King Saud University
College of Sciences
Department of Mathematics

Semester I, 1427-1428
Time: 2 hours

Mid-Term Examination
Math 585

Exercise 1.

Evaluate the following integral after justifying its convergence

$$\int_0^\infty \frac{Lnx}{1+x^3}dx \ .$$

Exercise 2.

We denote by Δ the unit disc in $\mathbb{C}$. Let f be a holomorphic function in Δ and continuous in $\overline{\Delta}$. Assume that there exists α, $0 < \alpha < 2\pi$ such that $f(e^{it}) = 0$, for all $t \in [0, \alpha]$

1) a) State the principle of Schwarz symmetrie.

b) We consider the function $h(z) = i(\frac{1+z}{1-z})$. Prove that h is holomorphic in Δ and caracterise $\Omega = h(\Delta)$.

c) Prove that $f \circ h^{-1}$ is holomorphic in Ω and continuous in $\overline{\Omega}$.

d) Use a) to prove that $f \circ h^{-1} \equiv 0$. Deduce that $f \equiv 0$.

2) Let $n \in \mathbb{N}$ such that $n\alpha > 2\pi$. Consider the function

$$F(z) = f(z)f(ze^{i\alpha})...f(ze^{in\alpha})$$

Prove that $F \equiv 0$ and deduce again that $f \equiv 0$

Exercise 3.

Let $a, b \in D(0,1)$ and $p, q \in \mathbb{N}^*$.

Prove that the function

$$f(z) = z^p(\frac{z-a}{1-\bar{a}z})^q - b$$

has exactly $p + q$ roots in $D(0,1)$.