Assignment 2

COURSE: THEORY OF STATISTICS I
Stat 520

Solve the following questions:

[1] Let $\overline{X}$ denote the mean of a random sample of size 15 from a population with probability density function

$$f\left(x\right) = 3x^2, \quad 0 < x < 1.$$

Find approximately value of probability $P(\frac{3}{5} < \overline{X} < \frac{4}{5})$.

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[2] Prove that $\Phi_X(t) = \sum_{r=0}^{\infty} \frac{(it)^r}{r!}\mu_r$, and hence show that

$$\mu_r = (-1)^r \left[\frac{\partial^r}{\partial t^r}\Phi_X(t)\right]_{t=0}.$$

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[3] Show that the distribution for which the characteristic function $e^{-|t|}$ has the density function

$$f(x) = \frac{1}{\pi}\frac{dx}{(1+x^2)}.$$

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[4] Show that the distribution $df = \frac{1}{\pi}\frac{(1-\cos x)}{x^2}dx$,$-\infty < x < \infty$ has

$$\Phi_X(t) = \begin{cases} 1-|t| & |t| \preceq 1 \\ 0 & |t| > 1 \end{cases}$$

Note: $\left(\int_0^{\infty} \frac{1-\cos x}{x^2}dx = \frac{\pi}{2}\right)$.

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[5] Find the density of the random variable X for which $\Phi_X\left(t\right) = e^{\frac{-t^2}{2}}$.

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[6] Let a random variable X has the density function $f_X\left(x\right) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}$. Find the following:

(i) the characteristic function $\Phi_X\left(t\right)$.

(ii) the characteristic function of the distribution for which moments about the origin are given by $\mu_r = \frac{\Gamma(v+r)}{\Gamma(v)}$.

Good Luck !