Chapter 7
Using sample statistics to Test Hypotheses about population parameters
Pages 215-233

- **Key words :**

- *Null hypothesis H_0, Alternative hypothesis H_A , testing hypothesis , test statistic , P-value*

Hypothesis Testing

- One type of statistical inference, estimation, was discussed in Chapter 6 .
- The other type ,hypothesis testing ,is discussed in this chapter.

Definition of a hypothesis

- It is a statement about one or more populations .

It is usually concerned with the parameters of the population. e.g. the hospital administrator may want to test the hypothesis that the average length of stay of patients admitted to the hospital is 5 days

Definition of Statistical hypotheses

- They are hypotheses that are stated in such a way that they may be evaluated by appropriate statistical techniques.
- There are two hypotheses involved in hypothesis testing
- **Null hypothesis** H_0: It is the hypothesis to be tested .
- **Alternative hypothesis** H_A : It is a statement of what we believe is true if our sample data cause us to reject the null hypothesis

7.2 Testing a hypothesis about the mean of a population:

- We have the following steps:

1. **Data**: determine variable, sample size (n), sample mean($\bar{x}$) , population standard deviation or sample standard deviation (s) if is unknown

2. **Assumptions :** We have two cases:

- Case1: Population is normally or approximately normally distributed with known or unknown variance (sample size n may be small or large),
- Case 2: Population is not normal with known or unknown variance (n is large i.e. n≥30).

- 3.Hypotheses:
- we have three cases
- Case I : H_0: $\mu=\mu_0$

  H_A: $\mu \neq \mu_0$
- e.g. we want to test that the population mean is different than 50
- Case II : H_0: $\mu = \mu_0$

  H_A: $\mu > \mu_0$
- e.g. we want to test that the population mean is greater than 50
- Case III : H_0: $\mu = \mu_0$

  H_A: $\mu < \mu_0$
- e.g. we want to test that the population mean is less than 50

4.Test Statistic:

- Case 1: population is normal or approximately normal

σ² is known

(n large or small)

$$Z = \frac{\overline{X} - \mu_o}{\sigma / \sqrt{n}}$$

σ² is unknown

n large

$$Z = \frac{\overline{X} - \mu_o}{s / \sqrt{n}}$$

n small

$$T = \frac{\overline{X} - \mu_o}{s / \sqrt{n}}$$

- Case2: If population is not normally distributed and n is large
- i)If σ² is known

$$Z = \frac{\overline{X} - \mu_o}{\sigma / \sqrt{n}}$$

ii) If σ² is unknown

$$Z = \frac{\overline{X} - \mu_o}{s / \sqrt{n}}$$

5.Decision Rule:

i) If $H_A: \mu \neq \mu_0$

- Reject H_0 if $Z > Z_{1-\alpha/2}$ or $Z < -Z_{1-\alpha/2}$

  (when use Z - test)

Or Reject H_0 if $T > t_{1-\alpha/2,n-1}$ or $T < -t_{1-\alpha/2,n-1}$

(when use T- test)

- ___________________________________

- ii) If $H_A: \mu > \mu_0$
- Reject H_0 if $Z > Z_{1-\alpha}$ (when use Z - test)

Or Reject H_0 if $T > t_{1-\alpha,n-1}$ (when use T - test)

- iii) If $H_A: \mu < \mu_0$

Reject H_0 if $Z < -Z_{1-\alpha}$ (when use Z - test)

- Or

Reject H_0 if $T < -t_{1-\alpha,n-1}$ (when use T - test)

Note:

$Z_{1-\alpha/2}$, $Z_{1-\alpha}$, Z_α are tabulated values obtained from table D

$t_{1-\alpha/2}$, $t_{1-\alpha}$, t_α are tabulated values obtained from table E with (n-1) degree of freedom (df)

- **6.Decision :**
- If we reject H_0, we can conclude that H_A is true.
- If ,however ,we do not reject H_0, we may conclude that H_0 is true.

An Alternative Decision Rule using the p - value Definition

- The p-value is defined as the smallest value of α for which the null hypothesis can be rejected.
- If the p-value is less than or equal to α ,we reject the null hypothesis ($p \le \alpha$)
- If the p-value is greater than α ,we do not reject the null hypothesis ($p > \alpha$)

Example 7.2.1 Page 223

- Researchers are interested in the mean age of a certain population.
- A random sample of 10 individuals drawn from the population of interest has a mean of 27.
- Assuming that the population is approximately normally distributed with variance 20,can we conclude that the mean is different from 30 years ? (α=0.05) .
- If the p - value is 0.0340 how can we use it in making a decision?

Solution

1-**Data:** variable is age, n=10, $\bar{x}$ =27 ,σ^2=20,α=0.05

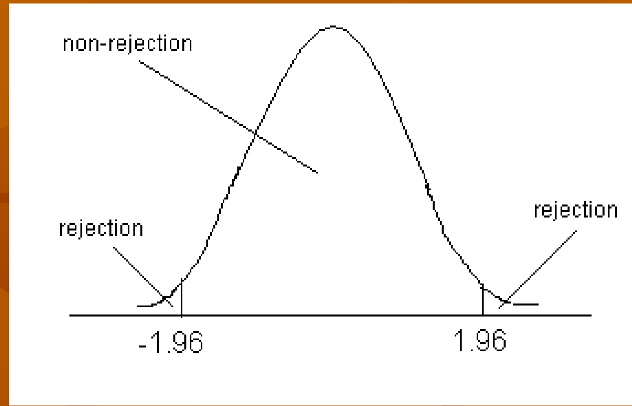
2-**Assumptions:** the population is approximately normally distributed with variance 20

3-Hypotheses:

- H_0 : μ=30
- H_A: $\mu \neq 30$

4-Test Statistic:

- **Z** $= -2.12$



5.Decision Rule

- **The alternative hypothesis is**
- $H_A: \mu > 30$
- Hence we reject H0 if $Z > Z_{1-0.025/2} = Z_{0.975}$
- or $Z < -Z_{1-0.025/2} = -Z_{0.975}$
- $Z_{0.975}$=1.96(from table D)

- **6.Decision:**

- **We reject H_0 ,since -2.12 is in the rejection region .**

- We can conclude that μ is not equal to 30

- Using the p value ,we note that p-value =0.0340< 0.05,therefore we reject H0

Example7.2.2 page227

- Referring to example 7.2.1.Suppose that the researchers have asked: Can we conclude that $\mu<30$.

1.Data.see previous example

2. Assumptions .see previous example

3.Hypotheses:

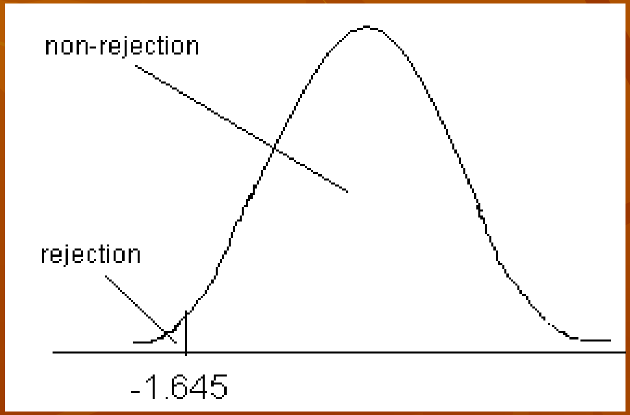
- H_0 $\mu = 30$
- H_A: $\mu < 30$

4.Test Statistic :

- $$Z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{27 - 30}{\sqrt{\frac{20}{10}}} = -2.12$$

5. **Decision Rule**: Reject H_0 if $Z < Z_\alpha$, where

- $Z_\alpha = -1.645$. (from table D)



6. **Decision**: Reject H_0 ,thus we can conclude that the population mean is smaller than 30.

Example7.2.4 page232

- Among 157 African-American men ,the mean systolic blood pressure was 146 mm Hg with a standard deviation of 27. We wish to know if on the basis of these data, we may conclude that the mean systolic blood pressure for a population of African-American is greater than 140. Use α=0.01.

Solution

1. **Data:** Variable is systolic blood pressure, n=157 , =146, s=27, α=0.01.

2. **Assumption:** population is not normal, σ^2 is unknown

3. **Hypotheses:** $H_0 : \mu=140$

   $H_A : \mu>140$

4. **Test Statistic:**

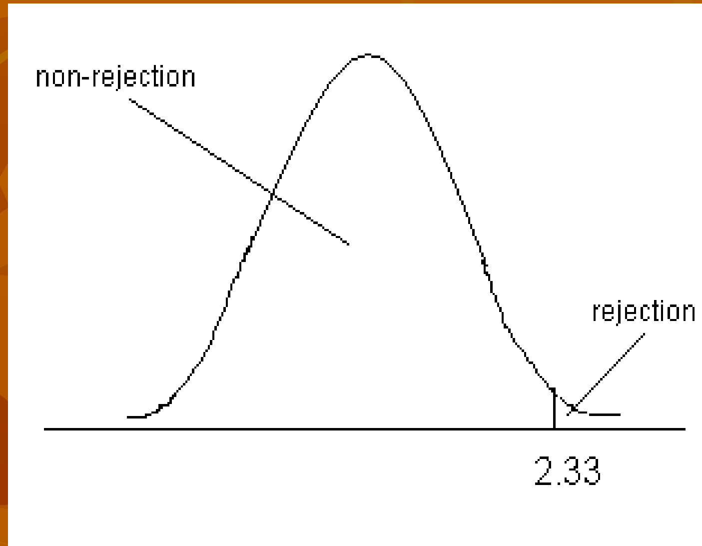
- $$Z = \frac{\bar{X} - \mu_o}{\frac{s}{\sqrt{n}}} = \frac{146-140}{\frac{27}{\sqrt{157}}} = \frac{6}{2.1548} = 2.78$$

5. Desicion Rule:

we reject H_0 if $Z>Z_{1-\alpha}$

$= Z_{0.99} = 2.33$

(from table D)



6. Desicion: We reject H_0.

Hence we may conclude that the mean systolic blood pressure for a population of African-American is greater than 140.