# 4.5 Continuous Probability Distribution

Pages 114 – 127

- Key words:

Continuous random variable, normal distribution , standard normal distribution , T-distribution

- Now consider distributions of continuous random variables.

# Properties of continuous probability Distributions:

1- Area under the curve = 1.

2- $P(X = a) = 0$ , where $a$ is a constant.

3- Area between two points $a$ , $b$ = $P(a<x<b)$.

# 4.6 The normal distribution:

- It is one of the most important probability distributions in statistics.
- The normal density is given by
- $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{(x-\mu)^2}{2\sigma^2}}$ $-\infty < X < \infty$, $-\infty < \mu < \infty$, $\sigma > 0$
- $\pi$, $e$ : constants
- $\mu$: population mean.
- $\sigma$ : Population standard deviation.

# Characteristics of the normal distribution: Page 111

- The following are some important characteristics of the normal distribution:

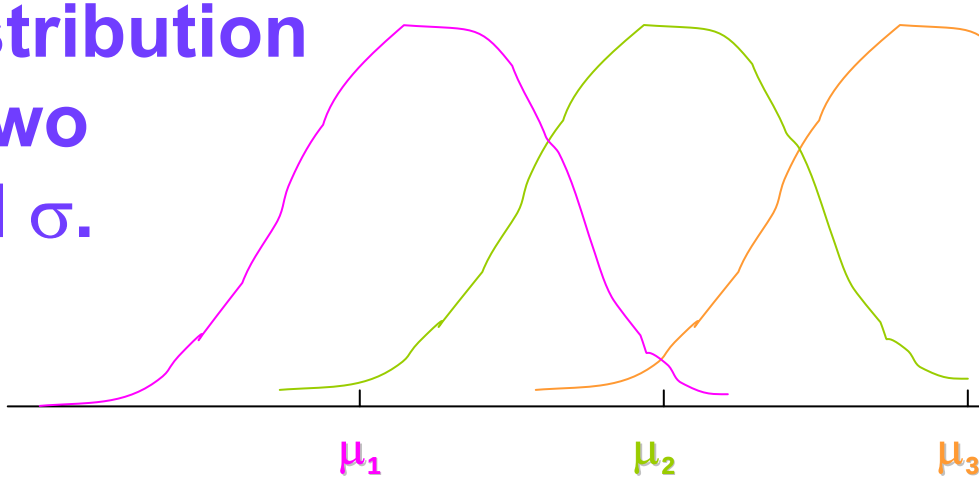
1- It is symmetrical about its mean, μ.

2- The mean, the median, and the mode are all equal.

3- The total area under the curve above the x-axis is one.

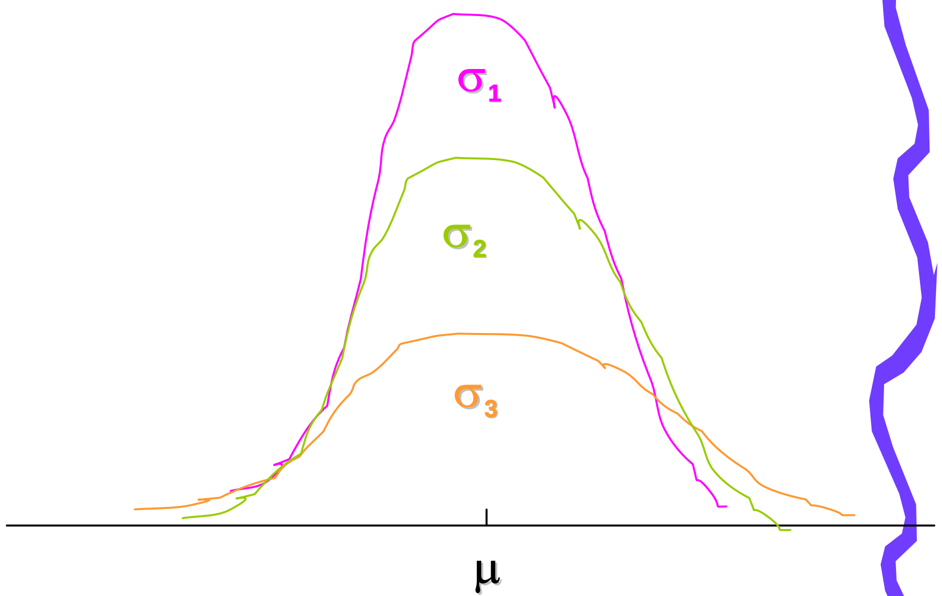
4- The normal distribution is completely determined by the parameters μ and σ.

**5- The normal distribution depends on the two parameters $\mu$ and $\sigma$.**
**$\mu$ determines the location of the curve.**
(As seen in figure 4.6.3) ,

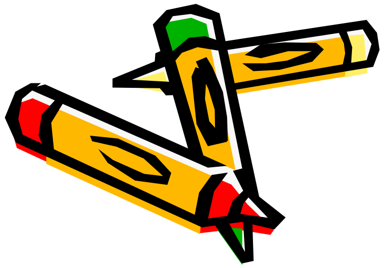



$\mu_1 < \mu_2 < \mu_3$

**But, $\sigma$ determines the scale of the curve, i.e. the degree of flatness or peaked ness of the curve.**
(as seen in figure 4.6.4)




$\sigma_1 < \sigma_2 < \sigma_3$



## Note that : (As seen in Figure 4.6.2)

1. $P(\mu - \sigma < x < \mu + \sigma) = 0.68$
2. $P(\mu - 2\sigma < x < \mu + 2\sigma) = 0.95$
3. $P(\mu - 3\sigma < x < \mu + 3\sigma) = 0.997$

# The Standard normal distribution:

- Is a special case of normal distribution with mean equal 0 and a standard deviation of 1.
- The equation for the standard normal distribution is written as
- $f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \quad , \quad -\infty < z < \infty$

# Characteristics of the standard normal distribution

1- It is symmetrical about 0.

2- The total area under the curve above the x-axis is one.

3- We can use table (D) to find the probabilities and areas.

# "How to use tables of Z"

**Note that**

**The cumulative probabilities $P(Z \leq z)$ are given in tables for $-3.49 < z < 3.49$. Thus,**

**$P(-3.49 < Z < 3.49) \cong 1$.**

**For standard normal distribution,**

**$P(Z > 0) = P(Z < 0) = 0.5$**

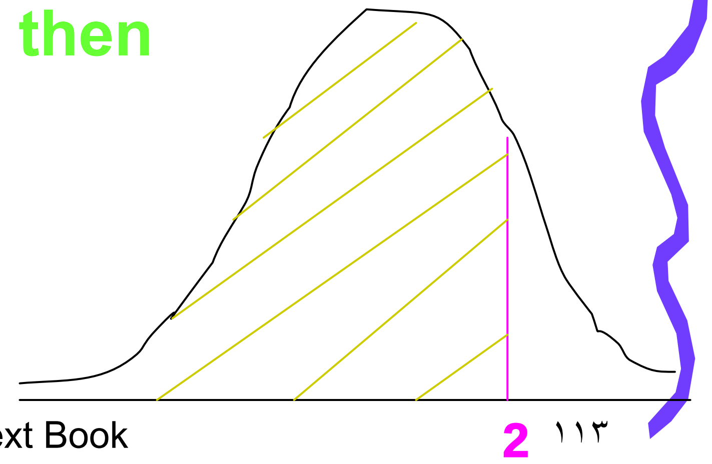
*Example 4.6.1:*

**If Z is a standard normal distribution, then**

**1) $P(Z < 2) = 0.9772$**

**is the area to the left to 2**

**and it equals 0.9772.**

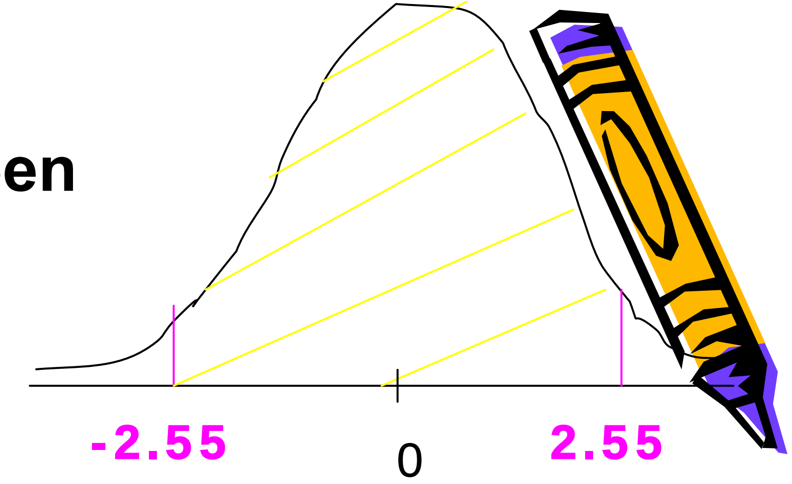


## *Example 4.6.2:*

**P(-2.55 < Z < 2.55) is the area between -2.55 and 2.55, Then it equals**

**P(-2.55 < Z < 2.55) =0.9946 – 0.0054**

**= 0.9892.**

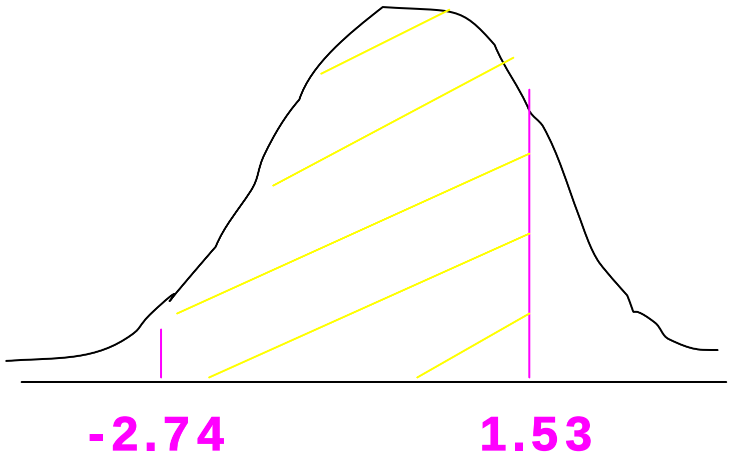



## *Example 4.6.2:*

**P(-2.74 < Z < 1.53) is the area between -2.74 and 1.53.**

**P(-2.74 < Z < 1.53) =0.9370 – 0.0031**

**= 0.9339.**

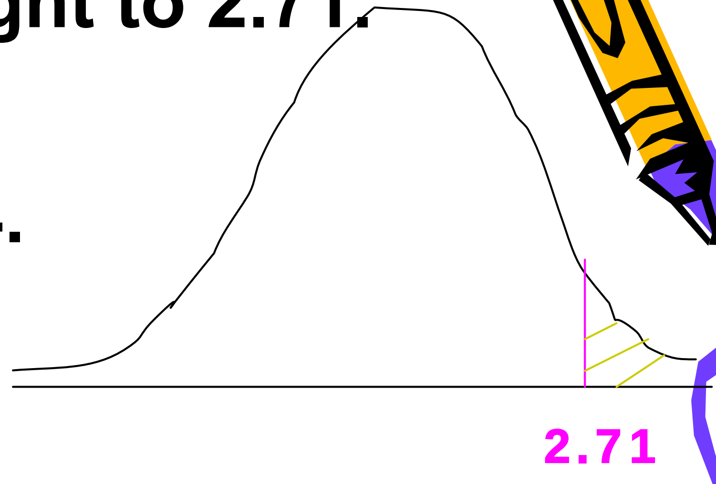


## Example 4.6.3:

**P(Z > 2.71) is the area to the right to 2.71.**

**So,**

**P(Z > 2.71) =1 – 0.9966 = 0.0034.**

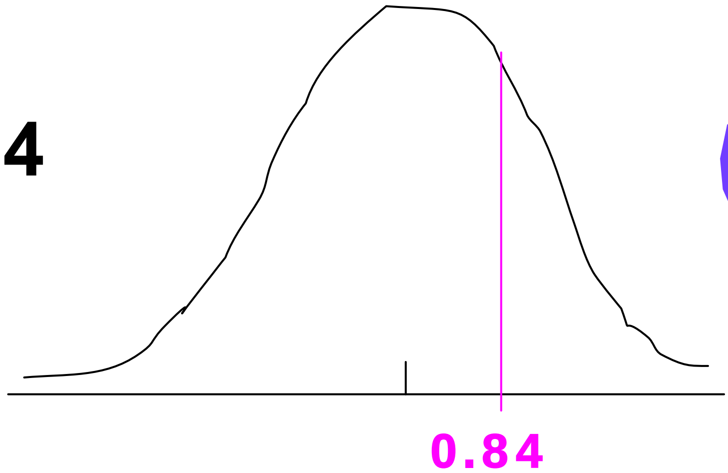



## Example :

**P(Z = 0.84) is the area at z = 2.71.**

**So,**

**P(Z = 0.84) =1 – 0.9966 = 0.0034**



# How to transform normal distribution (X) to standard normal distribution (Z)?

- This is done by the following formula:

$$z = \frac{x - \mu}{\sigma}$$

- **Example**:
- If X is normal with $\mu = 3$, $\sigma = 2$. Find the value of standard normal Z, If X= 6?
- **Answer:**

$$z = \frac{x-\mu}{\sigma} = \frac{6-3}{2} = 1.5$$

# *4.7 Normal Distribution Applications*

The normal distribution can be used to model the distribution of many variables that are of interest. This allow us to answer probability questions about these random variables.

## *Example 4.7.1:*

The 'Uptime 'is a custom-made light weight battery-operated activity monitor that records the amount of time an individual spend the upright position. In a study of children ages 8 to 15 years. The researchers found that the amount of time children spend in the upright position followed a normal distribution with Mean of 5.4 hours and standard deviation of 1.3.Find

**If a child selected at random ,then**

**1-The probability that the child spend less than 3 hours in the upright position 24-hour period**

$$P(X < 3) = P\left(\frac{X-\mu}{\sigma} < \frac{3-5.4}{1.3}\right) = P(Z < -1.85) = 0.0322$$

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**2-The probability that the child spend more than 5 hours in the upright position 24-hour period**

$$P(X > 5) = P\left(\frac{X-\mu}{\sigma} > \frac{5-5.4}{1.3}\right) = P(Z > -0.31)$$

$$= 1 - P(Z < -0.31) = 1 - 0.3520 = 0.648$$

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**3-The probability that the child spend exactly 6.2 hours in the upright position 24-hour period**

$$P(X = 6.2) = 0$$

**4-The probability that the child spend from 4.5 to 7.3 hours in the upright position 24-hour period**

$$P(4.5 < X < 7.3) = P\left(\frac{4.5-5.4}{1.3} < \frac{X-\mu}{\sigma} < \frac{7.3-5.4}{1.3}\right)$$

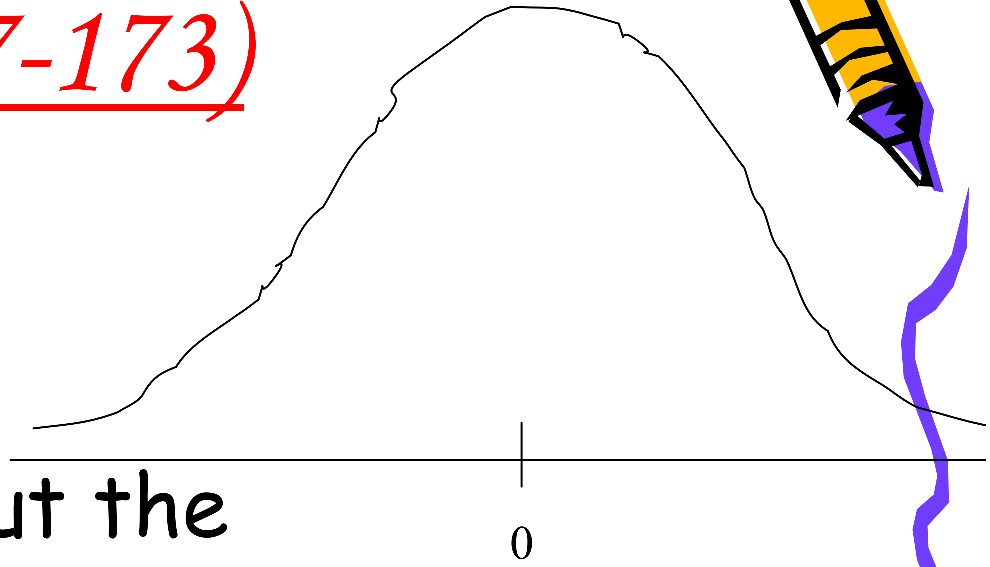
$$= P(-0.69 < Z < 1.46) = P(Z<1.46) - P(Z< -0.69)$$

$$= 0.9279 - 0.2451 = 0.6828$$

- Hw…EX. 4.7.2 – 4.7.3

# 6.3 The T Distribution:

# (167-173)




1- It has mean of zero.

2- It is symmetric about the mean.

3- It ranges from $-\infty$ to $\infty$.

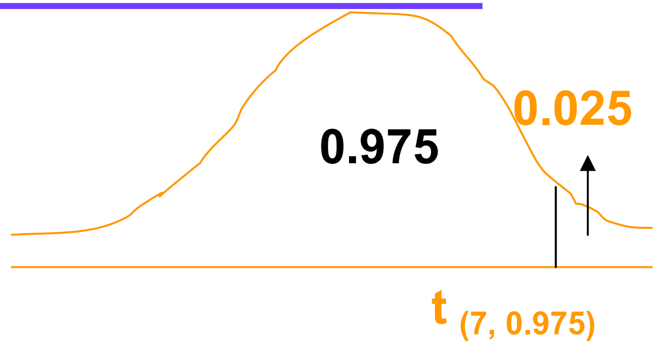
4- compared to the normal distribution, the t distribution is less peaked in the center and has higher tails.

5- It depends on the degrees of freedom (n-1).

6- The t distribution approaches the standard normal distribution as (n-1) approaches $\infty$.

# Examples

$t\ (7,\ 0.975) = 2.3646$

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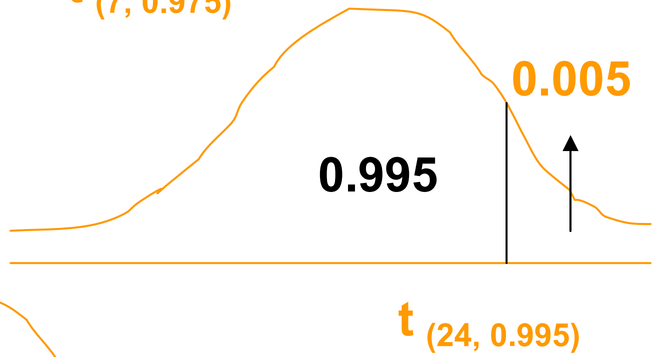
$t\ (24,\ 0.995) = 2.7696$

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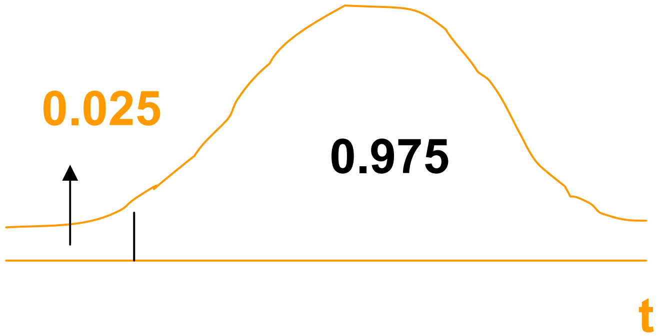
If $P\ (T_{(18)} > t) = 0.975$,

then $t = -2.1009$

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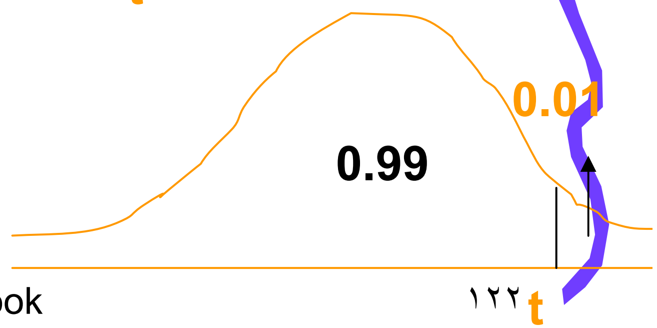
If $P\ (T_{(22)} < t) = 0.99$,

then $t = 2.508$




- <u>Exercise:</u>

- <u>Questions</u> : 4.7.1, 4.7.2
- H.W : 4.7.3, 4.7.4, 4.7.6