Chapter 2: Observation & Measures

- Definitions
- How to measure performance
- Key MOP
- Cumulative arrival & departure diagrams
- Averages and SD's of MOP

2.1 Measures of Performance (MOP)

Definitions

- <u>Arrival time:</u>

  The time at which a customer arrives to the queue

- <u>Departure time:</u>

  The time at which a customer completes service and leaves the system

- <u>Departure time from queue:</u>

  The time at which a customer leaves the queue to the service or to outside the system

- <u>Waiting time:</u>

  The period of time that a customer spends in the queue until he enters the service

- <u>Time in system:</u>

  The period of time that a customer spends in the system until he receives the service and leaves the system

2.1 Measures of Performance (MOP)

2.1.1 Customer MOP :

Waiting

- Time in queue : shorter better
- Service time: shorter better
- Customer waiting cost

Notes

- Random $\Rightarrow$ *average , standard deviation*
- Deterministic $\Rightarrow$ *function of time* $f(t)$
- Additional information
  - customers idle while waiting?
  - customers sit ?
  - customers know about how much to wait?
  - size of waiting room

2.1 Measures of Performance (MOP)

Cost of Service

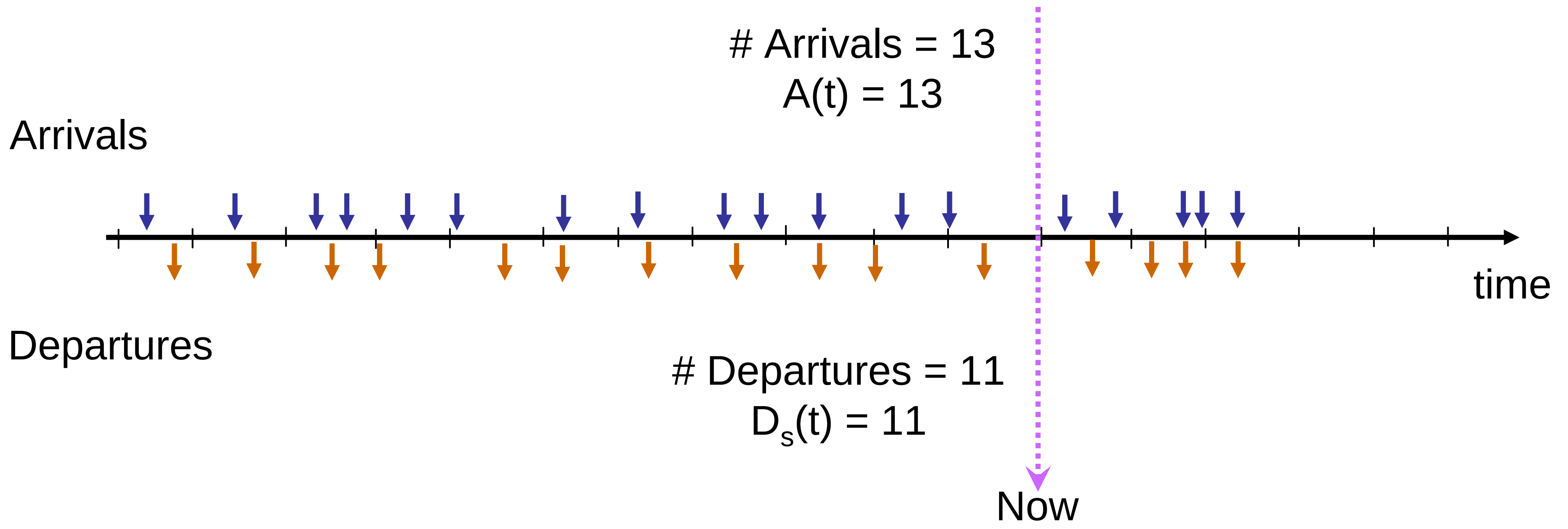
2.1.2 Server MOP

- Service time: shorter is more efficient
- Utilization : percentage of idle (less is better)
- Throughput: rate of departure (higher is better)
- Queue length: longer = more cost to accommodate

Notes

- Random $\Rightarrow$ *average , standard deviation*
- Deterministic $\Rightarrow$ *function of time* $f(t)$ or *function of queue*
- Additional information
  - server do something while idle while?
  - server idle while queue is present?
  - server productive (إنتاجي) ?

2.2 Cumulative Arrival and Departure



2.2 Cumulative Arrival and Departure

Definition:

$A(t)$ = cumulative arrivals *to the system from* 0 to t

$D_s(t)$ = cumulative departures *from the system* from 0 to t

$D_q(t)$ = cumulative departures *from the queue* from 0 to t

Notes:

- $A(t)$, $D_s(t)$, $D_q(t)$ are functions of time
- $A(t)$, $D_s(t)$, $D_q(t)$ are step functions
- $A(t)$, $D_s(t)$, $D_q(t)$ nondecreasing functions
- time 0 = starting of the analysis

2.2 Cumulative Arrival and Departure

Example

Consider the following data

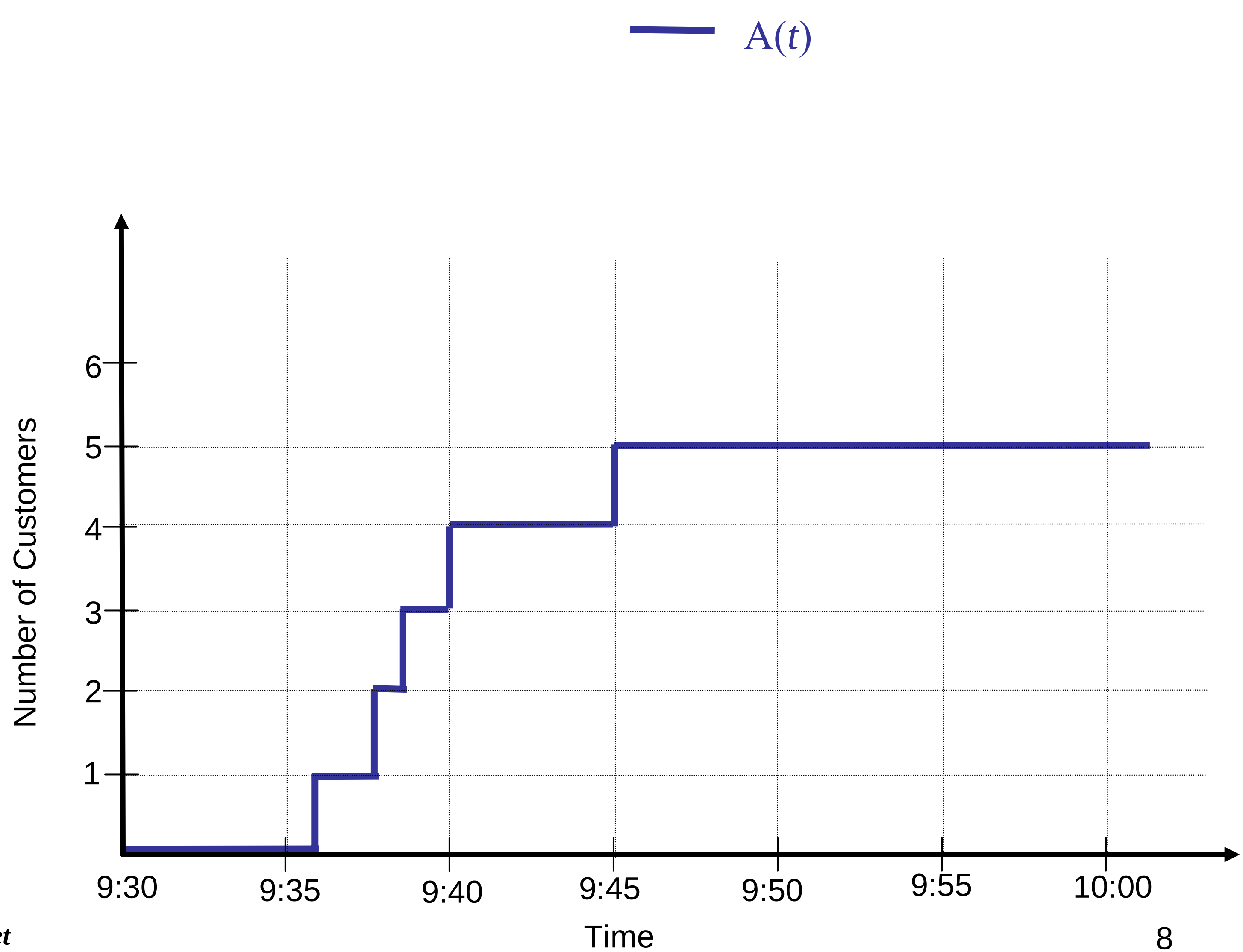
Customer	Arrival time	Departure queue	Departure system
1	9:36	9:36	9:40
2	9:37	9:40	9:44
3	9:38	9:44	9:48
4	9:40	9:48	9:52
5	9:45	9:52	9:56

2.2 Cumulative Arrival and Departure

Example

A(t) diagram

Cust.	Arrival time
1	9:36
2	9:37
3	9:38
4	9:40
5	9:45



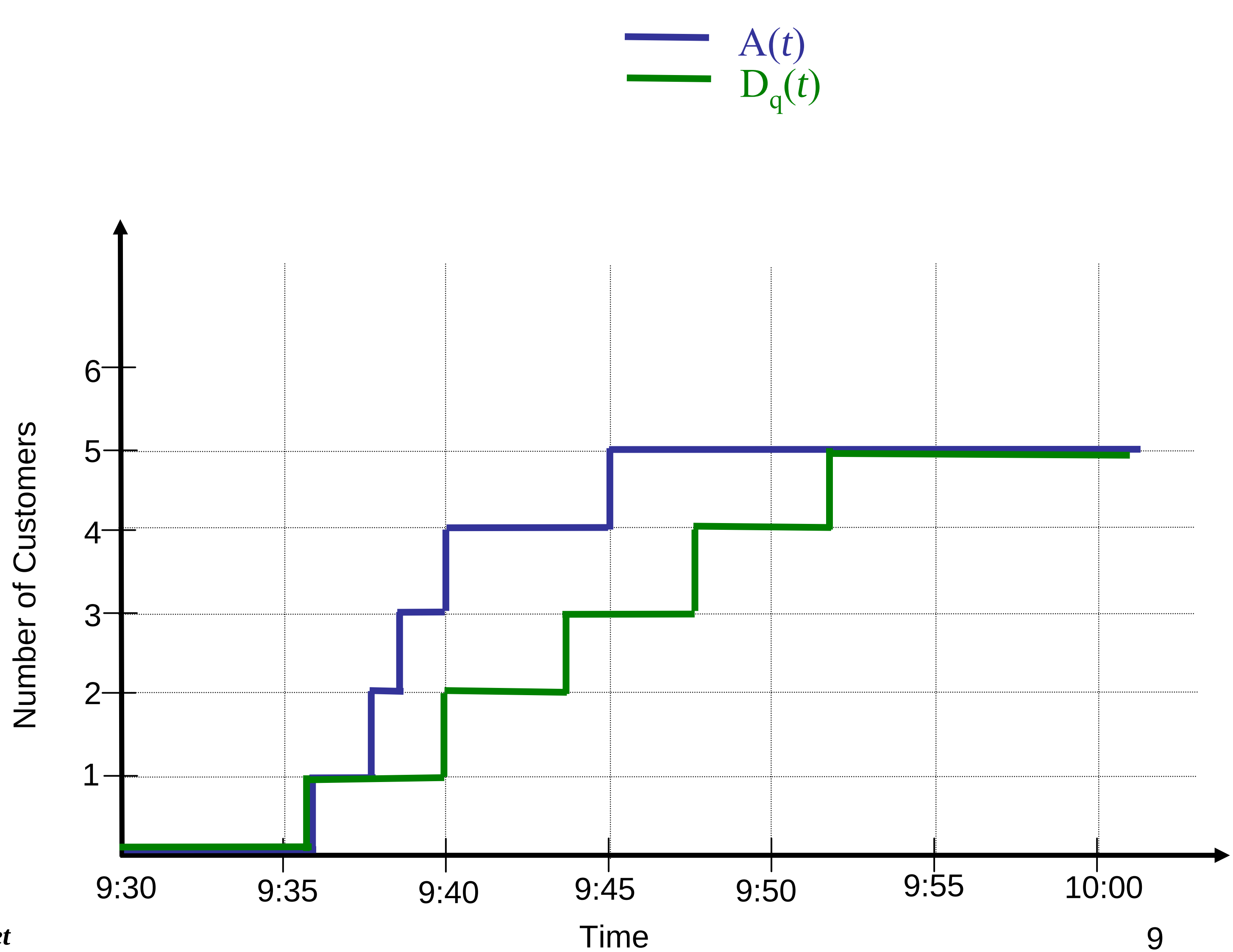
2.2 Cumulative Arrival and Departure

Example

A(t) diagram

$D_q(t)$ diagram

Cust.	Dep. queue
1	9:36
2	9:40
3	9:44
4	9:48
5	9:52



2.2 Cumulative Arrival and Departure

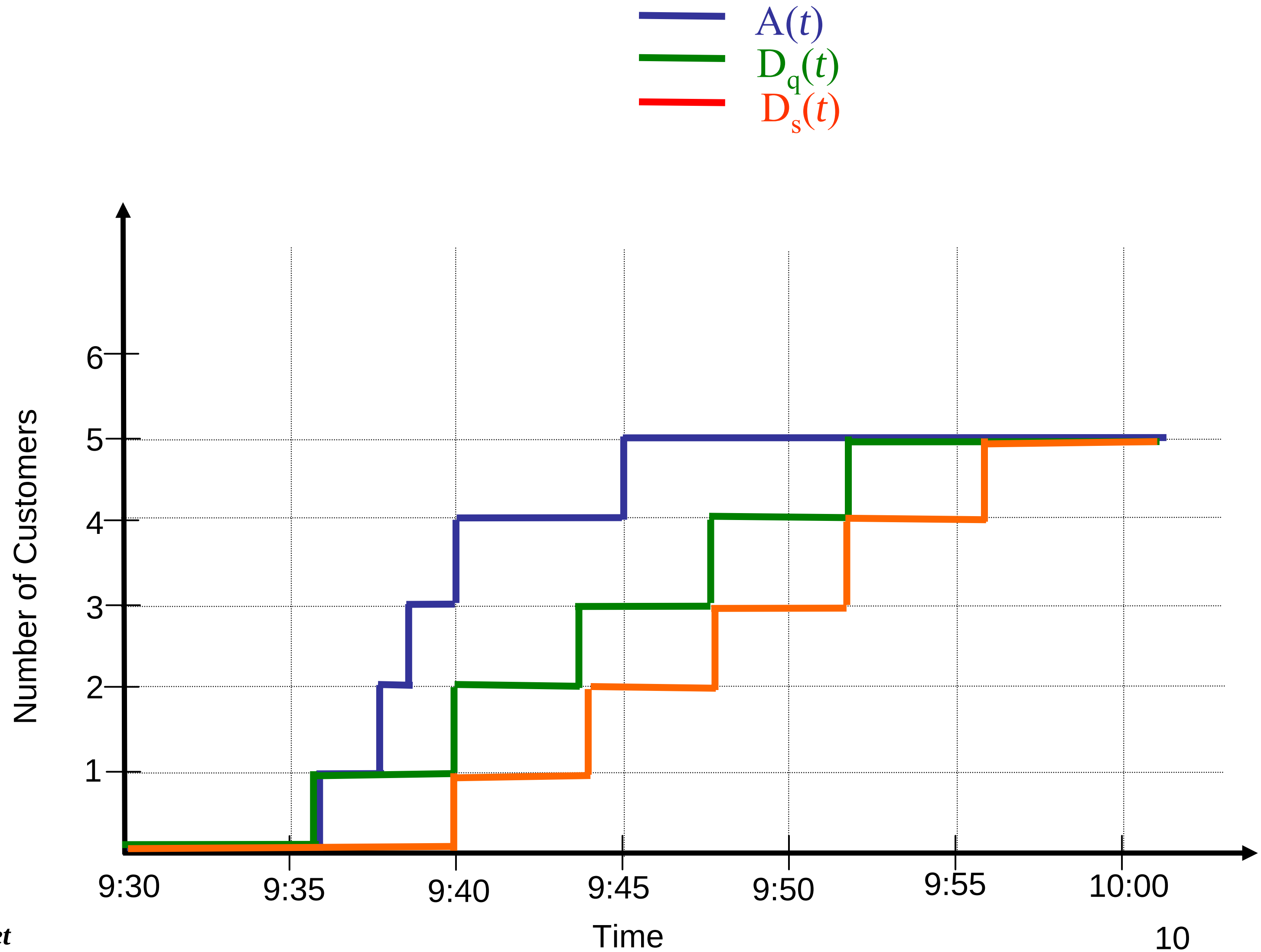
Example

A(t) diagram

$D_q(t)$ diagram

$D_s(t)$ diagram

Cust.	Dep. system
1	9:40
2	9:44
3	9:48
4	9:52
5	9:56



2.2 Cumulative Arrival and Departure

Exercise

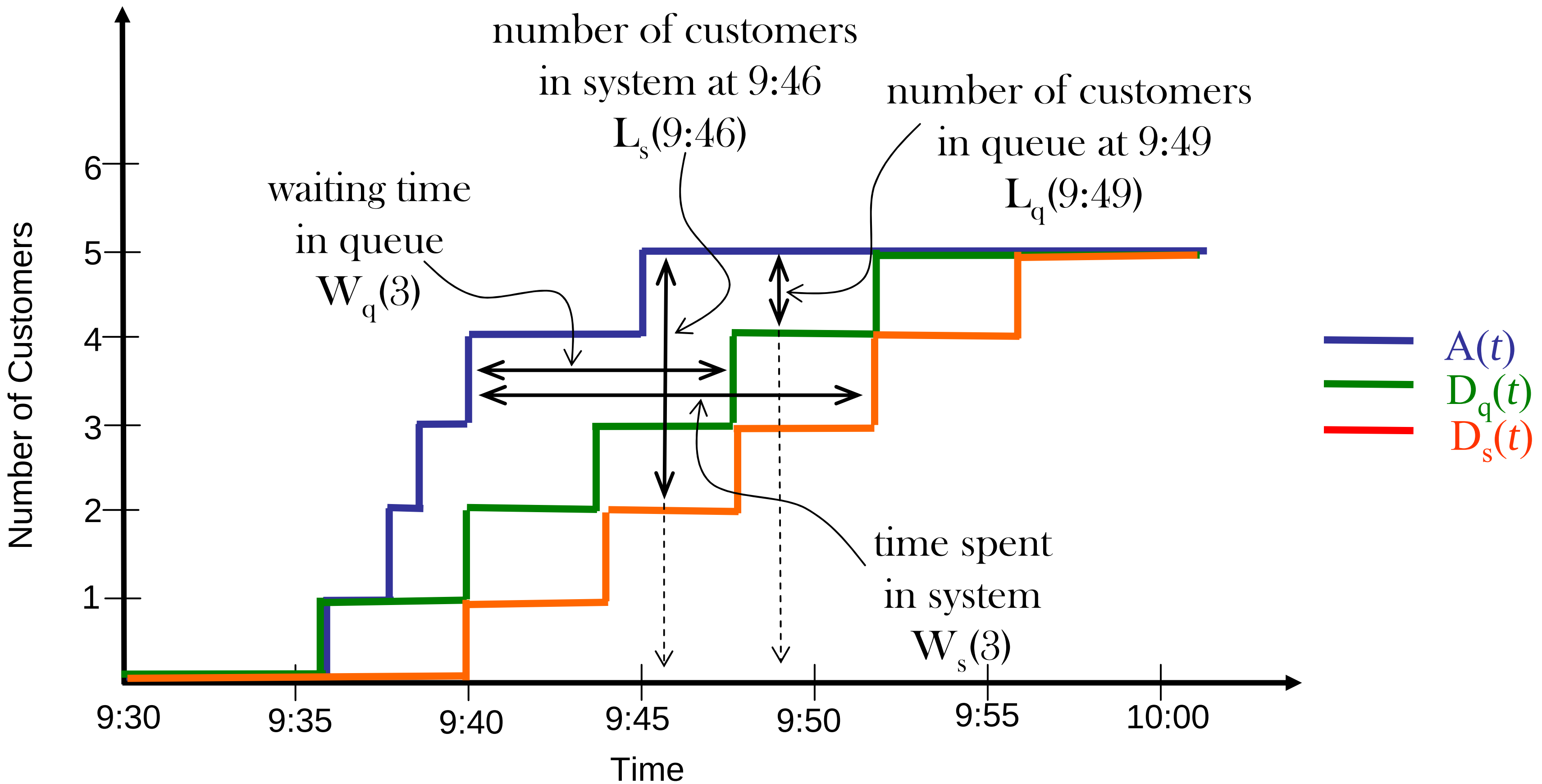
Draw $A(t)$, $D_q(t)$ and $D_s(t)$ for the following data

Cust.	Arrival time	Dep. queue	Dep. system
1	8:05	8:05	8:08
2	8:07	8:08	8:11
3	8:12	8:12	8:15
4	8:13	8:15	8:18
5	8:16	8:18	8:21
6	8:19	8:21	8:24
7	8:27	8:27	8:30

2.2 Cumulative Arrival and Departure

Example

A(t), $D_q(t)$,$D_s(t)$ diagram



2.2 Cumulative Arrival and Departure

2.2.1 Number of Customers

Definition

$L_q(t)$ = number of customers in queue at time t.

$L_s(t)$ = number of customers in system at time t.

Notes

- $L_q(t)$, $L_s(t)$ depend on time
- $L_q(t)$, $L_s(t)$ not increasing functions

2.2 Cumulative Arrival and Departure

2.2.1 Number of Customers

$$L_q(t) = A(t) - D_q(t)$$

$$L_s(t) = A(n) - D_s(t)$$

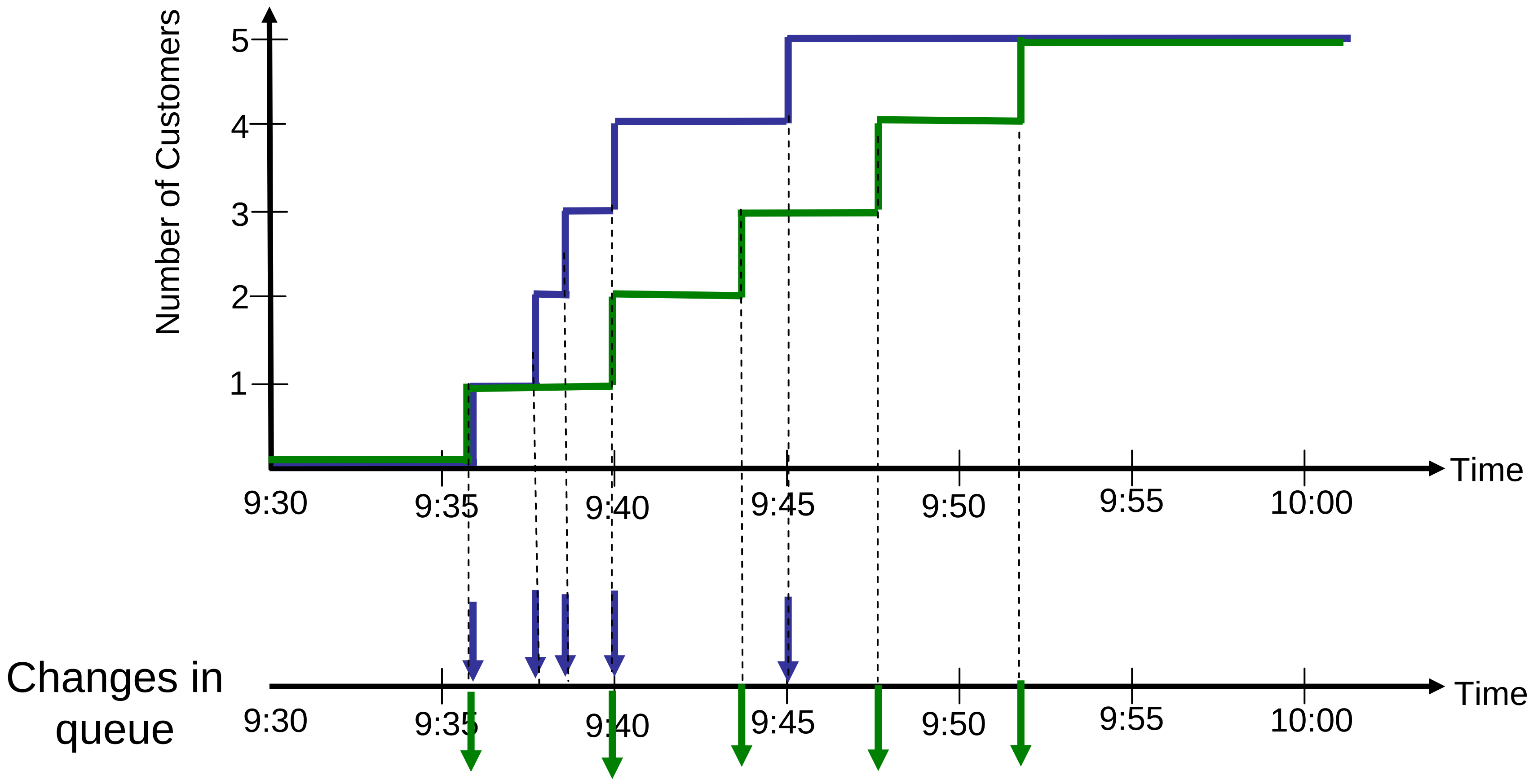
- $L_q(t)$ is defined from the $A(t)$ and $D_q(t)$ diagram
  - for every $A(t)$ change increase graph of $L_q(t)$
  - for every $D_q(t)$ change decrease graph of $L_q(t)$

- $L_s(t)$ is defined from the $A(t)$ and $D_s(t)$ diagram
  - for every $A(t)$ change increase graph of $L_s(t)$
  - for every $D_s(t)$ change decrease graph of $L_s(t)$

2.2 Cumulative Arrival and Departure

2.2.1 Number of Customers

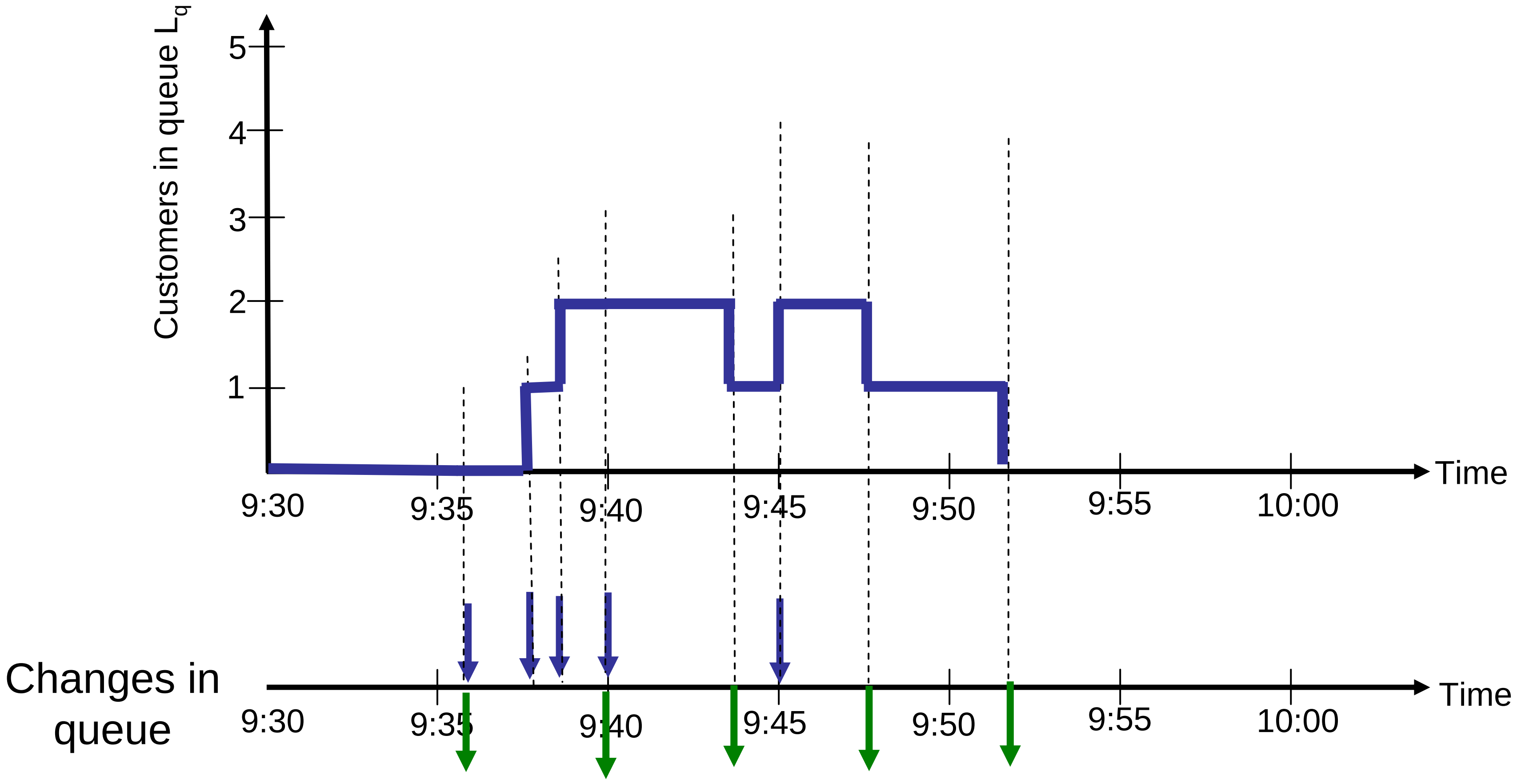
Example



2.2 Cumulative Arrival and Departure

2.2.1 Number of Customers

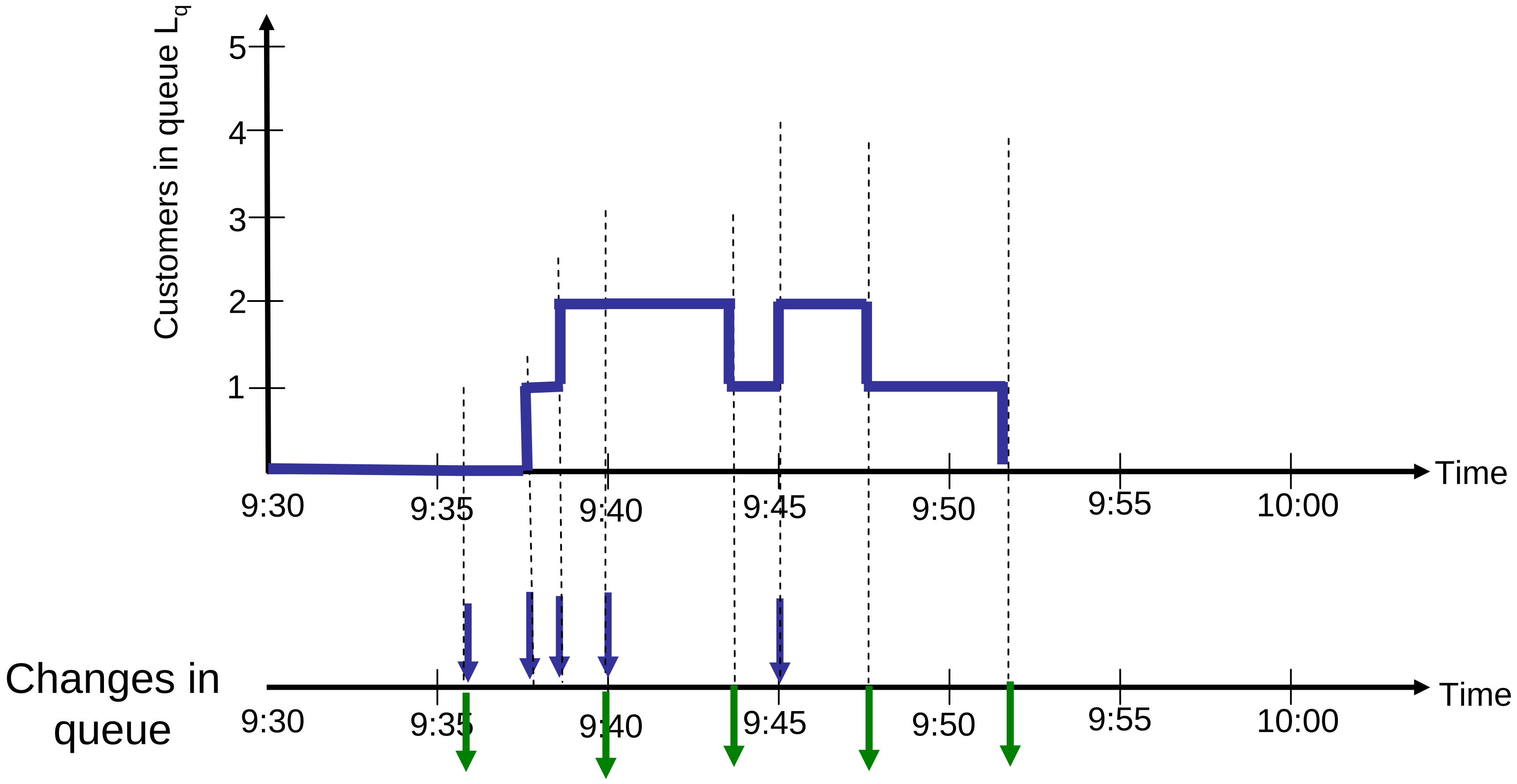
Example



2.2 Cumulative Arrival and Departure

2.2.1 Number of Customers

Example

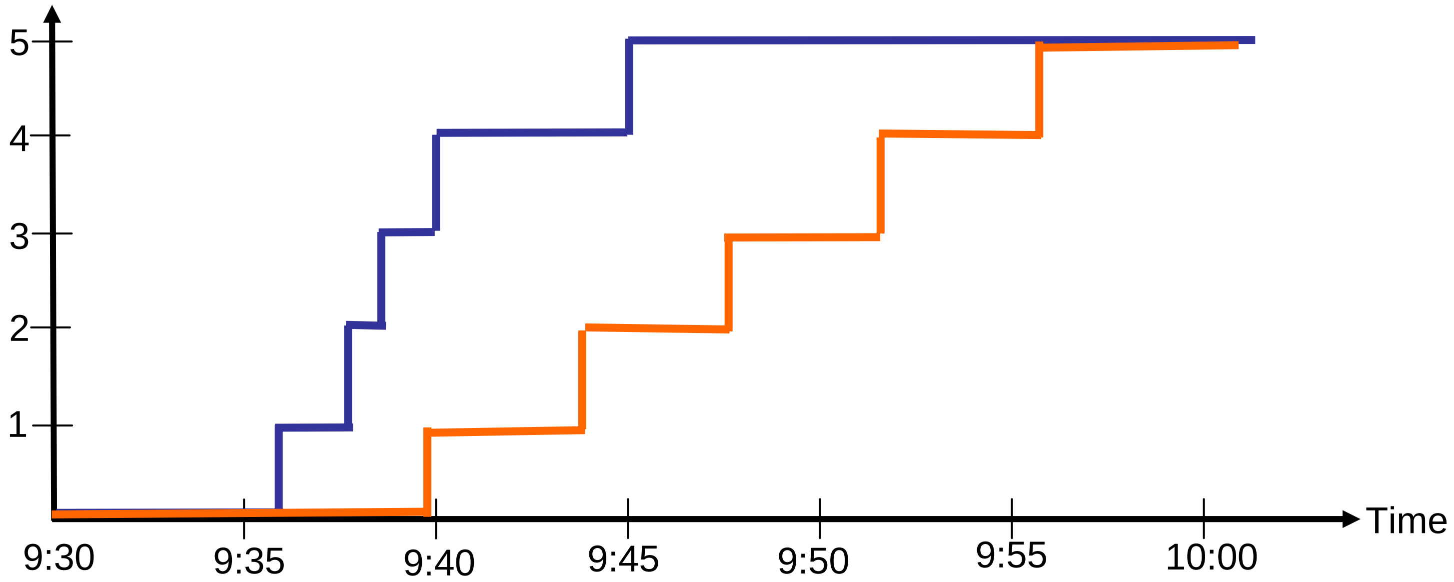


2.2 Cumulative Arrival and Departure

2.2.1 Number of Customers

Exercise

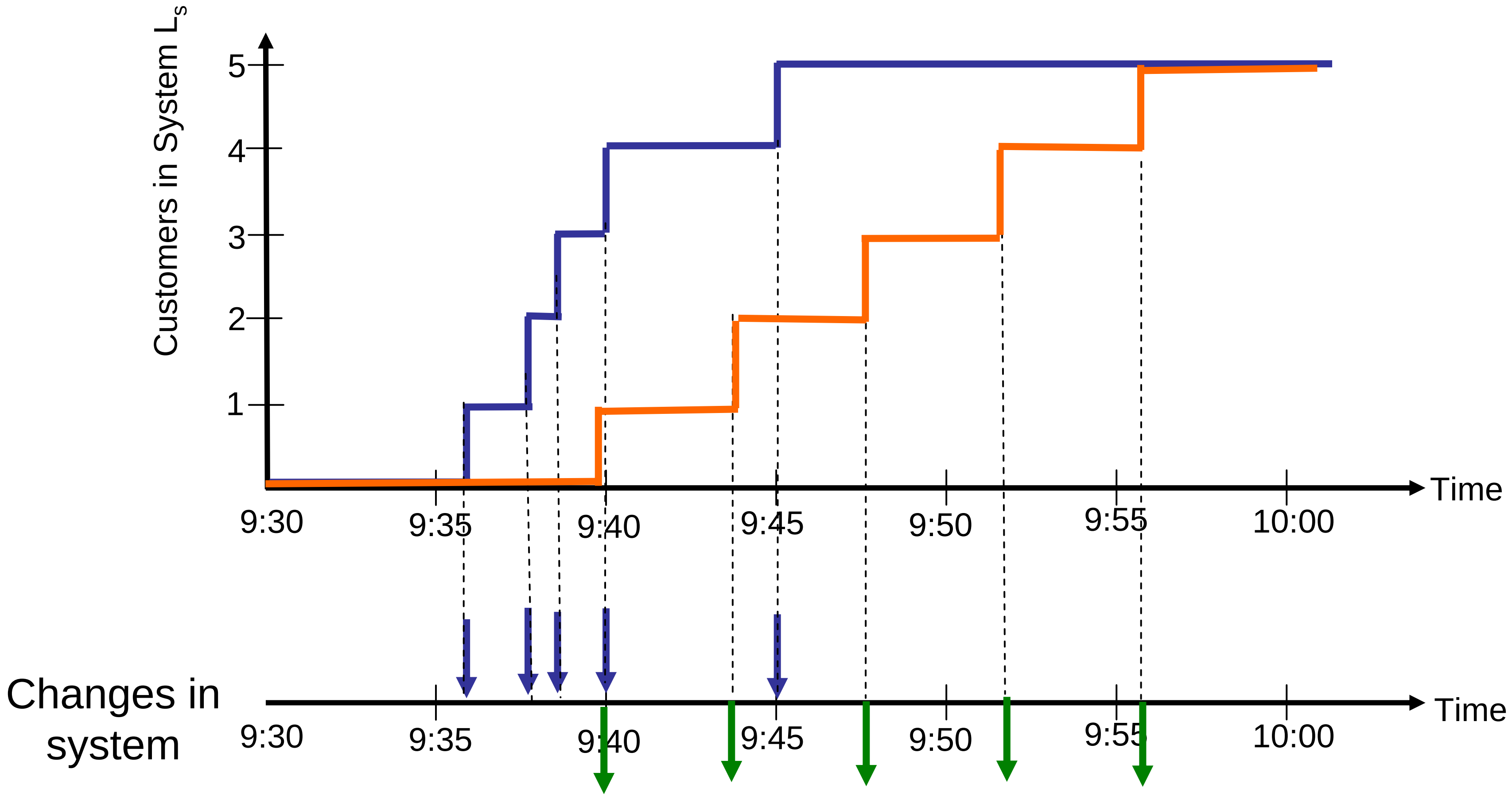
Draw $L_s(t)$ for the same data



2.2 Cumulative Arrival and Departure

2.2.1 Number of Customers

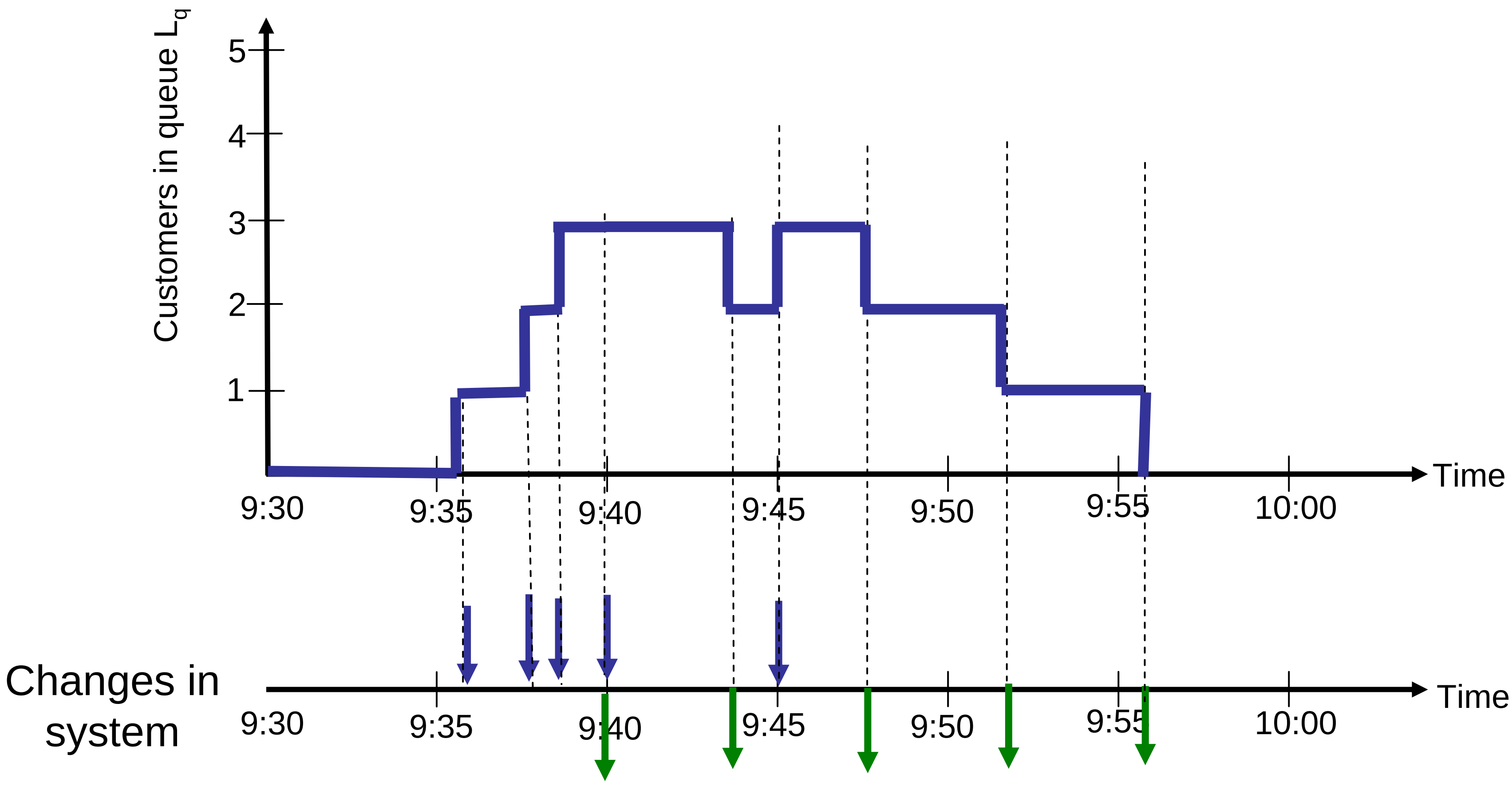
Exercise Draw $L_s(t)$ for the same data



2.2 Cumulative Arrival and Departure

2.2.1 Number of Customers

Exercise



2.2 Cumulative Arrival and Departure

2.2.2 Waiting Time

Definition

$T(n)$ = arrival time of customer number n.

$S(n)$ = Service time of customer number n.

$T_q(n)$ = departure time from queue of customer number n.

$T_s(n)$ = departure time from system of customer number n.

Notes

- $T(n)$, $T_q(n)$, $T_s(n)$ depend on customer number
- $T_q(n)$, $T_s(n)$ not always increasing

2.2 Cumulative Arrival and Departure

2.2.2 Waiting Time

Example:

n	$T(n)$	$T_q(n)$	$T_s(n)$
1	9:36	9:36	9:40
2	9:37	9:40	9:44
3	9:38	9:44	9:48
4	9:40	9:48	9:52
5	9:45	9:52	9:56

Definition

$W_q(n)$ = waiting time in queue for customer number n.

$W_s(n)$ = time in system for customer number n.

2.2 Cumulative Arrival and Departure

2.2.2 Waiting Time

Example:

n	$T(n)$	$T_q(n)$	$T_s(n)$
1	9:36	9:36	9:40
2	9:37	9:40	9:44
3	9:38	9:44	9:48
4	9:40	9:48	9:52
5	9:45	9:52	9:56

Definition

$W_q(n)$ = waiting time in queue for customer number n.

$W_s(n)$ = time in system for customer number n.

2.2 Cumulative Arrival and Departure

2.2.2 Waiting Time

FCFS

$$W_q(n) = T_q(n) - T(n)$$

$$W_s(n) = T_s(n) - T(n)$$

$$S(n) = W_s(n) - W_q(n)$$

Example

n	$T(n)$	$T_q(n)$	$T_s(n)$
1	9:36	9:36	9:40
2	9:37	9:40	9:44
3	9:38	9:44	9:48
4	9:40	9:48	9:52
5	9:45	9:52	9:56