Chapter 3

Probability

The Basis of the Statistical inference

- **Key words:**

- Probability, objective Probability, subjective Probability, equally likely Mutually exclusive, multiplicative rule Conditional Probability, independent events, Bayes theorem

3.1 Introduction

- The concept of probability is frequently encountered in everyday communication. **For example**, a physician may say that a patient has a 50-50 chance of surviving a certain operation.
  Another physician may say that she is 95 percent certain that a patient has a particular disease.
- Most people express probabilities in terms of percentages.
- But, it is more convenient to express probabilities as fractions. Thus, we may measure the probability of the occurrence of some event by a number between 0 and 1.
- The more likely the event, the closer the number is to one. An event that can't occur has a probability of zero, and an event that is certain to occur has a probability of one.

3.2 Two views of Probability objective and subjective:

- *** Objective Probability
- ** Classical and Relative
- Some definitions:

1.Equally likely outcomes:

Are the outcomes that have the same chance of occurring.

2.Mutually exclusive:

Two events are said to be mutually exclusive if they cannot occur simultaneously such that $A \cap B = \Phi$.

- **The universal Set** (S): The set all possible outcomes.
- **The empty set Φ** : Contain no elements.
- **The event ,E** : is a set of outcomes in S which has a certain characteristic.
- **Classical Probability** : If an event can occur in N mutually exclusive and equally likely ways, and if m of these possess a triat, E, the probability of the occurrence of event E is equal to m/ N .
- **For Example:** in the rolling of the die , each of the six sides is equally likely to be observed . So, the probability that a 4 will be observed is equal to 1/6.

- **Relative Frequency Probability:**
- **Def:** If some posses is repeated a large number of times, n, and if some resulting event E occurs m times , the relative frequency of occurrence of E , m/n will be approximately equal to probability of E . P(E) = m/n .
- *** **Subjective Probability** :
- Probability measures the confidence that a particular individual has in the truth of a particular proposition.
- **For Example** : the probability that a cure for cancer will be discovered within the next 10 years.

3.3 Elementary Properties of Probability:

- Given some process (or experiment) with n mutually exclusive events E_1, E_2, E_3,....., E_n, then
- 1-$P(E_i) \geq 0$, i= 1,2,3,.....n
- 2- $P(E_1) + P(E_2) + \ldots\ldots + P(E_n) = 1$
- 3- $P(E_i + E_J) = P(E_i) + P(E_J)$, E_i ,E_J are mutually exclusive
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Rules of Probability

- 1-Addition Rule
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- 2- If A and B are mutually exclusive (disjoint) ,then
- $P(A \cap B) = 0$
- Then , addition rule is
- $P(A\ B) = P(A) + P(B)$.
- 3- Complementary Rule
- $P(A') = 1 - P(A)$
- where, A' = = complement event
- Consider example 3.4.1 Page 63

Table 3.4.1 in Example 3.4.1

Family history of Mood Disorders	Early = 18 (E)	Later >18 (L)	Total
Negative(A)	28	35	63
Bipolar Disorder(B)	19	38	57
Unipolar (C)	41	44	85
Unipolar and Bipolar(D)	53	60	113
Total	141	177	318

**Answer the following questions:

Suppose we pick a person at random from this sample.

1-The probability that this person will be 18-years old or younger?

2-The probability that this person has family history of mood orders Unipolar(C)?

3-The probability that this person has no family history of mood orders Unipolar( )?

4-The probability that this person is 18-years old or younger or has no family history of mood orders Negative (A)?

5-The probability that this person is more than18-years old and has family history of mood orders Unipolar and Bipolar(D)?