Assignment 3

COURSE: THEORY OF STATISTICS I
Stat 520

Solve the following questions:

[1] Let $f(x_1, x_2) = 4x_1x_2$, $0 < x_1 < 1$, $0 < x_2 < 1$, zero elsewhere, be the *pdf* of X_1, X_2. Find

1. $P_r\left[0 < X_1 < \frac{1}{2},\ \frac{1}{4} < X_2 < 1\right]$,
2. $P_r[X_1 = X_2]$,
3. $P_r[X_1 < X_2]$.

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[2] Let $F(X, Y)$ be the distribution function of X and Y. Show that

$$P_r(a < X < b, c < Y < d) = F(b, d) - F(b - c) - F(a, d) + F(a, c),$$

for all real constants $a < b, c < d$.

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[3] Let $f(x, y) = e^{-x-y}$, $0 < x < \infty$, $0 < y < \infty$, zero elsewhere, be the *pdf* of X and Y. If $Z = X + Y$. compute

(i) $P_r(Z \leq 0)$,

(ii) $P_r(Z \leq 6)$.

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[4] If X and Y have the *pdf* $f(x, x) = \frac{1}{3}$, $(x, y) = (0, 0), (0, 1), (1, 1)$, zero elsewhere, find

$$E\left[\left(X - \frac{1}{3}\right)\left(Y - \frac{2}{3}\right)\right].$$

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[5] Let the *pdf* of X and Y be $f(x, y) = e^{-(x+y)}$; $0 < x < \infty, 0 < y < \infty$, zero elsewhere. Let

$$u(X, Y) = X, \quad v(X, Y) = Y,$$

and

$$w(X,Y) = XY.$$

Show that

$$E[u(X,Y)].E[v(X,Y)] = E[w(X,Y)].$$

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[6] Let $f(x_1, x_2) = 2X_{\ 1},\ \ 0 < x_1 < 1;\ 0 < x_2 < 1$, zero elsewhere, be the *pdf* of X_1, and X_2. Find the following:

(i) $E[X_1 + X_2]$.

(ii) $E\left[(X_1 + X_2 - E[X_1 + X_2])^2\right]$.

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[7] Let X_1 and X_2 have the joint *pdf* $f(x_1, x_2) = x_1 + x_2;\ 0 < x_1 < 1; 0 < x_2 < 1$, zero elsewhere. Find the conditional mean and variance of X_2.

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[8] Let $f(x_1 \mid x_2) = c_1 x_1 / x_2^2,\ 0 < x_1 < x_2;\ \ 0 < x_2 < 1$, zero elsewhere and $f_2(x_2) = c_2 x_2^4,\ 0 < x_2 < 1$, zero elsewhere, denotes, respectively, the conditional *pdf* of X_1 given $X_2 = x_2$, and the marginal *pdf* of X_2. Find

(i) The constants c_1 and c_2.

(ii) The joint *pdf* of $X_1$and X_2.

(iii) $P_r\left[\frac{1}{4} < X_1 < \frac{1}{2} \mid X_2 = \frac{5}{8}\right]$.

(iv) $P_r\left[\frac{1}{4} < X_1 < \frac{1}{2}\right]$.

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[9] Let X_1 and X_2 have the joint *pdf* $f(x_1, x_2) = x_1 + x_2;\ 0 < x_1 < 1; 0 < x_2 < 1$, zero elsewhere. Find the conditional mean and variance of X_2 given $X_1 = x_1$; $0 < x_1 < 1$.

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[10] Let $f(x, y) = 2;\ 0 < x < y;\ 0 < y < 1$, zero elsewhere, be the joint *pdf* of X and Y. Find

(i) Conditional mean of X given Y.

(ii) Conditional mean Y given X.

(iii). Correlation coefficient of X and Y.

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[11] Let X_1 and X_2 have the joint *pdf* $f(x_1, x_2) = e^{-y};\ \ 0 < x < y < \infty$, zero elsewhere. Find moment generation function of joint distribution.

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[12] Let X and Y have a bivariate normal distribution. Prove the covariance of X and Y is $\rho\sigma_1\sigma_2$, where ρ is Correlation coefficient of X and Y.

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[13]. Let X and Y have a bivariate normal distribution with parameter $\mu_1 = 20$, $\mu_2 = 40$, $\sigma_1^2 = 9$, σ_2^2, and $\rho = 0.6$. Find the shortest interval for which 0.90 is the conditional probability that Y is in this interval, given that $x = 22$.

G*ood Luck* !
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