2.4 Relationship between MOP

MOP

$$\mathbf{L_s , L_q , W_s , W_q , U , \lambda , \mu}$$

1. $L_s \Longleftrightarrow L_q$

$$E[\text{ \# in System}] = E[\text{\# in Queu}] + E[\text{\# in service}]$$

$$L_s = L_q + E[\text{\# in service}]$$

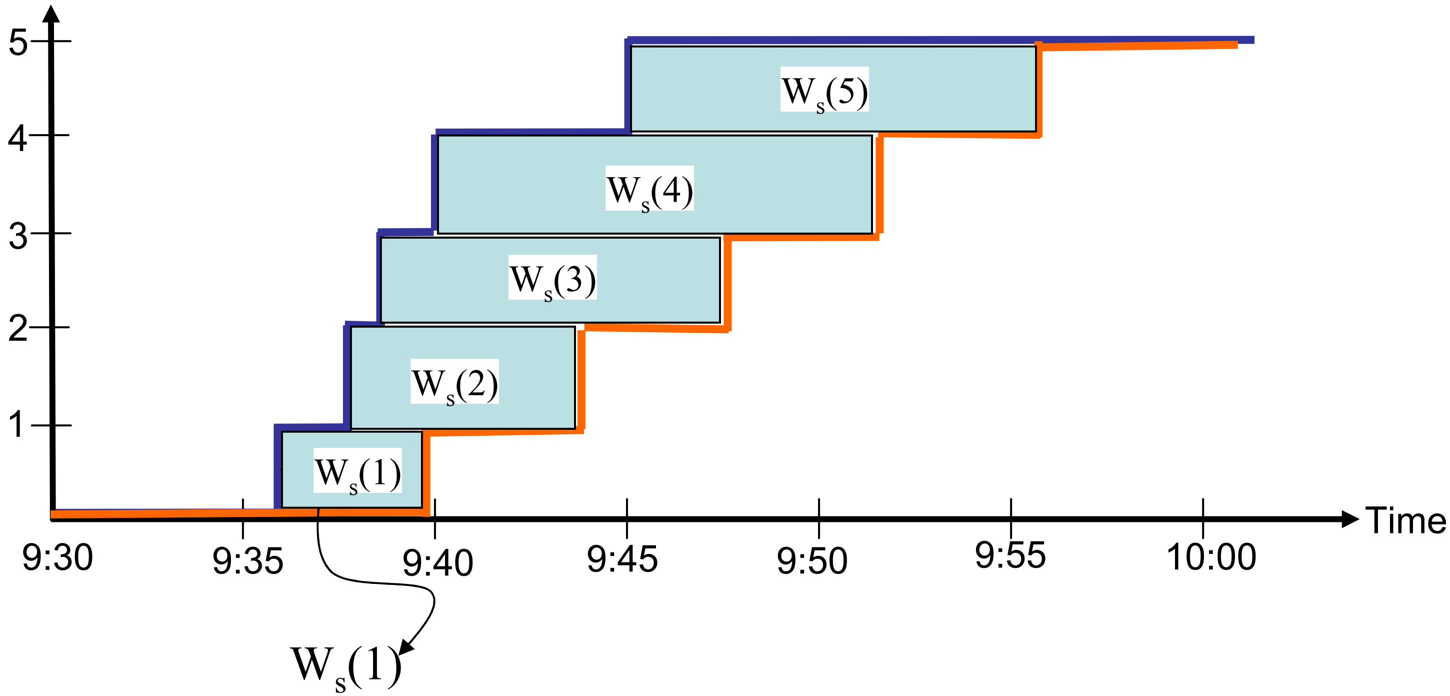
2. $W_s \Longleftrightarrow W_q$

$$E[\text{ time in System}] = E[\text{time in Queu}] + E[\text{time in service}]$$

$$W_s = W_q + S$$

$$W_s = W_q + (1/\mu)$$

2.4 Relationship between MOP

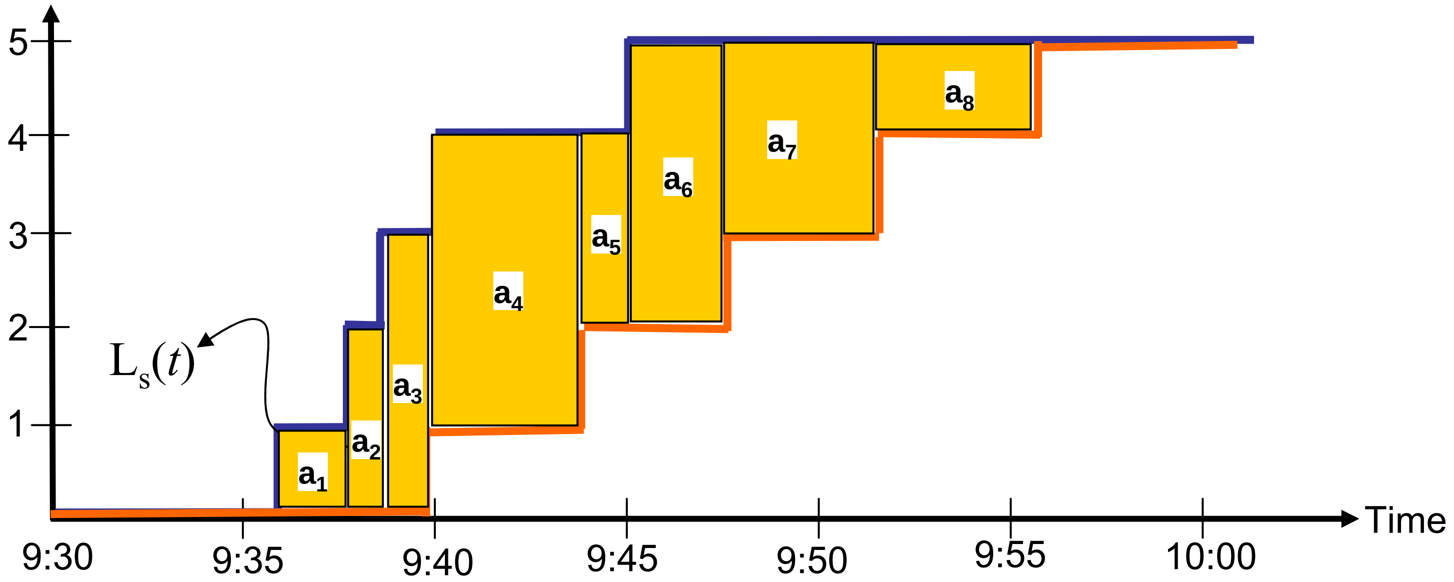


total time in system

$$= W_s(1) + W_s(2) + W_s(3) + W_s(4) + W_s(5)$$

$$= \overline{\overline{W}}_s$$

2.4 Relationship between MOP



$$\int_a^b L_s(t)\,dt = a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8$$

$$= \text{Area between } A(t) \text{ and } D_s(t)$$

$$= \overline{\overline{W}}_s$$

2.4 Relationship between MOP

$$\int_a^b L_s(t) \quad dt \quad = \quad \overline{\overline{W}}_s$$

Multiply LHS by $\dfrac{b-a}{b-a}$ and RHS by $\dfrac{N}{N}$

$$\frac{b-a}{b-a}\int_a^b L_s(t) \quad dt \quad = \quad \overline{\overline{W}}_s \frac{N}{N}$$

$L_s =$ $\qquad\qquad\qquad\qquad$ $= W_s$

$$(b-a)\ \mathbf{L_s} \quad = \quad N\ \mathbf{W_s}$$

$$\mathbf{L_s} \quad = \quad \underbrace{\frac{N}{b-a}}_{=\lambda} \quad \mathbf{W_s}$$

2.4 Relationship between MOP

Little's Formula

$$L_s = \lambda W_s$$

$$L_q = \lambda W_q$$

Stable Systems

2.4 Relationship between MOP

Example

n	$\mathrm{AT}(n)$	$\mathrm{DT_q}(n)$	$\mathrm{DT_s}(n)$	$\mathrm{W_q}(n)$	$\mathrm{W_s}(n)$	$\mathrm{S}(n)$
1	9:36	9:36	9:40	0:00	0:04	0:04
2	9:37	9:40	9:44	0:03	0:07	0:04
3	9:38	9:44	9:48	0:06	0:10	0:04
4	9:40	9:48	9:52	0:08	0:12	0:04
5	9:45	9:52	9:56	0:07	0:11	0:04
Total				24 min	44 min	20 min

$$\mathrm{E}[\text{time in queue}] = \mathrm{W_q} = (0+3+6+8+7)/5 = \mathbf{4.8\ min}$$

$$\mathrm{E}[\text{time in service}] = \mathrm{S} = (4+4+4+4+4)/5 = \mathbf{4\ min}$$

$$\mathrm{W_s} = \mathrm{W_q} + \mathrm{S} = 4.8 + 4 = 8.8$$

$$\mathrm{E}[\text{time in system}] = \mathrm{W_s} = (4+7+10+12+11)/5 = \mathbf{8.8\ min}$$

2.4 Relationship between MOP

Example

Time	$L_q(t)$	Interval	$T.L_q$
9:30−9:37	0	7	0
9:37−9:38	1	1	1
9:38−9:44	2	6	12
9:44− 9:45	1	1	1
9:45− 9:48	2	3	6
9:48− 9:52	1	4	4
9:52− 10:00	0	8	0
Total			**24**

Time	$L_s(t)$	Interval	$T.L_s$
9:30−9:36	0	6	0
9:36−9:37	1	1	1
9:37−9:38	2	1	2
9:38−9:44	3	6	18
9:44− 9:45	2	1	2
9:45− 9:48	3	3	9
9:48− 9:52	2	4	8
9:52− 9:56	1	4	4
9:56−10:00	0	4	0
Total			**44**

E[# in queue] = $L_q = 24/(30 \text{ min.}) = \mathbf{0.8}$

E[# in service]= $0.P_{\{n=0\}} + 1.P_{\{n>1\}} = 1 - P_0 = 1 - 0.333 = \mathbf{0.667}$

$$L_s = L_q + E[\text{\# in service}] = 0.8 + 0.667 = 1.467$$

E[# in system] = $L_s = 44/(30 \text{ min.}) = \mathbf{1.467}$

2.4 Relationship between MOP

Example

n	$AT(n)$	$DT_q(n)$	$DT_s(n)$	$W_q(n)$	$W_s(n)$	$S(n)$
1	9:36	9:36	9:40	0:00	0:04	0:04
2	9:37	9:40	9:44	0:03	0:07	0:04
3	9:38	9:44	9:48	0:06	0:10	0:04
4	9:40	9:48	9:52	0:08	0:12	0:04
5	9:45	9:52	9:56	0:07	0:11	0:04
Total				24 min	44 min	20 min

E[time in queue] = W_q = (0 + 3 + 6 + 8 + 7)/5 = **4.8 min**

E[time in system] = W_s = (4 + 7 + 10 + 12 + 11)/5 = **8.8 min**

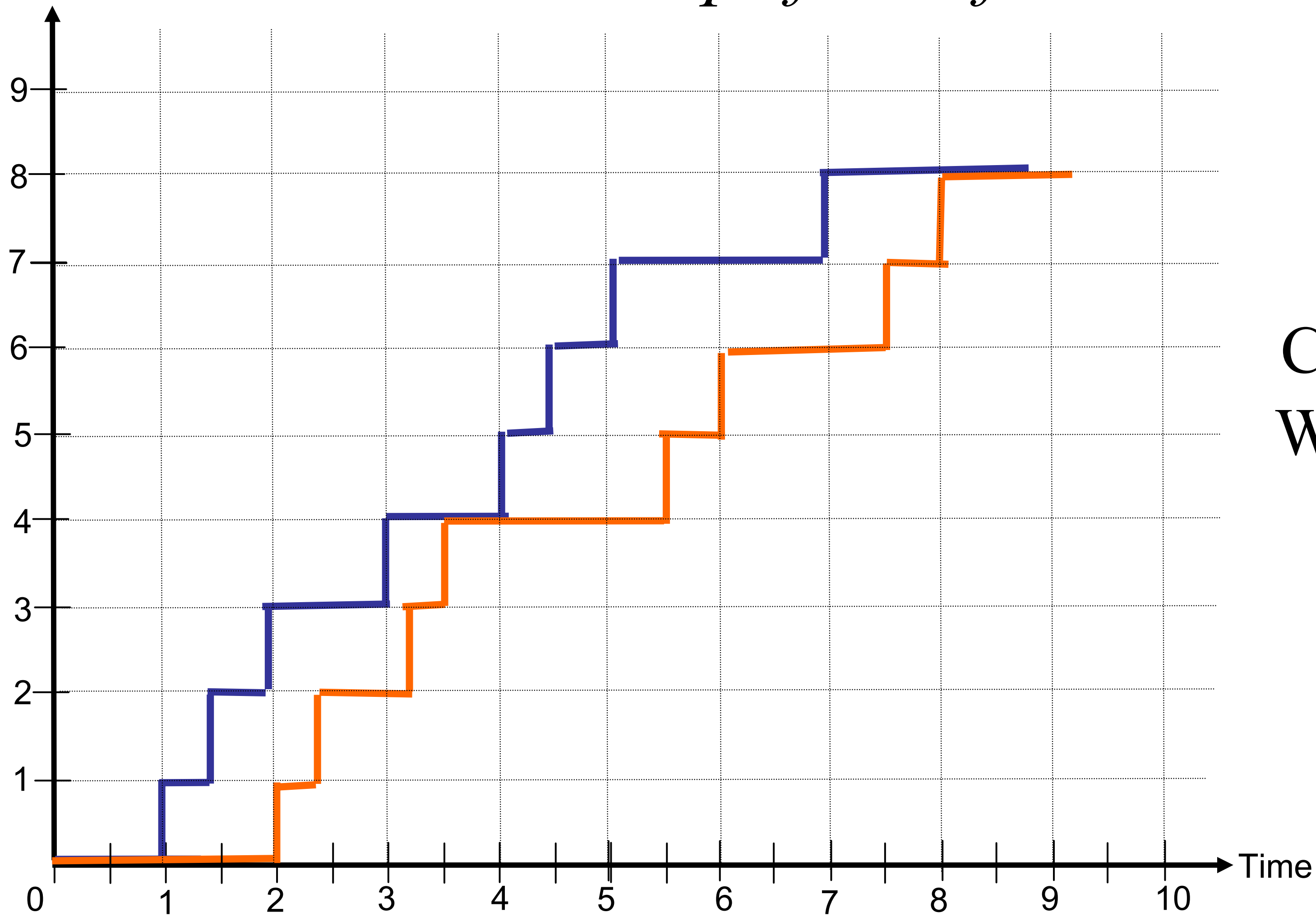
Arrival rate = λ = 1/E[T] = 1/3 customers per min

By Little's Formula: $L_s = \lambda W_s = (1/3)8.8 = 2.93$ customers

By Little's Formula: $L_q = \lambda W_q = (1/3)4.8 = 1.6$ customers

2.4 Relationship between MOP

Exercise : *use relationships for to find all MOP*



Compute
W , L , λ