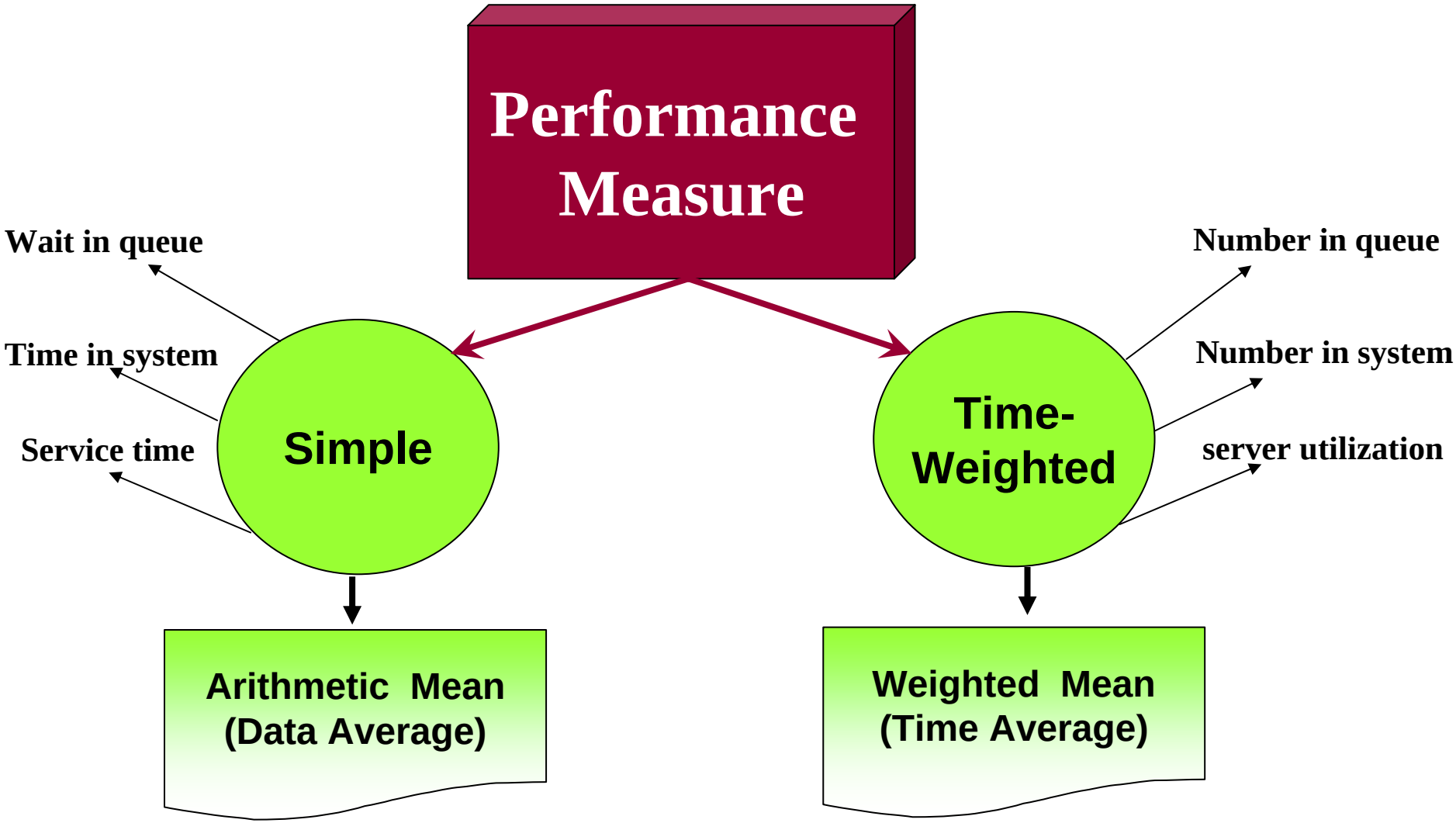
2.3 Calculating Measures of Performance



2.3 Calculating Measures of Performance

2.3.1 Average Waiting Time in Queue (W_q)

- W_q = Expected waiting time in queue
- Simple observations
- Random variable
- Observation: $W_q(1)$, $W_q(2)$, . . . , $W_q(N)$

$$W_q = \frac{\sum_{n=1}^{N} W_q(n)}{N}$$

$$\sigma^2_{Wq} = \frac{\sum_{n=1}^{N} \left(W_q(n) - W_q\right)^2}{N}$$

2.3 Calculating Measures of Performance

2.3.2 Average Waiting Time in System (W_s)

- W_s = Expected time spent in system
- Simple observations
- Random variable
- Observation: $W_s(1)$, $W_s(2)$, . . . , $W_s(N)$

$$W_s = \frac{\sum_{n=1}^{N} W_s(n)}{N}$$

$$\sigma^2_{Ws} = \frac{\sum_{n=1}^{N} \left(W_s(n) - W_s\right)^2}{N}$$

2.3 Calculating Measures of Performance

Example

n	AT(n)	$DT_q(n)$	$DT_s(n)$	$W_q(n)$	$W_s(n)$	S(n)
1	9:36	9:36	9:40	0:00	0:04	0:04
2	9:37	9:40	9:44	0:03	0:07	0:04
3	9:38	9:44	9:48	0:06	0:10	0:04
4	9:40	9:48	9:52	0:08	0:12	0:04
5	9:45	9:52	9:56	0:07	0:11	0:04
Total				24 min	44 min	20 min

$$W_q = (0 + 3 + 6 + 8 + 7)/5 = \mathbf{4.8\ min}$$

$$\sigma^2_{Wq} = [(4.8)^2 + (1.8)^2 + (1.2)^2 + (3.2)^2 + (2.2)^2]\ /5 = 8.41$$

$$W_s = (4 + 7 + 10 + 12 + 11)/5 = \mathbf{8.8\ min}$$

$$\sigma^2_{Ws} = [(-4.8)^2 + (-1.8)^2 + (1.2)^2 + (3.2)^2 + (2.2)^2]\ /5 = 8.41$$

2.3 Calculating Measures of Performance

2.3.3 Average Service Time (S)

- S = Expected service time
- Simple observations
- Random variable
- Observation: S(1) , S(2) , . . . , S(N)

$$S = \frac{\sum_{n=1}^{N} S(n)}{N} \qquad \sigma_s^2 = \frac{\sum_{n=1}^{N} \left(S(n) - S\right)^2}{N}$$

- Speed of service = Rate of service = μ
  = number of served customers per unit-time
  = 1/S customers pet unit-time

2.3 Calculating Measures of Performance

Example

n	AT(n)	$DT_q(n)$	$DT_s(n)$	$W_q(n)$	$W_s(n)$	S(n)
1	9:36	9:36	9:40	0:00	0:04	0:04
2	9:37	9:40	9:44	0:03	0:07	0:04
3	9:38	9:44	9:48	0:06	0:10	0:04
4	9:40	9:48	9:52	0:08	0:12	0:04
5	9:45	9:52	9:56	0:07	0:11	0:04
Total				24 min	44 min	20 min

$$S = (4 + 4 + 4 + 4 + 4)/5 = \mathbf{4\ min}$$

$$\sigma^2_{Wq} = [(0)^2 + (0)^2 + (0)^2 + (0)^2 + (0)^2]\ /5 = 0$$

2.3 Calculating Measures of Performance

2.3.4 Average Queue Length (L_q)

- Time-weighted observation
- Random variable
- Observation: function $L_q(t)$ and $t \in [a, b]$

$$L_q = \frac{\int_a^b L_q(t)\ dt}{b-a}$$

$$\sigma^2_{Lq} = \frac{\int_a^b \left(L_q(t) - L_q\right)^2\ dt}{b-a}$$

2.3 Calculating Measures of Performance

2.3.5 Average Number of Customers in System (L_s)

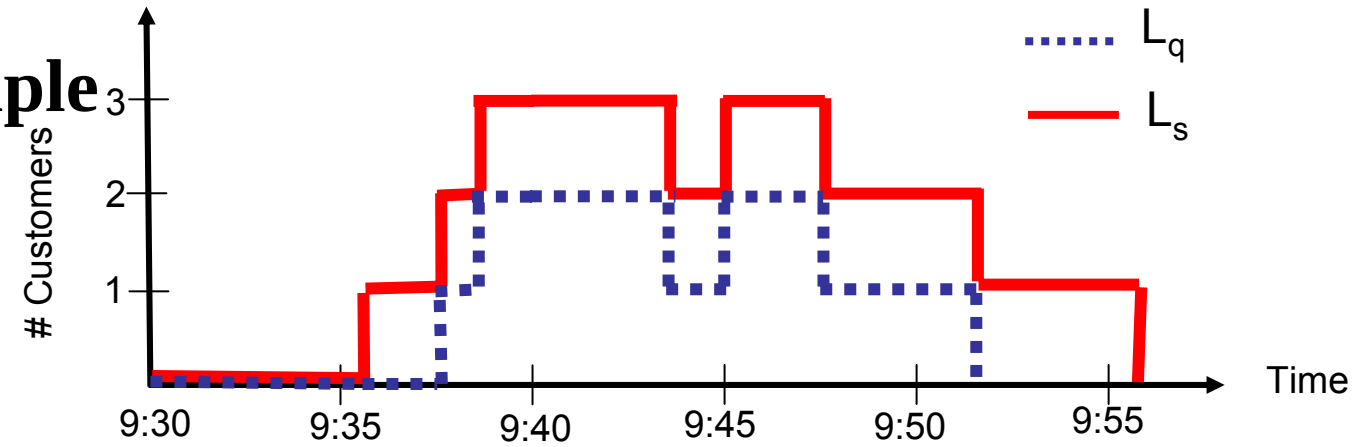
- Time-weighted observation
- Random variable
- Observation: function $L_s(t)$ and $t \in [a, b]$

$$L_s = \frac{\int_a^b L_s(t)\ dt}{b-a}$$

$$\sigma^2_{Ls} = \frac{\int_a^b \left(L_s(t) - L_s\right)^2\ dt}{b-a}$$

2.3 Calculating Measures of Performance

Example



Time	$L_q(t)$	Interval
9:30−9:37	0	7
9:37−9:38	1	1
9:38−9:44	2	6
9:44− 9:45	1	1
9:45− 9:48	2	3
9:48− 9:52	1	4
9:52− 10:00	0	8

Time	$L_s(t)$	Interval
9:30−9:36	0	6
9:36−9:37	1	1
9:37−9:38	2	1
9:38−9:44	3	6
9:44− 9:45	2	1
9:45− 9:48	3	3
9:48− 9:52	2	4
9:52− 9:56	1	4
9:56−10:00	0	4

2.3 Calculating Measures of Performance

Example

Time	$L_q(t)$	Interval	$T.L_q$
9:30−9:37	0	7	0
9:37−9:38	1	1	1
9:38−9:44	2	6	12
9:44− 9:45	1	1	1
9:45− 9:48	2	3	6
9:48− 9:52	1	4	4
9:52− 10:00	0	8	0
Total			**24**

$$L_q = 24/(10{:}00 - 9{:}30)$$
$$= 24 / 30 = \mathbf{0.8} \text{ Customers}$$

Time	$L_s(t)$	Interval	$T.L_s$
9:30−9:36	0	6	0
9:36−9:37	1	1	1
9:37−9:38	2	1	2
9:38−9:44	3	6	18
9:44− 9:45	2	1	2
9:45− 9:48	3	3	9
9:48− 9:52	2	4	8
9:52− 9:56	1	4	4
9:56−10:00	0	4	0
Total			**44**

$$L_s = 44/(10{:}00 - 9{:}30)$$
$$= 44 / 30 = \mathbf{1.467} \text{ Customers}$$

2.3 Calculating Measures of Performance

Example

Time	$L_q(t)$	T	$L_q(t)-L_q$	$T.(L_q(t)-L_q)^2$
9:30−9:37	0	7	-0.8	4.48
9:37−9:38	1	1	0.2	0.04
9:38−9:44	2	6	1.2	8.64
9:44− 9:45	1	1	0.2	0.04
9:45− 9:48	2	3	1.2	4.32
9:48− 9:52	1	4	0.2	0.16
9:52− 10:00	0	8	-0.8	5.12
Total				**22.8**

$$\sigma^2_{Lq} = 22.8/(10{:}00 - 9{:}30)$$
$$= 22.8 / 30 = \mathbf{0.76} \text{ Customers}$$

Time	$L_s(t)$	T	$L_s(t)-L_s$	$T(L_s(t)-L_s)^2$
9:30−9:36	0	6	-1.467	12.913
9:36−9:37	1	1	-0.467	0.218
9:37−9:38	2	1	0.533	0.284
9:38−9:44	3	6	1.533	14.101
9:44− 9:45	2	1	0.533	0.284
9:45− 9:48	3	3	1.533	7.051
9:48− 9:52	2	4	0.533	1.136
9:52− 9:56	1	4	-0.467	0.873
9:56−10:00	0	4	-1.467	8.608
Total				**45.468**

$$\sigma^2_{Ls} = 45.468/(10{:}00 - 9{:}30)$$
$$= 45.468 / 30 = \mathbf{1.52} \text{ Customers}$$

2.3 Calculating Measures of Performance

2.3.5 Average Interarrival Time (E[T])

- T_n = time between arrivals n and $n-1$
  = AT(n)−AT($n-1$)
- Simple observation
- Random variable

$$E[T] = \frac{\sum_{n=1}^{N} T(n)}{N} = \frac{b-a}{N} \qquad \sigma_T^2 = \frac{\sum_{n=1}^{N} (T_n - E[T])^2}{N}$$

- Rate of arrivals = λ
  = number of arrivals per unit-time
  = 1/E[T] arrival per unit-time

2.3 Calculating Measures of Performance

Example

n	AT(n)	$DT_q(n)$	$DT_s(n)$	$W_q(n)$	$W_s(n)$	S(n)
1	9:36	9:36	9:40	0:00	0:04	0:04
2	9:37	9:40	9:44	0:03	0:07	0:04
3	9:38	9:44	9:48	0:06	0:10	0:04
4	9:40	9:48	9:52	0:08	0:12	0:04
5	9:45	9:52	9:56	0:07	0:11	0:04
Total				24 min	44 min	20 min

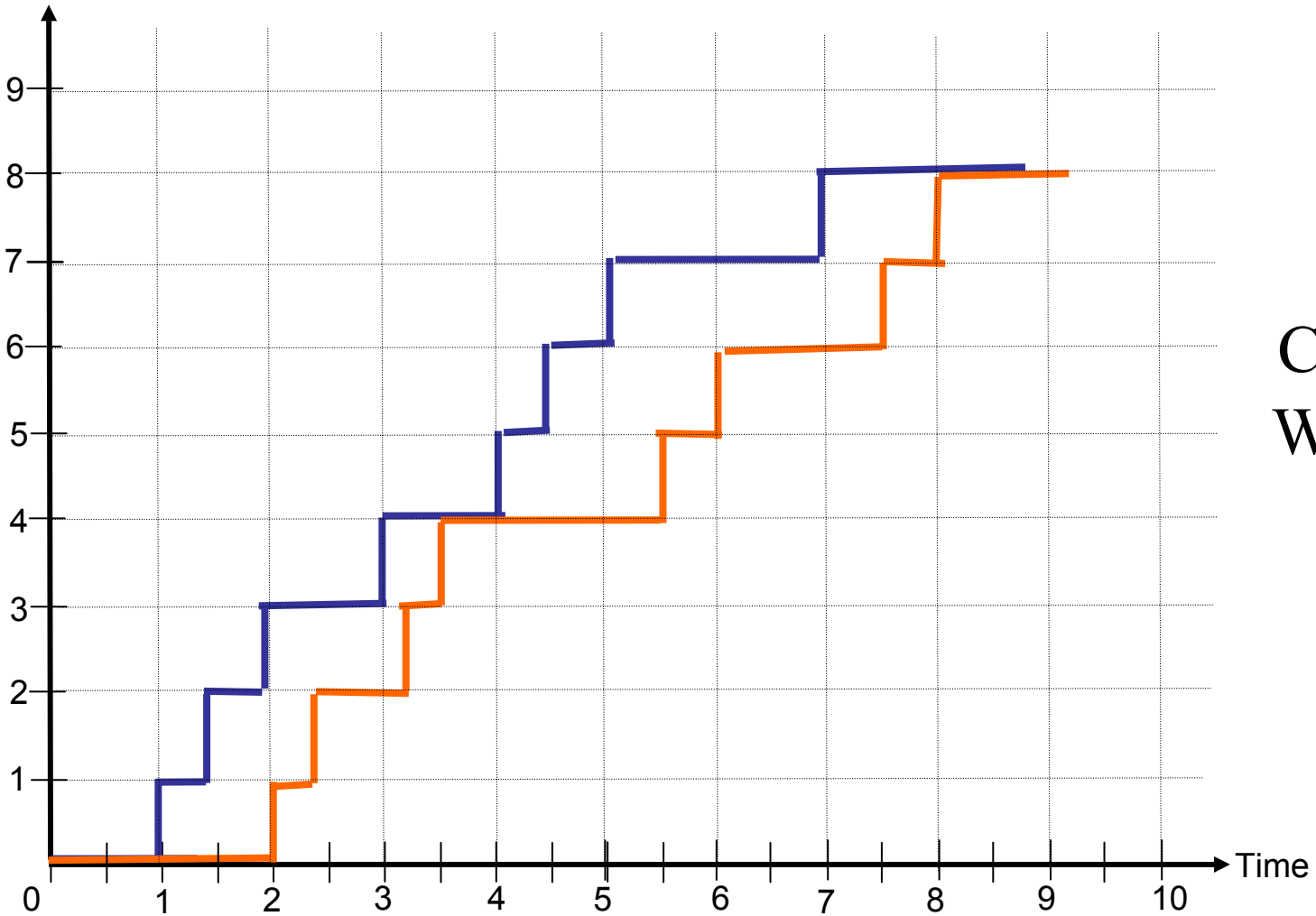
total interarrival time = 15

Average interarrival time =E[T] = 15/5 = **3 min.**

Arrival rate = λ = 1/3 customers per min.

2.3 Calculating Measures of Performance

Exercise



Compute
W , L , λ

2.3 Calculating Measures of Performance

2.3.6 Utilization of Service (U)

- U = percentage of time the server is busy
  = P{system not empty} = 1 − P{system is empty}
- Time-weighted observation
- Observation: intervals of empty system and $t \in [a\ ,\ b]$

$$P_0 = \frac{\sum_{\forall T} T_{(Ls=0)}}{b - a}$$

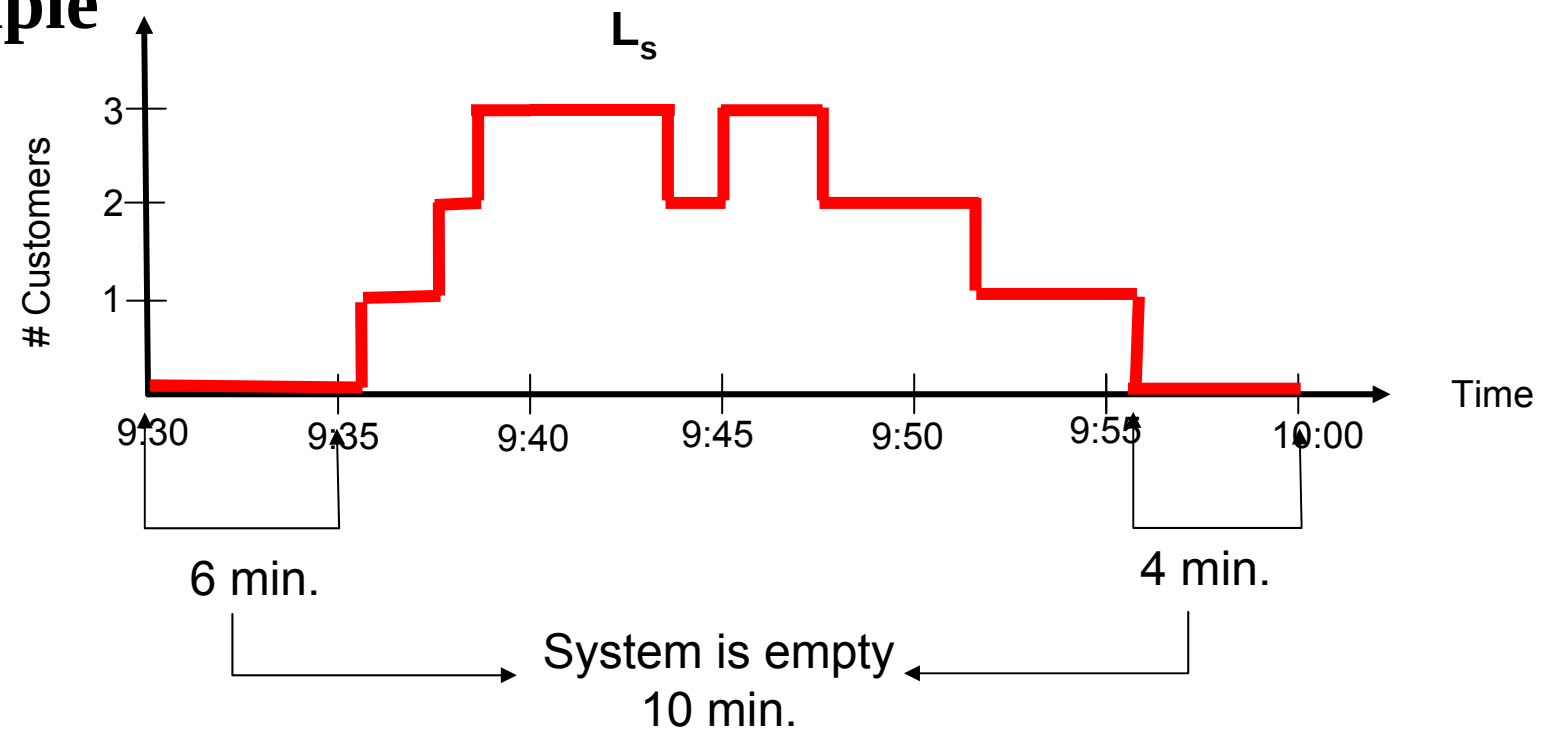
$T_{(Ls=0)}$ = period of time system is empty

P_0 = probability that system is empty

$$U = 1 - P_0$$

2.3 Calculating Measures of Performance

Example



$$P_0 = (6 + 4) / 30 = 0.333$$

$$U = 1 - P_0 = 0.667$$

Server is active 66.7% of the time

2.3 Calculating Measures of Performance

2.3.7 Probability of *n* Cusomers in System (P_n)

- Time-weighted observation
- Observation: L_s graph and $t \in [a, b]$

$$P_n = \frac{\sum_{\forall T} T_{(Ls=n)}}{b-a}$$

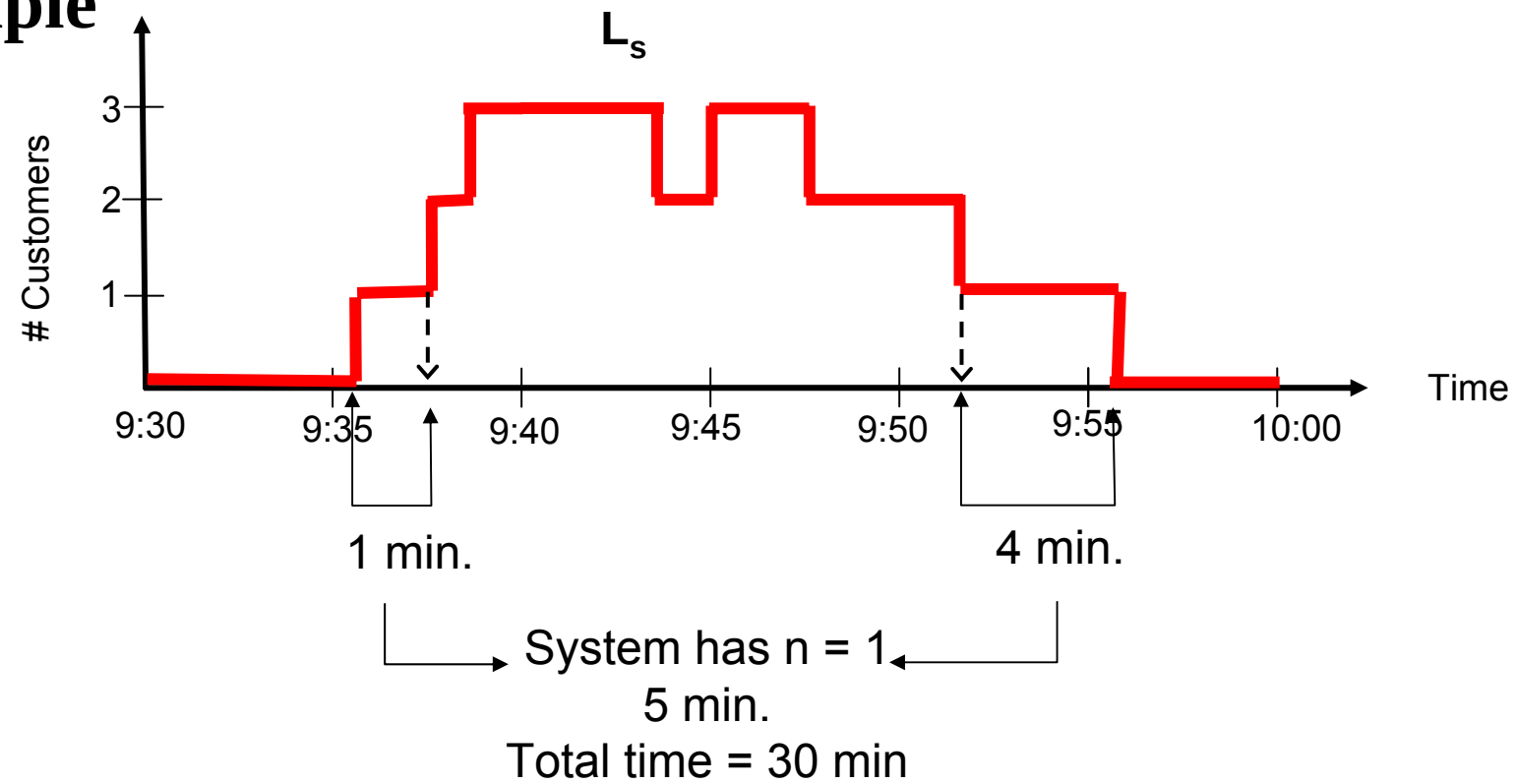
$T_{(Ls=n)}$ = period of time system has *n* customers

$$\sum_{n=0}^{\infty} P_n = 1$$

2.3 Calculating Measures of Performance

Example

n = 1

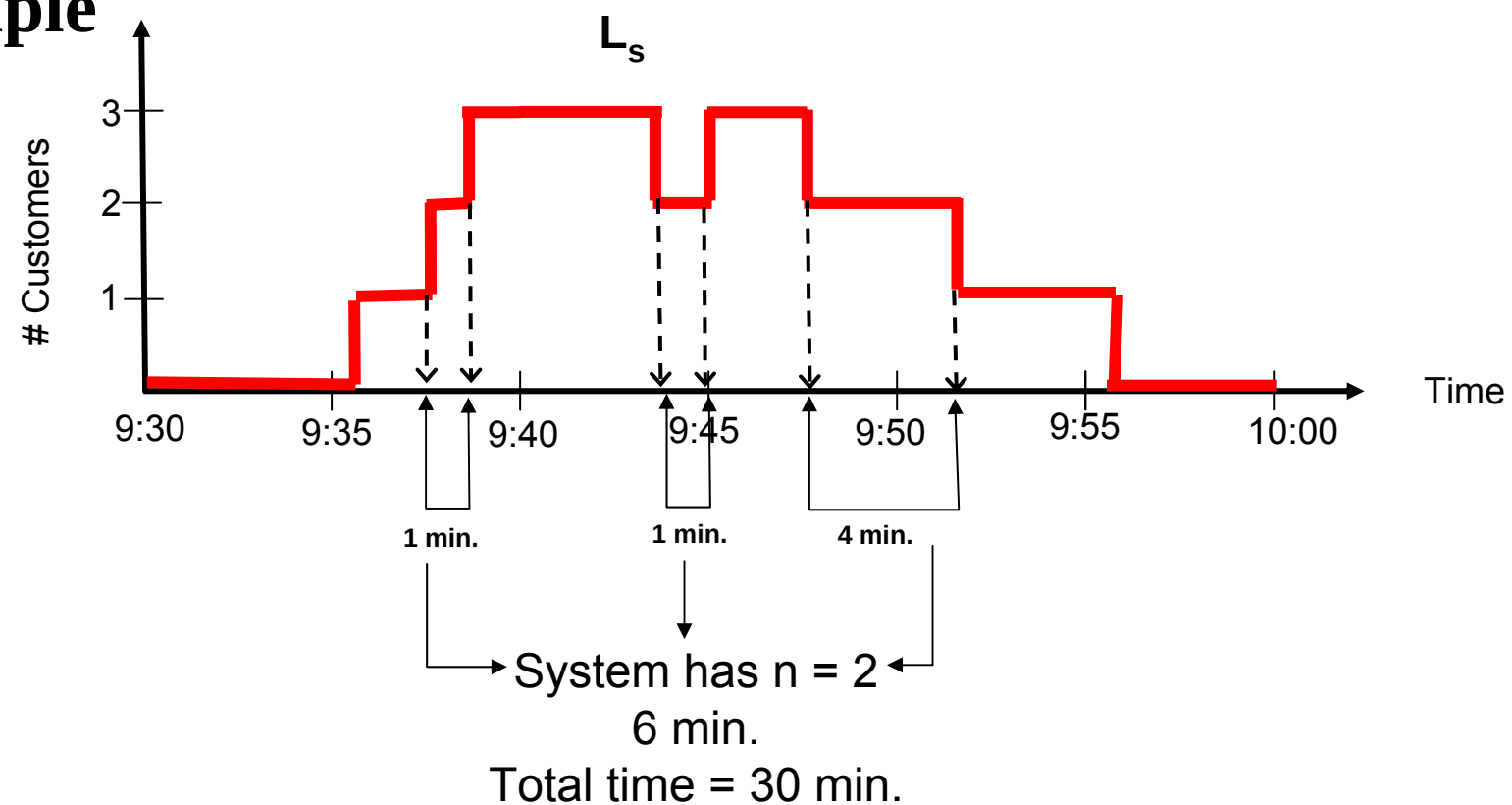


$$P_1 = (1 + 4) / 30 = 0.1667$$

2.3 Calculating Measures of Performance

Example

***n* = 2**



$$P_2 = (1 + 1 + 4) / 30 = 0.2$$