KING SAUD UNIVERSITY
College of Sciences
Department of Mathematics

Mid-Term examination/ Semester I/ 1428/1429

Math 683, Time: 2 hours

Exercise 1.

1) Use the Cauchy formula to prove that any holomorphic function f on $\Delta^2(0,1)\backslash\Delta^2(0,\frac{1}{2})$ extends holomorphically to $\Delta^2(0,1)$.

2) Let $P = \{z \in \mathbb{C}^n : |z_j| < 1\}$, $n > 1$. Let V be a neighborhood of ∂P such that $V \cap P$ is connected. Prove that any holomorphic function f on V extends holomorphically to $P \cup V$.

Exercise 2.

1) Let $\Omega \subset \mathbb{C}^n$ an open set and $K \subset\subset \Omega$. State the definition of the psh hull of K, the holomorphic hull of K and the convex hull of K. Prove that $\hat{K}^{\text{psh}} \subset \hat{K}_{\mathcal{O}(\Omega)} \subset \text{conv}(K)$.

2) Prove that Ω is convex iff $\forall\ K \subset\subset K$, $\text{conv}(K) \subset\subset \Omega$.

3) State three equivalent definitions of pseudoconvex domain. Prove that any convex domain is pseudoconvex.

4) Let $f : \Omega \to D$ be a biholomophic map between domains in $\mathbb{C}^n$. Prove that Ω is pseudoconvex iff D is pseudonvex.

5) Let $D = \{(z, w) \in \mathbb{C} \times \mathbb{C}^n : \mathcal{R}e(z) + P(w) < 0\}$, where P is a real analytic function in $\mathbb{C}^n$. Prove that D is pseudoconvex iff P is plurisubharmonic in $\mathbb{C}^n$.

Exercise 3.

1) State the definition of a strongly pseudoconvex domain and verify that the unit ball in $\mathbb{C}^n$ is strongly pseudoconvex.

2) Let Ω be a strongly pseudoconvex domain in $\mathbb{C}^n$ and $\varphi : \Delta(0,1) \to \partial\Omega$ be an analytic disc. Prove that φ is constant.

3) Let Let $f : \Omega \to D$ be a proper holomorphic map between domains in $\mathbb{C}^n$. Prove that Ω is strongly pseudoconvex iff D is strongly pseudonvex.