بســـــم ا الرحمن الرحـــــيم

King Saud University
College of Science
Department of Statistics
& Operations Research

OR 372 - Queueing Theory
Sunday : 28 – 11 - 1425H
Time : 8:00 – 11:00 am
Dr. Khalid Al-Nowibet

OR-372 Final Exam

ملحوظة : يتكون الاختبار من **6 أسئلة** فأحرص على ألا يستغرق منك كل سؤال أكثر من **30 دقيقة** ، مع خالص تمنياتي لكم بالتوفيق

Question #1

Customers arrive to a restaurant according to a Poisson process with rate 10 customers/hour. The restaurant opens daily at 9:00 am. Find the following:

(a) When the restaurant opens at 8:00 am, the workers need 30 min to arrange the tables and chairs. What is the probability that they will finish the arrangement before the arrival of a customer?

(b) What is the probability that there are 15 customers in the restaurant at 1:00 pm, given that there were 12 customers in the restaurant at 12:50 pm.

(c) Given that a new customer arrived at 9:13 am, what is the expected arrival time of the next customer?

(d) If a customer arrive to the restaurant at 2:00 pm what is the probability that the next customer will arrive before 2:10 pm.

Question #2

Consider a queueing system with Poisson arrival process and exponential service time having the following rates:

$$\lambda_0 = \lambda \qquad ; \qquad n = 0$$

$$\lambda_n = \frac{1}{n}\lambda \qquad ; \qquad n = 1,2,3,4$$

$$\mu_n = \mu \qquad ; \qquad n = 1,2,3,4,5$$

1. Draw the rate diagram for this system.
2. Write the balance equations for each state.
3. Solve the balance equations to get the steady state distribution of the system P_n.

Question #3

Consider the following applications:

1. Call center with five lines and two operators.
2. A gas station having 4 gas pumps and one worker where cars can wait on the street.
3. A maintenance department having one worker in a factory with 2 machines.

Determine the following elements: the arrivals, the type of service, queueing rule, and number of servers, the size of the system, and the queueing model (assuming Poisson arrivals and exponential service).

Question #4

Consider a small bank with one teller. Customers arrive to the bank according to a Poisson process with rate 8 customers per hour. The teller provides all kinds services for the customers. Each customer takes about 5 minutes from the teller's time. Assume that the service is exponential. In steady-state, find:

1. What is the probability that the teller is idle?
2. What is the average number of customers waiting for service?
3. On average, how long will a customer spend in the bank to complete his service?
4. What is the probability that there are more than 5 customers in the bank?

5. Assume that any customer is either going to withdraw money from his account, with probability 0.6, or deposit money to his account, with probability 0.4. In steady-state, what is the average number of customers requesting withdrawal and average number of customers requesting deposit?

<u>Question #5</u>

Consider a gas station located on a highway with ***five*** pumps. Cars arrive to the gas station according to a Poisson process at rate 50 cars/hour. Any car able to enter the gas station stops by one of the available pumps. If all pumps are occupied, the driver will not enter the gas station. The gas station has ***three*** workers to service the cars. Each car takes an exponential amount of time for service with average of 5 minutes. The workers remember the order in which cars arrived so they service the cars on a first come first serviced basis. In the long run:

1. What is the probability that all workers are idle?
2. What is the probability that an arriving car will be able to enter the gas station?
3. What is the probability that a car will have to wait for a worker?
4. On average, how many cars will find all pumps occupied in one hour?
5. On average, how many cars will be in the station?
6. On average, how many cars waiting for service in the station?
7. Assume that a driver is in a hurry, so he will enter the gas station if and only if he will be serviced immediately. What is the probability that he will enter this gas station?
8. On average, how long a car will have to wait for service?

<u>Question #6</u>

Authorities at KKI Airport have established a new terminal for departure. It is estimated that passengers will arrive to the new terminal at rate 20 passenger per hour. Each passenger needs an average of 8 minutes for check-in. The authorities want to decide how many check-in counters should be opened. Assume that the arrival process is Poisson and the service time is exponential. Find:

1. The minimum number of counters so that the average queue length is finite?
2. The minimum number of counters such that the average number of passengers in line is less than **five**.
3. The minimum number of counters such that the average time for check-in and receiving the boarding pass is less than **15 minutes**.

<u>Helpful Formulas:</u>

$$p_n = \left(1-\frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^n \qquad n = 0,1,2,\ldots \qquad L = \frac{\lambda}{\mu-\lambda}$$

$$p_n = \frac{\lambda^n}{n!\mu^n}p_0 \quad \text{if} \quad 1 \le n \le s \quad \underline{\text{and}} \quad p_n = \frac{\lambda^n}{s!\, s^{n-s}\, \mu^n}p_0 \quad \text{if} \quad n > s$$

$$\text{where} \quad p_0 = \left[\frac{\lambda^s}{s!\mu^s}\frac{s\mu}{s\mu-\lambda} + \sum_{n=0}^{s-1}\frac{\lambda^n}{n!\mu^n}\right]^{-1} \quad , \quad L = \frac{\lambda}{\mu} + \left[\frac{(\lambda/\mu)^s\,\lambda\mu}{(s-1)!\,(s\mu-\lambda)^2}\right]p_0$$

$$p_n = \frac{\lambda^n}{n!\mu^n}p_0 \quad \text{if} \quad 1 \le n \le s \quad \underline{\text{and}} \quad p_n = \frac{\lambda^n}{s!\, s^{n-s}\, \mu^n}p_0 \quad \text{if} \quad s < n \le K$$

$$\text{where} \quad p_0 = \left[\sum_{n=0}^{s-1}\frac{\lambda^n}{n!\mu^n} + \sum_{n=s}^{K}\frac{\lambda^n}{s!\, s^{n-s}\mu^n}\right]^{-1}$$