Self Assessment Quiz 10
Section 2.2

1. Let

$$\mathbf{v}_1 = \begin{bmatrix} 4 \\ 0 \\ -1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$$

   (a) Show that $\mathbf{v} = \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix}$ is a linear combination of the vectors $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_3$.

   (b) Show that $\mathbf{v} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ is not a linear combination of the vectors $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_3$.

   (c) Describe all vectors $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ that can be written as a linear combination of the vectors $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_3$.

2. Let

$$A = \begin{bmatrix} 1 & -1 & 5 \\ 1 & 0 & 2 \\ 0 & 1 & -3 \end{bmatrix}$$

   (a) Write the matrix equation $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \mathbf{0}$ as a vector equation using the column vectors of A.

   (b) Find $\det(A)$. Does the vector equation found in part (a) have a unique solution?

   (c) Find all ways in which the zero vector can be written as a linear combination of the column vectors of A.

   (d) Describe the set of solutions found in part (c) geometrically.

3. Answer each part True or False.

   (a) If $\mathbf{v} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3$, then the scalars are unique.

   (b) Every vector in $\mathbb{R}^n$ can be written uniquely as a linear combination of the vectors $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

   (c) If S is the set of all linear combinations of the vectors $\mathbf{v}_1, \dots, \mathbf{v}_k$ and there are scalars $c_1, \dots, c_{k-1}$ such that $\mathbf{v}_k = c_1\mathbf{v}_1 + \cdots + c_{k-1}\mathbf{v}_{k-1}$, then S is equal to the set of all linear combinations of $\mathbf{v}_1, \dots, \mathbf{v}_{k-1}$.