56 \text{ yr}\end{aligned}$$

In order to solve this problem, we must also convert the first-order half-life to a rate constant, k , expressed in units of reciprocal years. The half-life of 45 days is equal to a half-life of 0.12 years.

$$\begin{aligned}\ln \frac{C}{C_0} &= -kt \\ \ln (0.5) &= -k(0.12 \text{ yr}) \\ k &= 5.78 / \text{yr}\end{aligned}$$

From the previous problem, the concentration after 1 year is

$$\begin{aligned}C &= \frac{W}{\beta V} (1 - e^{-\beta t}) \\ \text{where } \beta &= \frac{1}{t_0} + k\end{aligned}$$

$$\beta = \frac{1}{t_0} + k = \frac{1}{5.56} + 5.78 = 5.96$$

$$C = \frac{W}{\beta V} (1 - e^{-\beta t})$$

$$C = \frac{(18250 \text{ kg} / \text{yr})(1000 \text{ g} / \text{kg})(10^3 \text{ mg} / \text{g})}{5.96(250000 \text{ m}^3)(1000 \text{ L} / \text{m}^3)} (1 - e^{-5.96 \cdot 1})$$

$$C = 12.25 (1 - e^{-5.96 \cdot 1})$$

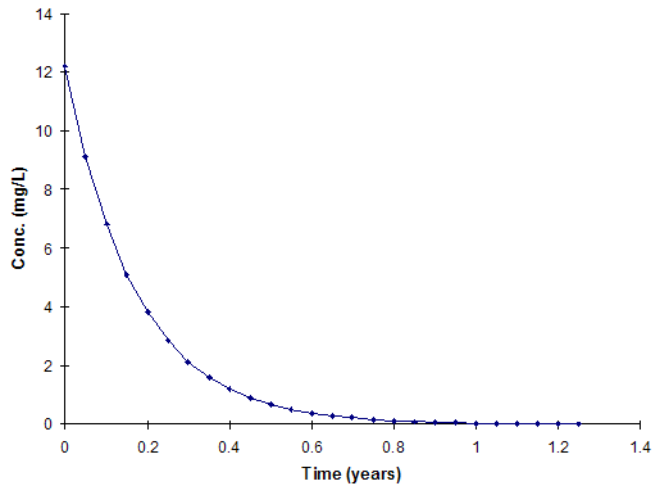
$$C = 12.25 - 12.25(2.58 \times 10^{-3})$$

$$C = 12.2 \text{ mg} / \text{L}$$

(1) Now, for this problem calculate the insecticide concentration as a function of time. The equation governing this process is

$$\begin{aligned}C(t) &= C_0 e^{-\left(\frac{Q_0}{V} + k\right)t}, \text{ or} \\ C(t) &= C_0 e^{-\left(\frac{1}{t_0} + k\right)t}\end{aligned}$$

where $C_0 = 12.2 \text{ mg/L}$. A plot of this equation is shown below.



(2) Calculate the time required to reach an insecticide concentration of 100 mg/L which happens to be the detection limit for this compound.

$$C(t) = C_0 e^{-\left(\frac{Q_0}{V} + k\right)t}, \text{ or}$$

$$C(t) = C_0 e^{-\left(\frac{1}{t_0} + k\right)t}$$

$$\frac{C(t)}{C_0} = e^{-\left(\frac{1}{t_0} + k\right)t}$$

$$\frac{0.100 \text{ mg/L}}{12.2 \text{ mg/L}} = e^{-\left(\frac{1}{5.56} + 5.78\right)t}$$

$$\ln \frac{0.100 \text{ mg/L}}{12.2 \text{ mg/L}} = -\left(\frac{1}{5.56} + 5.78\right)t$$

$$4.80 = 5.96 t$$

$$t = 0.81 \text{ yr}$$

Thus, it is seen that the lake will recover very rapidly after the input of the insecticide is halted.

Problem: Groundwater: Pulse Release of Pollutant

A 100 m³ tanker containing a 100 mg/m³ solution of 2,4-dinitotoluene wrecks and empties its entire contents above an aquifer. The cross-sectional area of the spill is 50 m². The aquifer has a porosity of 30 percent, a bulk density of 1.6 g/cm³, a velocity of 10 m/yr, and a dispersion coefficient of 10 m²/yr. The distribution coefficient of 2,4-dinitotoluene for this aquifer material has been measured to be 2.5 mL/g. 2,4-dinitotoluene biodegrades through a first order reaction at a rate of 0.693 yr⁻¹. How far and how fast will 2,4-dinitotoluene migrate through the aquifer?

Solution:

1. Calculate the retardation factor from the distribution coefficient.

$$R = 1 + \frac{\rho_b K_d}{n}$$

$$= 1 + \frac{\left(1.6 \frac{\text{g}}{\text{cm}^3}\right) \left(2.5 \frac{\text{mL}}{\text{g}}\right)}{0.3} = 14.33.$$

2. Correct the cross-sectional area of the spill site to the void volume rather than the total volume.

$$\text{Cross-sectional area} \times \text{porosity} = 50 \text{ m}^2 \times 0.3 = 15 \text{ m}^2.$$

3. Calculate the total mass of chlordane spilled.

$$\text{Volume spilled} \times \text{concentration of solution} =$$

$$\left(100 \text{ m}^3\right) \left(1000 \frac{\text{mg}}{\text{m}^3}\right) = 100,000 \text{ mg or } 100 \text{ g}.$$

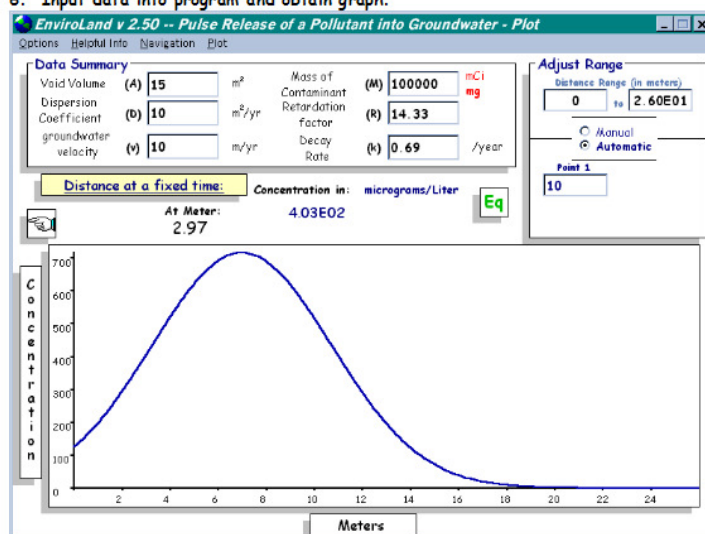
4. If necessary, calculate the first order degradation rate from the half-life.

$$-\frac{\ln 0.5}{t_{1/2}} = k = -\frac{-0.693}{0.09625(\text{yr})} = 0.693 \text{ yr}^{-1}.$$

5. Arrange data into proper units

groundwater velocity, $v = 10 \text{ m/yr}$,
 the retardation factor, $R = 14.33$,
 the mass of contaminant, $M = 100,000 \text{ mg}$
 the dispersion coefficient, $D = 10 \text{ m}^2/\text{yr}$,
 the reaction rate constant, $k = 0.693 \text{ yr}^{-1}$, and
 the cross-sectional area, $A = 15 \text{ m}^2$.

6. Input data into program and obtain graph.



7. Calculate the concentration 10 meters downgradient from the lagoon ten years after the input.

$$\begin{aligned}
 C(x,t) &= \frac{M}{A \sqrt{4\pi \frac{D}{R} t}} * e^{-\frac{\left(x - \left(\frac{v}{R}\right)t\right)^2}{4 \frac{D}{R} t}} * e^{-kt} \\
 &= \frac{100,000 \text{mg}}{15 \text{m}^2 \sqrt{4\pi \left(\frac{10 \frac{\text{m}^2}{\text{yr}}}{14.33}\right) * 10 \text{yr}}} * e^{-\frac{\left(10 \text{m} - \left(\frac{10 \frac{\text{m}}{\text{yr}}}{14.33}\right) 10 \text{yr}\right)^2}{4 * \left(\frac{10 \frac{\text{m}^2}{\text{yr}}}{14.33}\right) * 10 \text{yr}}} * e^{-\left(0.693 \frac{1}{\text{yr}}\right) 10 \text{yr}} \\
 &= 1,971 \text{ mg/m}^3 \text{ or } 1.971 \text{ mg/L 2,4-dinitotoluene.}
 \end{aligned}$$

Problem: Groundwater: Step Input of Pollutant

A waste lagoon containing 100 µg/L benzene is contaminating the underlying aquifer. The aquifer has a porosity of 30 percent, a bulk density of 1.6 g/cm³, a velocity of 10 m/yr, and a dispersivity of 10 m. The distribution coefficient of benzene for this aquifer material has been measured to be 5 mL/g. Benzene biodegrades through a first order reaction at a rate of 0.025 yr⁻¹. How far and how fast will benzene migrate through the aquifer?

Solution:

1. Calculate the retardation factor from the distribution coefficient.

$$R = 1 + \frac{\rho_b K_d}{n}$$

$$= 1 + \frac{\left(1.6 \frac{\text{g}}{\text{cm}^3}\right) \left(5 \frac{\text{mL}}{\text{g}}\right)}{0.3} = 27.67.$$

2. If necessary, calculate the dispersivity from the dispersion coefficient and the velocity.

$$D = \alpha_x v, \text{ so } \alpha_x = D/v = \frac{100 \left(\frac{\text{m}^2}{\text{yr}}\right)}{10 \left(\frac{\text{m}}{\text{yr}}\right)} = 10 \text{ m}$$

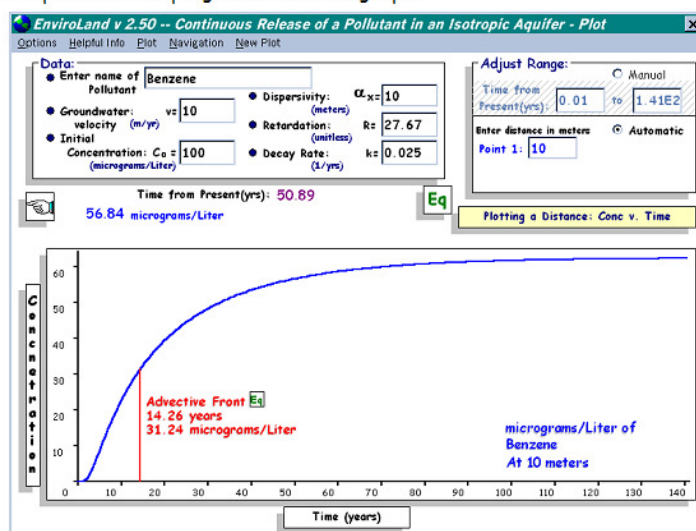
3. If necessary, calculate the first order degradation rate from the half-life.

$$-\frac{\ln 0.5}{t_{1/2}} = k = -\frac{-0.693}{27.72(\text{yr})} = 0.025 \text{ yr}^{-1}.$$

4. Convert data into proper units

$C_0 = 100 \text{ µg/L}$,
 $v = 10 \text{ m/yr}$,
 $\alpha_x = 10 \text{ m}$,
 $R = 27.67$, and
 $k = 0.025 \text{ yr}^{-1}$.

5. Input data into program and obtain graph.



6. Calculate the concentration 10 meters downgradient from the lagoon ten years after the input.

$$C(x,t) = \left(\frac{100 \frac{\mu g}{L}}{2} \right) e^{\left(\left(\frac{10m}{2(10m)} \right) \left[1 + \frac{(27.27)(0.025 \frac{1}{yr})(10m)}{10m} \right]^{\frac{1}{2}} \right)} \left(\frac{10m - \left(\frac{10 \frac{m}{yr}}{27.67} \right) (10yr) \left[1 + (4) \left(0.025 \frac{1}{yr} \right) \left(\frac{(27.67)(10m)}{10 \frac{m}{yr}} \right)^{\frac{1}{2}} \right]}{2 \left((10m) \left(\frac{10 \frac{m}{yr}}{27.67} \right) (10yr) \right)^{\frac{1}{2}}} \right)^{\frac{1}{2}}$$

= 0.78 $\mu g/L$ benzene.