7.6 Hypothesis Testing :The Difference between two population proportion:

- Testing hypothesis about two population proportion (P_1, P_2) is carried out in much the same way as for difference between two means when condition is necessary for using normal curve are met
- We have the following steps:

1.Data: sample size (n_1 ,n_2), sample proportions($\hat{P}_1 , \hat{P}_2$), Characteristic in two samples (x_1 , x_2), $\bar{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$

2- Assumption : Two populations are independent .

- 3.Hypotheses:
- we have three cases
- Case I : H_0: $P_1 = P_2 \rightarrow P_1 - P_2 = 0$

  H_A: $P_1 \neq P_2 \rightarrow P_1 - P_2 \neq 0$
- Case II : H_0: $P_1 = P_2 \rightarrow P_1 - P_2 = 0$

  H_A: $P_1 > P_2 \rightarrow P_1 - P_2 > 0$
- Case III : H_0: $P_1 = P_2 \rightarrow P_1 - P_2 = 0$

  H_A: $P_1 < P_2 \rightarrow P_1 - P_2 < 0$

4.Test Statistic:

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\overline{p}(1-\overline{p})}{n_1} + \frac{\overline{p}(1-\overline{p})}{n_2}}}$$

Where H_0 is true ,is distributed approximately as the standard normal

5.Decision Rule:

i) If $H_A: P_1 \neq P_2$

- Reject H_0 if $Z > Z_{1-\alpha/2}$ or $Z < -Z_{1-\alpha/2}$
- ______________________________
- ii) If $H_A: P_1 > P_2$
- Reject H_0 if $Z > Z_{1-\alpha}$
- ______________________________
- iii) If $H_A: P_1 < P_2$
- Reject H_0 if $Z < -Z_{1-\alpha}$

Note: $Z_{1-\alpha/2}$, $Z_{1-\alpha}$, Z_α are tabulated values obtained from table D

6. Conclusion: reject or fail to reject H_0

Example7.6.1 page 262

Noonan is a genetic condition that can affect the heart growth, blood clotting and mental and physical development. Noonan examined the stature of men and women with Noonan. The study contained 29 Male and 44 female adults. One of the cut-off values used to assess stature was the third percentile of adult height .Eleven of the males fell below the third percentile of adult male height ,while 24 of the female fell below the third percentile of female adult height .Does this study provide sufficient evidence for us to conclude that among subjects with Noonan ,females are more likely than males to fall below the respective of adult height? Let α=0.05

Solution:

1.Data: $n_M = 29$, $n_F = 44$, $x_M = 11$, $x_F = 24$, $\alpha = 0.05$

$$\overline{p} = \frac{x_M + x_F}{n_M + n_F} = \frac{11 + 24}{29 + 44} = 0.479 \quad \hat{p}_M = \frac{x_m}{n_M} = \frac{11}{29} = 0.379, \hat{p}_F = \frac{x_F}{n_F} = \frac{24}{44} = 0.545$$

2- Assumption : Two populations are independent .

3.Hypotheses:

- Case II : $H_0: P_F = P_M \rightarrow P_F - P_M = 0$

  $H_A: P_F > P_M \rightarrow P_F - P_M > 0$

- 4.Test Statistic:

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\overline{p}(1-\overline{p})}{n_1} + \frac{\overline{p}(1-\overline{p})}{n_2}}} = \frac{(0.545 - 0.379) - 0}{\sqrt{\frac{(0.479)(0.521)}{44} + \frac{(0.479)(0.521)}{29}}} = 1.39$$

5.Decision Rule:

Reject H_0 if $Z > Z_{1-\alpha}$, Where $Z_{1-\alpha} = Z_{1-0.05} = Z_{0.95} = 1.645$

_6. Conclusion: Fail to reject H_0

Since $Z = 1.39 > Z_{1-\alpha=} 1.645$

Or , If P-value = 0.0823 $\rightarrow$ fail to reject H_0 $\rightarrow P > \alpha$

- Exercises:
- Questions : Page 234 -237
- 7.2.1,7.8.2 ,7.3.1,7.3.6 ,7.5.2 ,,7.6.1

- H.W:
- 7.2.8,7.2.9, 7.2.11, 7.2.15,7.3.7,7.3.8,7.3.10
- 7.5.3,7.6.4