

Section (2.4) :
Descriptive Statistics
Measures of Central
Tendency
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key words:

Descriptive Statistic, measure of central tendency ,statistic, parameter, mean (μ) ,median, mode.

The Statistic and The Parameter

- **A Statistic:**

It is a descriptive measure computed from the data of a **sample**.

- **A Parameter:**

It is a a descriptive measure computed from the data of a **population**.

Since it is difficult to measure a parameter from the population, a **sample** is drawn of size n , whose values are $\chi_1, \chi_2, \dots, \chi_n$. From this data, we measure the **statistic**.

Measures of Central Tendency

A measure of central tendency is a measure which indicates where the **middle** of the data is.

The three most commonly used measures of central tendency are:

The Mean, the Median, and the Mode.

The Mean:

It is the average of the data.

The Population Mean:

$\mu = \frac{\sum_{i=1}^N X_i}{N}$ which is usually unknown, then we use the

sample mean to estimate or approximate it.

The Sample Mean:

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

Example:

Here is a random sample of size 10 of ages, where

$x_1 = 42, x_2 = 28, x_3 = 28, x_4 = 61, x_5 = 31,$
 $x_6 = 23, x_7 = 50, x_8 = 34, x_9 = 32, x_{10} = 37.$

$$\bar{x} = (42 + 28 + \dots + 37) / 10 = 36.6$$

Properties of the Mean:

- **Uniqueness.** For a given set of data there is one and only one mean.
- **Simplicity.** It is easy to understand and to compute.
- **Affected by extreme values.** Since all values enter into the computation.

Example: Assume the values are 115, 110, 119, 117, 121 and 126. The mean = 118.

But assume that the values are 75, 75, 80, 80 and 280. The mean = 118, a value that is not representative of the set of data as a whole.

The Median:

When **ordering** the data, it is the observation that divide the set of observations into **two equal parts** such that half of the data are before it and the other are after it.

* If n is **odd**, the median will be the middle of observations. It will be the $(n+1)/2^{\text{th}}$ ordered observation.

When $n = 11$, then the median is the 6th observation.

* If n is **even**, there are two middle observations. The median will be the mean of these two middle observations. It will be the $(n+1)/2^{\text{th}}$ ordered observation.

When $n = 12$, then the median is the 6.5th observation, which is an observation halfway between the 6th and 7th ordered observation.

Example:

For the same random sample, the ordered observations will be as:

23, 28, 28, 31, 32, 34, 37, 42, 50, 61.

Since $n = 10$, then the median is the 5.5th observation, i.e. $= (32+34)/2 = 33$.

Properties of the Median:

- **Uniqueness.** For a given set of data there is one and only one median.
- **Simplicity.** It is easy to calculate.
- **It is not affected by extreme values** as is the mean.

The Mode:

It is the value which occurs most **frequently**.

If all values are different there is **no mode**.

Sometimes, there are **more than one mode**.

Example:

For the same random sample, the value 28 is repeated two times, so it is the mode.

Properties of the Mode:

- Sometimes, it is not **unique**.
- It may be used for **describing qualitative data**.