

KING SAUD UNIVERSITY
College of Sciences
Department of Mathematics

Final examination/ Summer semester / 1427/1428

Math 203, Time: 3 hours

• **Question 1.** [Marks: 3+4+5]

a) Check the convergence or the divergence of the series : $\sum_{n=0}^{\infty} [\frac{1}{2^n} + (-\frac{3}{4})^n]$.
Find the sum if it is convergent.

b) Determine whether the series $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\ln n}$ is absolutely convergent, conditionally convergent or divergent.

c) Find the interval of convergence and the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n3^n} (x-1)^n$.

• **Question 2.** [Marks: 4+4+4]

a) Find the Maclaurin series of the function $\sin x$ and use its first three non-zero terms to approximate the improper integral $\int_{-1}^1 \frac{\sin x}{x} dx$.

b) Evaluate the integral $\int_1^e \int_{\frac{1}{e}}^{\frac{1}{y}} \cos(x - \ln x) dx dy$.

c) Find the surface area of the solid bounded above by the surface $z = 9 - x^2 - y^2$ and below by the xy-plane.

• **Question 3.** [Marks: 4+4+4]

a) A solid is bounded by the paraboloid $z = x^2 + y^2$, the cylinder $x^2 + y^2 = 4$ and the xy-plane. Find its centroid.

b) Using spherical coordinates, evaluate the integral

$$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} z^2 \sqrt{x^2 + y^2 + z^2} dz dy dx.$$

c) Show that the following line integral is independent of path, and find its value :

$$\int_{(-4,3)}^{(5,2)} (y^2 + 2xy) dx + (x^2 + 2xy) dy.$$

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• **Question 4.** [Marks: 4+5+5]

a) Use Green's theorem to evaluate the line integral $\oint_C y^3 dx + (x^3 + 3xy^2) dy$, where C is the path from $(0, 0)$ to $(1, 1)$ along the graph of $y = x^3$ and from $(1, 1)$ to $(0, 0)$ along the graph of $y = x$.

b) Use the divergence theorem to find $\iiint_S \vec{F} \cdot \vec{n} \, ds$ if $\vec{F}(x, y, z) = (x^2 + \sin z) \vec{i} + (xy + \cos z) \vec{j} + e^y \vec{k}$, S is the surface of the region bounded by the cylinder $x^2 + y^2 = 4$, the plane $x + z = 6$ and the xy -plane.

c) Use Stokes's theorem to evaluate $\oint_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = 2z \vec{i} + x \vec{j} + y^2 \vec{k}$ and S is the surface of the paraboloid $z = 4 - x^2 - y^2$ and C is the trace of S in the xy -plane.

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