

# Assignment 1

COURSE "THEORY OF STATISTICS I"

Stat 520

M. Sc. Program

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**Solve the Following Problems:**

- (1) Let  $Y_1 = \frac{1}{2}(X_1 - X_2)$  where  $X_1$  and  $X_2$  are stochastically independent random variables each being  $\chi_{(2)}^2$ . Find the probability distribution function of  $Y_1$ .
- (2) Let  $X_i$  have a Gamma density with parameter  $n_i$  and  $\lambda$  for  $i = 1, 2$ . Let  $X_1$  and  $X_2$  are independent. If  $Y_1 = \frac{X_1}{X_1 + X_2}$  find the joint probability distribution function of  $Y_1$ .
- (3) Let  $X_1$  and  $X_2$  are two independent standard normal random variables. Let  $Y_1 = X_1 + X_2$  and  $Y_2 = \frac{X_1}{X_2}$ . Find the joint probability distribution function of  $Y_1$  and  $Y_2$  and also the marginal of  $Y_2$ .
- (4) Let  $X_i$  have a Gamma density with parameter  $n_i$  and  $\lambda$  for  $i = 1, 2$ . Assume that  $X_1$  and  $X_2$  are independent. Let  $Y_1 = X_1 + X_2$  and  $Y_2 = \frac{X_1}{X_2}$ . Find the joint probability distribution function of  $Y_1$  and  $Y_2$ .
- (5) Let  $X$  have a Beta distribution in  $[0, 1]$ . If  $Y = \frac{X}{1-X}$  find the probability distribution function of  $Y$ .
- (6) Let  $X_1$  and  $X_2$  be independent normal variables with zero mean and unit variance obtain the density of  $Y = \frac{X_1}{X_2}$ .
- (7) If  $X$  and  $Y$  two independent random variables with  $f(x, y) = 2e^{-(x+y)}$ ,  $0 < x < y$  and  $0 < y < \infty$ . Find the joint probability distribution function of  $X$ ,  $X + Y$  and the marginal of  $X$  and the marginal of  $X + Y$ .