Baye's Theorem
Pages 79-83

Definition.1

The sensitivity of the symptom

This is the probability of a positive result given that the subject has the disease. It is denoted by P(T|D)

Definition.2

The specificity of the symptom

This is the probability of negative result given that the subject does not have the disease. It is denoted by

Definition.3

The predictive value positive of the symptom

This is the probability that a subject has the disease given that the subject has a positive screening test result

It is calculated using Bayes Theorem through the following formula

$$P(D \mid T) = \frac{P(T \mid D)P(D)}{P(T \mid D)P(D) + P(T \mid \overline{D})P(\overline{D})}$$

Where P(D) is the rate of the disease which is always given.

$$P(\overline{D}) = 1 - P(D)$$

$$p(T \mid \overline{D}) = 1 - P(\overline{T} \mid \overline{D})$$

Note that: the numerator is equal to the sensitivity times rate of the disease; while the denominator is equal to the sensitivity times the rate of the disease plus 1 minus the specifity times 1 minus the rate of the disease.

Definition.4

The predictive value negative of the symptom

This is the probability that a subject does not have the disease given that the subject has a negative screening test result
It is calculated using Bayes Theorem through the following formula

$$P(\overline{D}\,|\,\overline{T}) = \frac{P(\overline{T}\,|\,\overline{D})P(\overline{D})}{P(\overline{T}\,|\,\overline{D})P(\overline{D}) + P(\overline{T}\,|\,D)P(D)}$$

where,

$$p(\overline{T}\,|\,D) = 1 - P(T\,|\,D)$$

Example 3.5.1 page 82

A medical research team wished to evaluate a proposed screening test for Alzheimer's disease. The test was given to a random sample of 450 patients with Alzheimer's disease and an independent random sample of 500 patients without symptoms of the disease. The two samples were drawn from populations of subjects who were 65 years or older. The results are as follows.

Test Result	Yes (D)	No ($\overline{D}$)	Total
Positive(T)	436	5	441
Negativ( )$\overline{T}$	14	495	509
Total	450	500	950

In the context of this example

a)What is a false positive?

A false positive is when the test indicates a positive result (T) when the person does not have the disease $\overline{D}$

b) What is the false negative?

A false negative is when a test indicates a negative result ($\overline{T}$) when the person has the disease (D).

c) Compute the sensitivity of the symptom.

$$P(T \mid D) = \frac{436}{450} = 0.9689$$

d) Compute the specificity of the symptom.

$$P(\overline{T} \mid \overline{D}) = \frac{495}{500} = 0.99$$

e) Suppose it is known that the rate of the disease in the general population is 11.3%. What is the predictive value positive of the symptom and the predictive value negative of the symptom

The predictive value positive of the symptom is calculated as

$$P(D \mid T) = \frac{P(T \mid D)P(D)}{P(T \mid D)P(D) + P(T \mid \overline{D})P(\overline{D})}$$

$$= \frac{(0.9689)(0\ .113)}{(0.9689)(0\ .113) + (.01)(1 - 0.113)} = 0.925$$

The predictive value negative of the symptom is calculated as

$$P(\overline{D} \mid \overline{T}) = \frac{P(\overline{T} \mid \overline{D})P(\overline{D})}{P(\overline{T} \mid \overline{D})P(\overline{D}) + P(\overline{T} \mid D)P(D)}$$

$$= \frac{(0.99)(0.887)}{(0.99)(0.887) + (0.0311)(0.113)} = 0.996$$

Exercise:

- Page 83
- Questions :
- 3.5.1, 3.5.2
- H.W.:
- Page 87 : Q4,Q5,Q7,Q9,Q21