King Saud University
College of Science
Department of Statistics
& Operations Research

OR 372 - Queueing Theory
22 Shawal 1425H
Room # 1 B 13
Time : 8:00 – 10:00 pm

Second Midterm Exam

Question 1: In each of the following, choose the correct answer

(a) A queueing system is in the steady state when :

1. the arrival process and the service process are both Poisson
2. the departure rate = the arrival rate.
3. the arrival rate $\lambda \to 0$ as $t \to \infty$.
4. None of the above

(b) The queueing system $M/M/1$ means that:

1. the arrival process is Poisson.
2. the service time is exponential with mean $1/\mu$.
3. the system has infinite capacity.
4. all of the above.

(c) The *Forward Equations* for the rate of change in $P_n(t)$ becomes a set of linear equations:

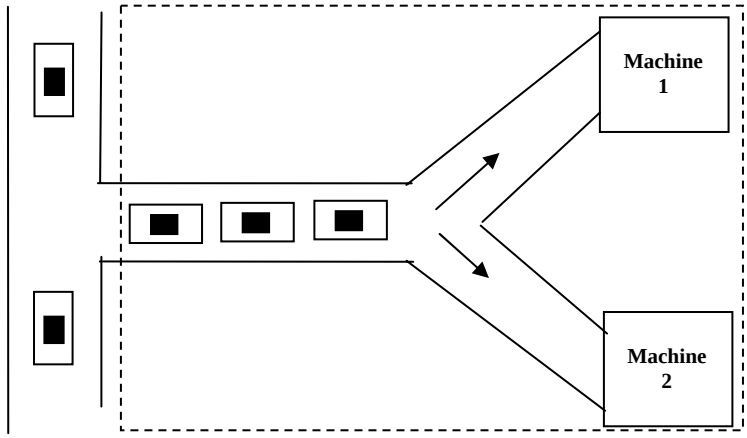
1. when λ is constant and $t \to \infty$.
2. when λ and μ are constant and $t \to 0$.
3. when $n \to \infty$.
4. None of the above.

(d) Consider a barber shop with **two** barbers and three waiting seats. In the steady state, probability that a new customer will wait to be served is :

1. $P_1 + P_2 + P_3 + P_4$.
2. $P_4 + P_5$
3. $1 - P_0 + P_1$
4. $P_0 + P_1 + P_2$

Question 2:

Consider an automatic car-wash station on a highway with two machines. Cars enter the station according to a Poisson process with rate λ car/hour. A machine takes an exponential time to wash a car with average of $1/\mu$ hours. Because of the limited space available in front of the station there is room for at most 3 cars to wait for the service. Any driver finds no empty parking space he will not enter (See the diagram).



1. Define the state of the system and all of its values.
2. What is the arrival rate of state n (λ_n) and the service rate of stat (μ_n)
3. Draw the rate diagram for this system.

***Hint**: there are two cars or more in the station the departure rate is doubled.*

4. Write the balance equations for each state.
5. If a care finds Customers arrive to a supermarket according to a Poisson process with rate $\lambda = 5$ customers per hour. The management estimated that 30% of the customers spend 10 SR of purchases, 40% of the customers spend 30 SR and 30% of the customers spend 50 SR. The supermarket opens at 8:00 am and closes at 8:00 pm.
   (a) What is the expected arrival time of the **first** customer?
   (b) What is the probability that he **first** customer will arrive within the first 15 min.?
   (c) If you enter the supermarket at 8:45 what is the probability that you will see **seven** customers shopping in the supermarket?
   (d) If you are in the supermarket and a customer just entered the supermarket what is the probability that he will spend at least 30 SR?
   (e) What is the expect number of customers entering the supermarket in one day?
   (f) A customer just entered the supermarket, what is the expected money that he is going to spend?
   (g) What is the expect earnings of the supermarket in one day?
   (h) What is the expected number of customers entering the supermarket and spending 50 SR in one day?
   (i) If you enter the supermarket and find 15 customers shopping what is the expected number of customers spending 40 SR?
   (j) If enter you the supermarket at 8:15 am and spend 30 min shopping without any new arrival, what is the probability that you will continue shopping until 9:00 am alone?
   (k) Given that you entered the supermarket at 10:30 am and found only one customer shopping. What is the probability that he arrived within 30 minutes ahead of you?

(l) Given that the next arrival has exactly 30 SR in his pocket, what is the probability that he will spend less than 10 SR.

Question 3:

The following observations have been made regarding the time between successive arrivals to a single server FCFS queueing process.

Customer Number	Interarrival Time	Service Time
1	–	3
2	8	8
3	8	6
4	7	8
5	5	9
6	9	9
7	8	4
8	6	5
9	12	3
10	6	6

(1) Calculate the arrival time, departure time and the waiting time of each customer.
(2) Plot the number of customers in the system as a function of time.
(3) Find the expected interarrival time?
(4) Find the expected number of arrivals per unit of time?
(5) What is the expected service time?
(6) What is the probability that the server is idle?
(7) Calculate the average waiting time in queue?
(8) Given the waiting time in queue in (7), find the average time spent in the system.