King Saud University
College of Science
Department of mathematic May -27 -2008

MID-TERM EXAMINATION
Math-687

Exercise 1.

1- Let $\Sigma = \{('z, z_n) \in \mathbb{C}^n : 2Re(z_n) + |'z|^2 < 0\}$. Verify that Σ is biholomorphic to $\mathbb{B}_n$, the unit ball in $\mathbb{C}^n$.

Prove that Σ is not bounded and the analytic automorphism group of Σ is non-compact. Deduce that $Aut(\mathbb{B}_n)$ is non-compact.

2- State the Wong-Rosay theorem. Describe the outline of the proof of this theorem.

3- a) Let D be a strongly pseudoconvex domain in $\mathbb{C}^n$, $B_\infty = \{z \in \mathbb{C}^n : |z|^2 + |z^2| < 1\}$ and $f : D \to B_\infty$ be a proper holomorphic mapping with branch locus equal to V. Use scaling technique to explain briefly that if $\{p_k\}_k$ is a sequence in V converging to a boundary point, then (after taking a subsequence) $\{f(p_k)\}_k$ converges to a boundary point in $\{z \in \partial B_\infty : z^2 = 0\}$.

b) Deduce (by the maximum principle) that $f(V) \subset \mathbb{H}$ where

$$\mathbb{H} = \{z \in B_\infty : z^2 = 0\}.$$

c) Prove that $f(V) = \mathbb{H}$ for $n \geq 3$.

Exercise 2.

1- Let Ω be a connected complex manifold and $A \subset \Omega$ be an analytic subset. Prove that if A contains a nonempty subset in Ω, then $A = \Omega$.

2- Define the regular part and the singular part of an analytic set. Prove that if A is a pure $n-1$-dimensional analytic subset of an n-dimensional complex manifold Ω, then the singular part sng(A) is also an analytic subset in Ω of dimension $\leq n-2$.

Exercise 3.

1) State the definition of a local holomorphic peak function at a boundary point of a domain in $\mathbb{C}^n$.

2) Let $f_n : D \to \Omega$ be a sequence of holomorphic functions between bounded domains in $\mathbb{C}^n$. Assume that $\{f_n(p)\}$ converges at some point $p \in D$ to a boundary point $q \in \partial\Omega$ and assume that there exists a local holomorphic peak function at q. Prove that the sequence $\{f_n\}$ converges uniformly on compact subsets of D to q.