7.3 Hypothesis Testing :The Difference between two population mean :

- We have the following steps:

1.Data: determine variable, sample size (n), sample means, population standard deviation or samples standard deviation (s) if is unknown for two population.

2. Assumptions : We have two cases:

- Case1: Population is normally or approximately normally distributed with known or unknown variance (sample size n may be small or large),
- Case 2: Population is not normal with known variances (n is large i.e. $n \geq 30$).

- 3.Hypotheses:
- we have three cases
- Case I : H_0: $\mu 1 = \mu 2 \rightarrow \mu_1 - \mu_2 = 0$
- H_A: $\mu_1 \neq \mu_2 \rightarrow \mu_1 - \mu_2 \neq 0$
- e.g. we want to test that the mean for first population is different from second population mean.
- Case II : H_0: $\mu 1 = \mu 2 \rightarrow \mu_1 - \mu_2 = 0$

  H_A: $\mu_1 > \mu_2 \rightarrow \mu_1 - \mu_2 > 0$
- e.g. we want to test that the mean for first population is greater than second population mean.
- Case III : H_0: $\mu 1 = \mu 2 \rightarrow \mu_1 - \mu_2 = 0$

  H_A: $\mu_1 < \mu_2 \rightarrow \mu_1 - \mu_2 < 0$
- e.g. we want to test that the mean for first population is greater than second population mean.

4.Test Statistic:

- **Case 1: Two population is normal or approximately normal**

σ^2 is known (n_1 , n_2 large or small)

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

σ^2 is unknown if (n_1 , n_2 small)

population Variances equal

$$T = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

population Variances not equal

$$T = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

where

$$\boxed{S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}}$$

- Case2: If population is not normally distributed
- and n_1, n_2 is large($n_1 \geq 0$,$n_2 \geq 0$)
- and population variances is known,

$$Z = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

5.Decision Rule:

i) If H_A: $\mu_1 \neq \mu_2 \rightarrow \mu_1 - \mu_2 \neq 0$

- Reject H_0 if $Z > Z_{1-\alpha/2}$ or $Z < -Z_{1-\alpha/2}$ (when use Z - test)

Or Reject H_0 if $T > t_{1-\alpha/2,(n_1+n_2-2)}$ or $T < -t_{1-\alpha/2,,(n_1+n_2-2)}$ (when use T- test)

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- ii) H_A: $\mu_1 > \mu_2 \rightarrow \mu_1 - \mu_2 > 0$
- Reject H_0 if $Z > Z_{1-\alpha}$ (when use Z - test)

Or Reject H_0 if $T > t_{1-\alpha,(n_1+n_2-2)}$ (when use T - test)

- iii) If H_A: $\mu_1 < \mu_2 \rightarrow \mu_1 - \mu_2 < 0$
  Reject H_0 if $Z < -Z_{1-\alpha}$ (when use Z - test)
- Or

Reject H_0 if $T < -t_{1-\alpha,\,(n_1+n_2-2)}$ (when use T - test)

Note:

$Z_{1-\alpha/2}$, $Z_{1-\alpha}$, Z_α are tabulated values obtained from table D

$t_{1-\alpha/2}$, $t_{1-\alpha}$, t_α are tabulated values obtained from table E with (n_1+n_2-2) degree of freedom (df)

6. Conclusion: reject or fail to reject H_0

Example7.3.1 page238

- Researchers wish to know if the data have collected provide sufficient evidence to indicate a difference in mean serum uric acid levels between normal individuals and individual with Down's syndrome. The data consist of serum uric reading on 12 individuals with Down's syndrome from normal distribution with variance 1 and 15 normal individuals from normal distribution with variance 1.5 . The mean are $\overline{X}_1 = 4.5mg/100$ and $\overline{X}_2 = 3.4mg/100$ $\alpha=0.05$.

Solution:

1. **Data:** Variable is serum uric acid levels, $n_1=12$, $n_2=15$, $\sigma^2_1=1$, $\sigma^2_2=1.5$,$\alpha=0.05$.

2. **Assumption:** Two population are normal, σ^2_1 , σ^2_2 are known

3. **Hypotheses:** H_0: $\mu 1 = \mu 2 \rightarrow \mu_1 - \mu_2 = 0$

- H_A: $\mu_1 \neq \mu_2 \rightarrow \mu_1 - \mu_2 \neq 0$

4. Test Statistic:

- $$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(4.5 - 3.4) - (0)}{\sqrt{\frac{1}{12} + \frac{1.5}{15}}} = 2.57$$

5. Desicion Rule:

Reject H_0 if $Z > Z_{1-\alpha/2}$ or $Z < -Z_{1-\alpha/2}$

$Z_{1-\alpha/2} = Z_{1-0.05/2} = Z_{0.975} = 1.96$ (from table D)

6-Conclusion: Reject H_0 since $2.57 > 1.96$

Or if p-value $=0.102 \rightarrow$ reject H_0 if $p < \alpha \rightarrow$ then reject H_0

Example7.3.2 page 240

The purpose of a study by Tam, was to investigate wheelchair Maneuvering in individuals with over-level spinal cord injury (SCI) And healthy control (C). Subjects used a modified a wheelchair to incorporate a rigid seat surface to facilitate the specified experimental measurements. The data for measurements of the left ischial tuerosity (عظام الفخذ وتأثيرها من الكرسي المتحرك) for SCI and control C are shown below

C	131	115	124	131	122	117	88	114	150	169
SCI	60	150	130	180	163	130	121	119	130	143

We wish to know if we can conclude, on the basis of the above data that the mean of left ischial tuberosity for control C lower than mean of left ischial tuerosity for SCI, Assume normal populations equal variances. $\alpha=0.05$, p-value = -1.33

Solution:

1. **Data:**, n_C=10 , n_{SCI}=10, S_C=21.8, S_{SCI}=133.1 ,α=0.05.

- $\bar{X}_C = 126.1$, $\bar{X}_{SCI} = 133.1$ (calculated from data)

2.**Assumption:** Two population are normal, σ^2_1 , σ^2_2 are unknown but <u>equal</u>

3. **Hypotheses:** H_0: $\mu_C = \mu_{SCI} \rightarrow \mu_C - \mu_{SCI} = 0$

H_A: $\mu_C < \mu_{SCI} \rightarrow \mu_C - \mu_{SCI} < 0$

4.**Test Statistic:**

- $$T = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{(126.1 - 133.1) - 0}{\sqrt{756.04}\sqrt{\frac{1}{10} + \frac{1}{10}}} = -0.569$$

Where,

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} = \frac{9(21.8)^2 + 9(32.3)^2}{10 + 10 - 2} = 756.04$$

5. Decision Rule:

Reject H_0 if $T < -T_{1-\alpha,(n_1+n_2-2)}$

$T_{1-\alpha,(n_1+n_2-2)} = T_{0.95,18} = 1.7341$ (from table E)

6-Conclusion: Fail to reject H_0 since $-0.569 \nless -1.7341$

Or

Fail to reject H_0 since $p = -1.33 > \alpha = 0.05$

Example7.3.3 page 241

Dernellis and Panaretou examined subjects with hypertension and healthy control subjects .One of the variables of interest was the aortic stiffness index. Measures of this variable were calculated From the aortic diameter evaluated by M-mode and blood pressure measured by a sphygmomanometer. Physics wish to reduce aortic stiffness. In the 15 patients with hypertension (Group 1),the mean aortic stiffness index was 19.16 with a standard deviation of 5.29. In the30 control subjects (Group 2),the mean aortic stiffness index was 9.53 with a standard deviation of 2.69. We wish to determine if the two populations represented by these samples differ with respect to mean stiffness index .we wish to know if we can conclude that in general a person with thrombosis have on the average higher IgG levels than persons without thrombosis at $\alpha=0.01$, p-value = 0.0559

Group	Mean LgG level	Sample Size	standard deviation
Thrombosis	59.01	53	44.89
No Thrombosis	46.61	54	34.85

Solution:

1. **Data:**, $n_1=53$, $n_2=54$, $S_1= 44.89$, $S_2= 34.85$ $\alpha=0.01$.

2. **Assumption:** Two population are not normal, σ^2_1 , σ^2_2 are unknown and sample size large

3. **Hypotheses:** H_0: $\mu_1 = \mu_2 \rightarrow \mu_1 - \mu_2 = 0$

   H_A: $\mu_1 > \mu_2 \rightarrow \mu_1 - \mu_2 > 0$

4. **Test Statistic:**

- $$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{(59.01 - 46.61) - 0}{\sqrt{\frac{44.89^2}{53} + \frac{34.85^2}{54}}} = 1.59$$

5. Decision Rule:

Reject H_0 if $Z > Z_{1-\alpha}$

$Z_{1-\alpha} = Z_{0.99} = 2.33$ (from table D)

6-Conclusion: Fail to reject H_0 since $1.59 \ngtr 2.33$

Or

Fail to reject H_0 since $p = 0.0559 > \alpha = 0.01$

7.5 Hypothesis Testing A single population proportion:

- Testing hypothesis about population proportion (P) is carried out in much the same way as for mean when condition is necessary for using normal curve are met
- We have the following steps:

1.Data: sample size (n), sample proportion($\hat{p}$) , P_0

$$\hat{p} = \frac{\text{no. of element in the sample with some charachtar istic}}{\text{Total no. of element in the sample}} = \frac{a}{n}$$

2. Assumptions :normal distribution ,

- 3.Hypotheses:
- we have three cases
- Case I : $H_0: P = P_0$

  $H_A: P \neq P_0$
- Case II : $H_0: P = P_0$

  $H_A: P > P_0$
- Case III : $H_0: P = P_0$

  $H_A: P < P_0$

4.Test Statistic:

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$$

Where H_0 is true ,is distributed approximately as the standard normal

5.Decision Rule:

i) If $H_A: P \neq P_0$

- Reject H_0 if $Z > Z_{1-\alpha/2}$ or $Z < -Z_{1-\alpha/2}$
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- ii) If $H_A: P > P_0$
- Reject H_0 if $Z > Z_{1-\alpha}$
- ____________________________________
- iii) If $H_A: P < P_0$

Reject H_0 if $Z < -Z_{1-\alpha}$

Note: $Z_{1-\alpha/2}$, $Z_{1-\alpha}$, Z_{α} are tabulated values obtained from table D

6. Conclusion: reject or fail to reject H_0

2. Assumptions : $\hat{p}$ is approximately normaly distributed

3.Hypotheses:

- we have three cases
- H_0: P = 0.063

  H_A: P > 0.063
- 4.Test Statistic :

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.08 - 0.063}{\sqrt{\frac{0.063\ (0.937)}{301}}} = 1.21$$

5.Decision Rule: Reject H_0 if $Z > Z_{1-\alpha}$

Where $Z_{1-\alpha} = Z_{1-0.05} = Z_{0.95} = 1.645$

6. Conclusion: Fail to reject H_0

Since

$$Z = 1.21 \ngtr Z_{1-\alpha} = 1.645$$

Or ,

If P-value = 0.1131,

fail to reject $H_0 \rightarrow P > \alpha$

Example7.5.1 page 259

Wagen collected data on a sample of 301 Hispanic women Living in Texas .One variable of interest was the percentage of subjects with impaired fasting glucose (IFG). In the study,24 women were classified in the (IFG) stage .The article cites population estimates for (IFG) among Hispanic women in Texas as 6.3 percent .Is there sufficient evidence to indicate that the population Hispanic women in Texas has a prevalence of IFG higher than 6.3 percent ,let $\alpha=0.05$

Solution:

1.Data: $n = 301$, $p_0 = 6.3/100=0.063$,a=24, $\hat{p} = \frac{a}{n} = \frac{24}{301} = 0.08$

$q_0 = 1 - p_0 = 1 - 0.063 = 0.937$, $\alpha=0.05$