

Assignment 3

COURSE: THEORY OF STATISTICS I
Stat 520

Solve the following questions:

[1] Let $f(X_1, X_2) = 4x_1x_2$, $0 < X_1 < 1$, $0 < X_2 < 1$, zero elsewhere, be the *pdf* of X_1, X_2 . Find

1. $P_r [0 < X_1 < \frac{1}{2}, \frac{1}{4} < X_2 < 1]$,
 2. $P_r [X_1 = X_2]$,
 3. $P_r [X_1 < X_2]$.
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[2] Let $F(X, Y)$ be the distribution function of X and Y . Show that

$$P_r (a < X < b, c < Y < d) = F(b, d) - F(b - c) - F(a, d) + F(a, c),$$

for all real constants $a < b, c < d$.

[3] Let $f(x, y) = e^{-x-y}$, $0 < x < \infty$, $0 < y < \infty$, zero elsewhere, be the *pdf* of x and y . If $z = x + y$.

compute

1. $P_r (z \leq 0)$,
 2. $P_r (z \leq 6)$.
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[4] If X and Y have the *pdf* $f(X, Y) = \frac{1}{3}$, $(x, y) = (0, 0), (0, 1), (1, 1)$, zero elsewhere, find

$$E \left[\left(X - \frac{1}{3} \right) \left(Y - \frac{2}{3} \right) \right].$$

[5] Let the *pdf* of X and Y be $f(X, Y) = e^{-(x+y)}$; $0 < x < \infty, 0 < y < \infty$, zero elsewhere. Let

$$u(x, y) = X, v(x, y) = Y,$$

and

$$w(x, y) = XY.$$

Show that

$$E[u(x, y)] \cdot E[v(x, y)] = E[w(x, y)].$$

[6] Let $f(X_1, X_2) = 2X_1$, $0 < x_1 < 1; 0 < x_2 < 1$, zero elsewhere, be the *pdf* of X_1 , and X_2 . Find the following:

- (i) $E[x_1 + X_2]$.
 - (ii) $E[(x_1 + X_2 - E[x_1 + X_2])^2]$.
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[7] Let X_1 and X_2 have the joint *pdf* $f(X_1, X_2) = X_1 + X_2$; $0 < x_1 < 1; 0 < x_2 < 1$, zero elsewhere. Find the conditional mean and variance of X_2 .

[8] Let $f(x_1 | x_2) = c_1 x_1 / x_2^2$, $0 < x_1 < x_2$; $0 < x_2 < 1$, zero elsewhere and $f_2(x_2) = c_2 x_2^4$, $0 < x_2 < 1$, zero elsewhere, denotes, respectively, the conditional *pdf* of x_1 given $X_2 = x_2$, and the marginal *pdf* of X_2 . Find

- (i). The constants c_1 and c_2 .
 - (ii). The joint *pdf* of X_1 and X_2 .
 - (iii). $P_r[\frac{1}{4} < X_1 < \frac{1}{2} | X_2 = \frac{5}{8}]$.
 - (iv). $P_r[\frac{1}{4} < X_1 < \frac{1}{2}]$.
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[9] Let X_1 and X_2 have the joint *pdf* $f(X_1, X_2) = X_1 + X_2$; $0 < x_1 < 1; 0 < x_2 < 1$, zero elsewhere. Find the conditional mean and variance of X_2 given $X_1 = x_1$; $0 < x_1 < 1$.

[10] Let $f(x, y) = 2$; $0 < x < y$; $0 < y < 1$, zero elsewhere, be the joint *pdf* of X and Y . Find

- (i) Conditional mean of X given Y .
 - (ii) Conditional mean Y given X .
 - (iii). Correlation coefficient of X and Y .
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[11] Let X_1 and X_2 have the joint *pdf* $f(X_1, X_2) = e^{-y}$; $0 < x < y < \infty$, zero elsewhere. Find moment generation function of joint distribution.

[12] Let X and Y have a bivariate normal distribution. Prove the covariance of X and Y is $\rho\sigma_1\sigma_2$, where ρ is Correlation coefficient of X and Y .

[13]. Let X and Y have a bivariate normal distribution with parameter $\mu_1 = 20$, $\mu_2 = 40$, $\sigma_1^2 = 9$, σ_2^2 , and $\rho = 0.6$. Find the shortest interval for which 0.90 is the conditional probability that Y is in this interval, given that $x = 22$.

Good Luck !
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