Question # 1. Marks: 4+6=10

(a) Use Gaussian elimination to solve the system
\[
\begin{align*}
2x + 4y - z &= 1 \\
-4x + y + 5z &= -1 \\
2x + y - 2z &= 2
\end{align*}
\]

(b) Write the following system of equations
\[
\begin{align*}
x + 2y &= 7 \\
x + y + z &= 6 \\
y - 2z &= 0
\end{align*}
\]
in the matrix form $AX = B$. Find $A^{-1}$ by adjoint method and hence solve the system for $x, y$ and $z$.

Question # 2. Marks: 2+2+2=6

(a) If $A = \begin{pmatrix} 1 & 0 & -2 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$
then evaluate the $\text{det}(A)$ by cofactor expansion along a column of your choice.

(b) A force is given by a vector $\mathbf{F} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ and moves a particle from the point $P(2, 1, 0)$ to the point $Q(4, 6, 2)$. Find the work done.

(c) Identify and sketch the surface $-4x^2 + y^2 - 4z^2 = 4$

Question # 3. Marks: 3+3=6

(a) Find an equation of the plane that passes through the points $P(1, 3, 2)$, $Q(3, -1, 6)$ and $R(5, 2, 0)$.

(b) Find the normal component of acceleration for the curve
given by \( \mathbf{r}(t) = t\mathbf{i} + \frac{1}{3}t^3\mathbf{j} + \frac{1}{2}t^2\mathbf{k} \) at \( t = 2 \).

**Question # 4. Marks: 5 + 5 + 5 + 5 = 20**

(a) Find the equation of the plane tangent to the surface 
\( z = 4x^2 - y^2 + 2y \) at the point \( P(-1, 2, 4) \).

(b) Let \( z = f(x, y) = x^2 + 3xy - y^2 \). Find the differential \( \mathrm{d}z \). 
If \( x \) changes from 2 to 2.05 and \( y \) changes from 3 to 2.96, find \( \Delta z \), the increment of \( z \).

(c) If \( z = e^x \sin y \), where \( x = st^2 \) and \( y = s^2t \), find \( \frac{\partial z}{\partial s} \) and \( \frac{\partial z}{\partial t} \).

(d) Let \( f(x, y, z) = \sqrt{x^2 + y^2 + z^2} \). Find the directional derivative \( D_{a}f(x, y, z) \) at \( (1, 2, -2) \) in the direction of the vector \( \mathbf{a} = -6\mathbf{i} + 6\mathbf{j} - 3\mathbf{k} \).

**Question # 5. Marks: 4+4=8**

(a) Find the local maximum, minimum, and saddle point (if any) for the function \( f(x, y) = y^2 + x^2y + x^2 - 2y \).

(b) Use the Lagrange multiplier method to find the extrema for the function \( f(x, y) = xy \) subject to the constraint \( x^2 + 2y^2 = 4 \).