Chapter 4: Birth and Death Queuing Models

- Definitions
- Steady-State Analysis
- Birth/Death Queueing Modes
- Applications

4.1 Birth and Death Processes

Definition:

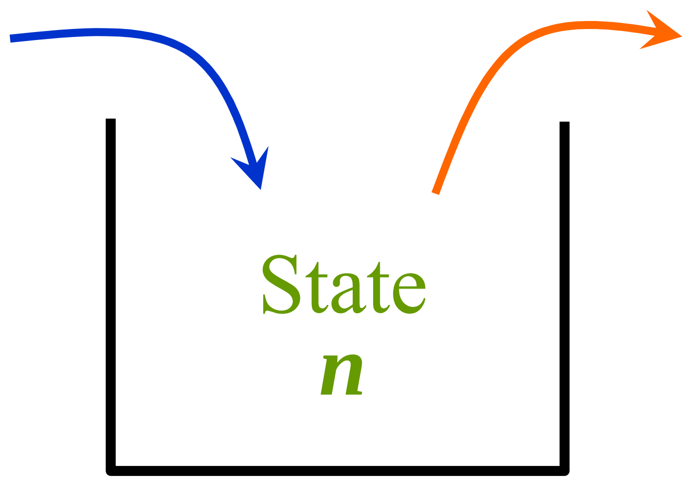
- Let *n* state of the process
- Two different Poisson Processes:
  - PP-1 with rate λ
  - PP-2 with rate μ
- Process PP-1 increases state *n*
  Process PP-2 decreases state *n*

4.1 Birth and Death Processes

Introduction:

Poisson Process
PP-1 with rate λ

Birth Process



Poisson Process
PP-2 with rate μ

Death Process

Probability of being in state *n* at time *t*

$P_n(t)$

4.1 Birth and Death Processes

Introduction:

- $P_n(t)$ a function of time
- Rates that changes values of $P_n(t)$ are known : λ , μ
- Function of $P_n(t)$ can be defined by

$$\frac{d\,P_n(t)}{dt}$$

- Rate of change in $P_n(t)$ is :

$$\frac{d\,P_n(t)}{dt} = \left[\begin{array}{c}\text{Average}\\ \text{Rate } \underline{\textbf{in to}}\\ \text{State } n\\ \text{at tim } t\end{array}\right] - \left[\begin{array}{c}\text{Average}\\ \text{Rate } \underline{\textbf{out of}}\\ \text{State } n\\ \text{at time } t\end{array}\right]$$

4.1 Birth and Death Processes

Steady-State analysis

- $P_n(t)$: prob. that there are **n** customers in the system
- System starts empty
- System operates for long time $t \to \infty$.
- Rates of process are fixed over time : λ , μ
- Function of $P_n(t)$ becomes fixed for all n

$$t \longrightarrow \infty$$

$$\frac{d\, P_n(t)}{dt} \longrightarrow 0$$

$$P_n(t) \longrightarrow P_n$$

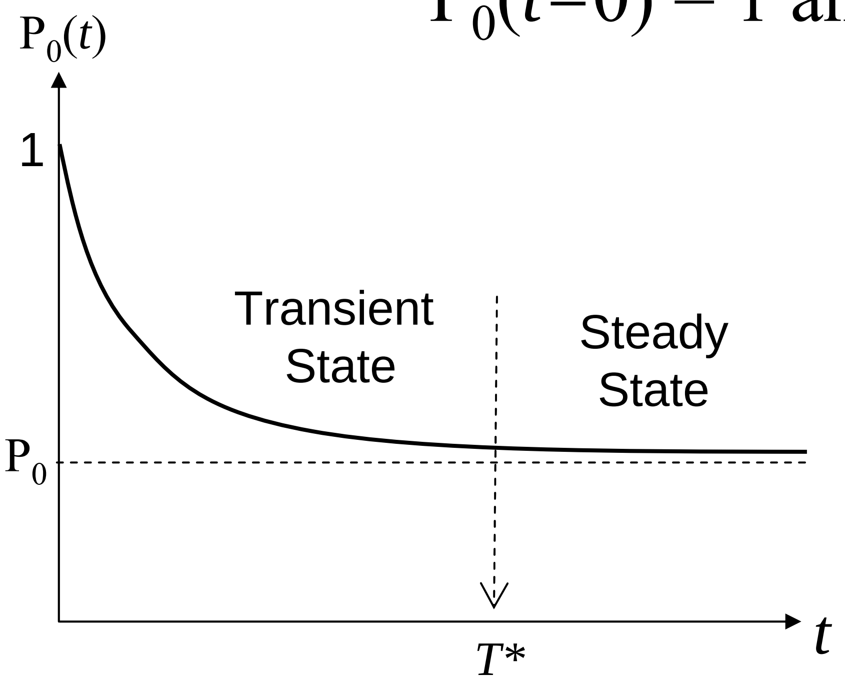
4.1 Birth and Death Processes

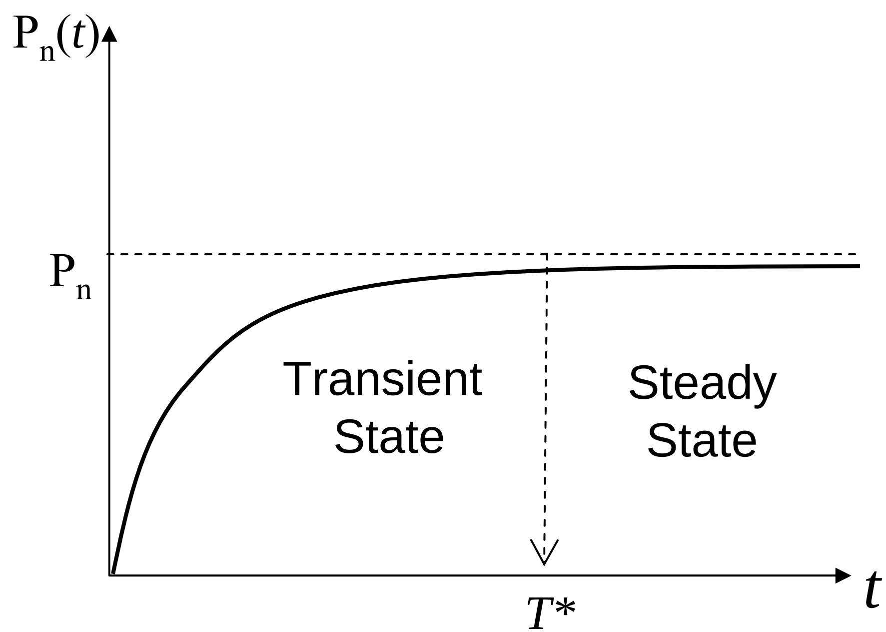
Steady-State analysis

T*
Is not Known

- Why must time $t \rightarrow \infty$?
- System starts empty (initial condition)

$$P_0(t=0) = 1 \text{ and } P_n(t=0) = 0$$





4.1 Birth and Death Processes

Steady-State analysis

- For $P_n(t)$: $\sum P_n(t) = 1$ for all t
- As $t \to \infty$:

  If P_n exists with $\sum P_n = 1$ for all n

  $\Rightarrow$ Steady-State Distribution exists

  $\Rightarrow$ Stationary Distribution exists

$$\mathbf{0} = \begin{bmatrix} \text{Average} \\ \text{Rate } \underline{\textbf{in to}} \\ \text{State } n \end{bmatrix} - \begin{bmatrix} \text{Average} \\ \text{Rate } \underline{\textbf{out of}} \\ \text{State } n \end{bmatrix}$$

4.1 Birth and Death Processes

Steady-State analysis

$$\mathbf{0} = \begin{bmatrix} \text{Average} \\ \text{Rate } \underline{\textbf{in to}} \\ \text{State } n \end{bmatrix} - \begin{bmatrix} \text{Average} \\ \text{Rate } \underline{\textbf{out of}} \\ \text{State } n \end{bmatrix}$$

Balance Equations

$$\begin{bmatrix} \text{Average} \\ \text{Rate } \underline{\textbf{out of}} \\ \text{State } n \end{bmatrix} = \begin{bmatrix} \text{Average} \\ \text{Rate } \underline{\textbf{in to}} \\ \text{State } n \end{bmatrix}$$

4.1 Birth and Death Processes

Kendall's Queueing Notation

$$A / B / X / Y / Z$$

- A : interarrival time distribution
- B : service time distribution
- X : number of servers
- Y : System size
- Z : queue discipline

4.1 Birth and Death Processes

Kendall's Queueing Notation

- A & B Distributions
  - M : Exponential with constant rates
  - D : Deterministic
  - E_k : Erlang with K stages
  - G : General distribution with $\overline{X}$ and S^2

- X : number of servers (integers 1 , 2 , 3 , …)
- Y : System size (integers 1 , 2 , 3 , …)
  - Default is ∞
  - Number in service + max. queue size
- Z : queue discipline (FCFS , LCLS , SRO , …)
  - Default is FCFS