

Some Aging Classes of Life Distributions at Specific Age

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Abstract

We introduce some properties of the new better than used in convex ordering at age t_0 ($NBUC - t_0$) and new better than used of second order(2) at age t_0 ($NBU(2)-t_0$) classes of life distributions, where the survival probability at age 0 is greater than or equal to the conditional survival probability at specified age $t_0 > 0$. Preservation properties of the two classes under various reliability operations and shock model are arriving according to homogeneous Poisson process are established.

Keywords: $NBUC - t_0$; $NBU(2)-t_0$; preservation properties; shock model; parallel system

1 Introduction

The notion of aging for engineering systems has been characterized by several classes of life distributions in reliability various classes of life distributions have been introduced to describe several types of deterioration (improvement) that accompany aging. In performing reliability analysis, it has been found very useful to classify life distributions using the concept of stochastic ordering. For definitions of several classes of distributions, e.g., IFR , $IFRA$, NBU , $NBUE$, $NBUC$ and $NBU(2)$. For more detailed discussions on properties and for some possible applications, we refer to *Bryson and Siddiqui (1969)*, *Rolski (1975)*, *Barlow and Proschan (1981)*, *Deshpand et al (1986)* and *Cao and Wang (1991)*. Unfortunately these types of classes are restricted to one type of applications since they present positive aging or negative aging for their dual classes throughout their life span of the underlying components or their systems. *Hollander et al. (1986)* have introduced the new better than used at specific age at t_0 ($NBU - t_0$) and its dual ($NWU - t_0$). They have listed some types of situation where ($NBU - t_0$) aging or its dual might arise.

There are many situation in real life where the components of the system gradually deteriorate up time t_0 which is warranty guarantee time provided by most manufacturers, then maintenance through repairs or spare parts replacement take place after tomet t_0 . Here maintenance is expected to improve the performance of the system but can not bring it back to a better situation than it was at age t_0 . For some interesting examples, see *Hollander at al.*(1986).

In this paper we further develop some of the above mentioned classes. In *Section 2* we will recall some definitions and facts about characteristics of lifetime distributions and about stochastic ordering. In *Section 3* we discuss whether the $NBUC - t_0$ and $NBU(2) - t_0$ are preserved under convolution. In *Section 4*, we show that $NBUC - t_0$ class is closed under the formation of parallel sytems. Finally, $NBUC - t_0$ under shock model is discussed in *Section 5*.

2 Definitions and relationships

In reliability theory, aging life is usually characterized by a nonnegative random variable $X \geq 0$ with cumulative distribution function (cdf) F and survival function (sf) $\bar{F} = 1 - F$. For any random variable X , let

$$X_t = [X - t | X > t], \quad t \in \{x : F(x) < 1\},$$

denote a random variable whose distribution is the same as the conditional distribution of $X - t$ given that $X > t$. When X is the lifetime of a device, X_t can be regarded as the residual lifetime of the device at time t , given that the device has survived up to time t . Its survival function is (see, for instance, *Deshpand et al.* (1986))

$$\bar{F}_t(x) = \frac{\bar{F}(t+x)}{\bar{F}(t)}, \quad \bar{F}(t) > 0,$$

where $\bar{F}(x)$ is the survival function of X . It is well-known fact that when \bar{F} is an exponential distribution then $X_t \stackrel{st}{=} X$ or $\bar{F}_t(x) = \bar{F}(x)$. Comparing X and X_t in various forms and types create classes of aging useful in many biomedical, engineering and statistical studies, cf. *Barlow and Proschan* (1981). It is well-known that the relation $X_t \stackrel{st}{\leq} X$ or $\bar{F}_t(x) \stackrel{st}{\leq} \bar{F}(x)$ defines the class of new better than used (NBU).

Definition 2.1.

A non-negative random variable X , or it is distribution F , is said to be:

(i) new better than used (denoted by $X \in NBU$) if

$$\bar{F}(t+y) \leq \bar{F}(t) \bar{F}(y), \quad \text{for all } t \geq 0 ;$$

(ii) new better than used in the convex order (denoted by $X \in NBUC$) if

$$\int_x^\infty \bar{F}(t+y) dy \leq \bar{F}(t) \int_x^\infty \bar{F}(y) dy \quad \text{for all } x, t \geq 0.$$

(iii) new better than used of second order(2) (denoted by $X \in NBU(2)$) if

$$\int_0^y \bar{F}(t+x) dx \leq \bar{F}(t) \int_0^y \bar{F}(x) dx$$

We have the following chain of implications of the above mentioned classes as follows:

$$NBUC \Leftarrow NBU \Rightarrow NBU(2)$$

Hollander et al (1986) introduced the larger class of life distributions called new better than used of age t_0 defined as follows: A non-negative random variable X is said to be new better than used of age t_0 if

$$\bar{F}(t_0+x) \leq \bar{F}(t_0)\bar{F}(x), \quad \text{for all } x \geq 0.$$

Thus the NBU property states that a used item of any age has stochastically smaller residual life length than does a new item, whereas the $NBU - t_0$ property states that a used item of age t_0 has stochastically smaller residual life length than does a new item.

According to the above definition we introduce the concept of new better than used in convex ordering and new better than used of second order (2) classes of age t_0 as follows.

Definition 2.2.

A life distribution F is said to be

(i) new better than used in convex ordering of age t_0 ($NBUC - t_0$) if

$$\int_x^\infty \bar{F}(y+t_0) dy \leq \bar{F}(t_0) \int_x^\infty \bar{F}(y) dy \quad \text{for all } x \geq 0, t_0 > 0.$$

(ii) new better than used of second order(2) of age t_0 ($NBU(2) - t_0$) if

$$\int_0^x \bar{F}(y+t_0) dy \leq \bar{F}(t_0) \int_0^x \bar{F}(y) dy, \quad \text{for all } x \geq 0.$$

Remark: It is seen from the above results that the $NBU(2) - t_0$ class is a middle class between the $NBU - t_0$ and $NBU E - t_0$ classes. This

of course would prove useful in applications since it is less restrictive than $NBU - t_0$ and easier to verify in practice than $NBU E - t_0$. Note that the $NBU - t_0$ notion of aging compares a new unit with used units of all possible age in specific age t_0 . One might look at the $NBU(2) - t_0$ property as comparing the average performance of corresponding used units in specific age t_0 which is more appealing in practice.

3 Closure properties

As an important reliability operation, convolution of life distributions of certain class is often paid much attention. It has been shown that both the $NBUC - t_0$ and $NBU(2) - t_0$ classes are closed under this operation. In the next theorems we established the closure property of the $NBUC - t_0$ and $NBU(2) - t_0$ under the convolution operation.

Theorem(3.1). Suppose that F_1 and F_2 are two independent $NBUC - t_0$ life distributions. Then their convolution is also

$NBUC - t_0$.

Proof: The survival function of the convolution of two life distributions F_1 and F_2 is

$$\bar{F}(z) = \int_0^\infty \bar{F}_1(z-u) dF_2(u) \quad \text{for all } z \geq 0.$$

For any fixed $x \geq 0$,

$$\begin{aligned} \int_x^\infty \bar{F}(y+t_0) dy &= \int_0^\infty \int_0^\infty \bar{F}_1(x+t_0+y-u) dF_2(u) dy \\ &= \int_0^\infty \int_0^x \bar{F}_1(x+t_0+y-u) dF_2(u) dy + \int_0^\infty \int_x^\infty \bar{F}_1(x+t_0+y-u) dF_2(u) dy \\ &= \int_0^x \left[\int_0^\infty \bar{F}_1(x+t_0+y-u) dy \right] dF_2(u) \\ &\quad + \int_0^\infty \left[\int_0^\infty \bar{F}_1(t_0+y-w) d_w F_2(x+w) \right] dy \\ &= A_1 + A_2 \end{aligned}$$

Not that $\bar{F}(t_0) \geq \bar{F}_i(t_0)$ and $i = 1, 2$. It follows from the $NBUC - t_0$ property of F_1 that

$$\begin{aligned} A_1 &\leq \overline{F}_1(t_0) \int_0^x \left[\int_0^\infty \overline{F}_1(x+y-u) dy F_2(u) \right] dF_2(u) \\ &\leq \overline{F}(t_0) \int_0^\infty \int_0^x \overline{F}_1(x+y-u) dF_2(u) dy. \end{aligned}$$

Integrating by parts in the inner integral of A_2 we have

$$\begin{aligned} A_2 &= \int_0^\infty \left[\overline{F}_1(t_0+y)\overline{F}_2(x) + \int_0^\infty \overline{F}_2(x+w) d_w \overline{F}_1(y+t_0-w) \right] dy \\ &= \overline{F}_2(x) \int_0^\infty \overline{F}_1(t_0+y) dy + \int_0^\infty \left[\int_0^{y+t_0} \overline{F}_2(x+w) d_w \overline{F}_1(y+t_0-w) \right] dy \\ &= \phi_1 + \int_0^\infty \left[\int_0^{y+t_0} \overline{F}_2(x+t_0+y-u) dF_1(u) \right] dy \\ &= \phi_1 + \int_{t_0}^\infty \left[\int_{u-t_0}^\infty \overline{F}_2(x+t_0+y-u) dy \right] dF_1(u) \\ &\quad + \int_0^{t_0} \left[\int_0^\infty \overline{F}_2(x+t_0+y-u) dy \right] dF_1(u) \\ &= \phi_1 + \phi_2 + \phi_3 \end{aligned} \tag{3.1}$$

where

$$\phi_1 \leq \mu_1 \overline{F}_2(x)\overline{F}_1(t_0) \leq \mu_1 \overline{F}_2(x)\overline{F}(t_0) \tag{3.2}$$

$$\phi_2 = \overline{F}_1(t_0) \int_0^\infty \overline{F}_2(x+u) du \tag{3.3}$$

$$\begin{aligned} \phi_3 &\leq \int_0^{t_0} \left[\overline{F}_2(t_0-u) \int_0^\infty \overline{F}_2(x+y) dy \right] dF_1(u) \\ &= \int_0^\infty \overline{F}_2(x+y) dy \int_0^{t_0} \overline{F}_2(t_0-u) dF_1(u) \\ &= [\overline{F}(t_0) - \overline{F}_1(t_0)] \int_0^\infty \overline{F}_2(x+u) du. \end{aligned} \tag{3.4}$$

by the *NBUC* - t_0 property of F_1, F_2 and $\mu_1 = \int_0^\infty \overline{F}_1(u) du$. then from (3.1) - (3.4)

we get

$$A_2 \leq \bar{F}(t_0) \left[\mu_1 \bar{F}_2(x) + \int_0^\infty \bar{F}_2(x + u) du \right].$$

On the other hand

$$\begin{aligned} & \bar{F}(t_0) \int_x^\infty \bar{F}(y) dy \\ = & \bar{F}(t_0) \left[\int_0^\infty \int_0^x \bar{F}_1(x + y - u) dF_2(u) dy + \int_0^\infty \bar{F}_2(x + u) du + \mu_1 \bar{F}_2(x) \right] \\ & \geq A_1 + A_2 = \int_x^\infty \bar{F}(y + t_0) dy. \end{aligned}$$

This proves that F is $NBUC - t_0$.

Theorem(3.2). Suppose that F_1 and F_2 are two independent $NBU(2) - t_0$ life distributions. Then their convolution is also $NBU(2) - t_0$.

Proof: The survival function of the convolution of two life distributions F_1 and F_2 is

$$\bar{F}(y) = \int_0^\infty \bar{F}_1(y - u) dF_2(u) du$$

By Fubini's theorem and the $NBU(2) - t_0$ property, we have

$$\begin{aligned} \int_0^x \bar{F}(y + t_0) dy &= \int_0^x \int_0^\infty \bar{F}(y + t_0 - u) dF_2(u) dy \\ &= \int_0^\infty \int_0^x \bar{F}_1(y + t_0 - u) dy dF_2(u) \\ &\leq \int_0^\infty \bar{F}_1(t_0) \int_0^x \bar{F}_1(y - u) dy dF_2(u) \\ &= \bar{F}_1(t_0) \int_0^x \int_0^\infty \bar{F}_1(y - u) dF_2(u) dy \\ &\leq \bar{F}(t_0) \int_0^x \bar{F}_1(y) dy, \end{aligned} \tag{3.5}$$

the last inequality following from the fact that the convolution of two independent nonnegative random variables is stochastically larger than each of them. This proves that F is also $NBU(2) - t_0$.

4 Closure of the NBUC- t_0 class under formation of parallel system.

Hendi et al (1993) have shown that the NBUC class is closed under formation of parallel system of i.i.d.components. Here it may be of interest to prove the closure of the NBUC - t_0 class under formation of parallel systems in the following theorem.

Theorem(4.1). Let X_1, X_2, \dots, X_n be i.i.d. random variables with distribution F and $F \in NBUC - t_0$. Then the random variable $X_n = \max(X_1, X_2, \dots, X_n)$ has distribution $F_n \in NBUC - t_0$.

Proof:

The survival function of X_n is $\overline{F}_{(n)}(t_0) = P(X_n > t_0) = 1 - F^n(t_0)$. We need to show that

$$\int_x^\infty \overline{F}_{(n)}(t_0 + y) dy \leq \overline{F}_{(n)}(t_0) \int_x^\infty \overline{F}_{(n)}(y) dy, \quad x \geq 0 \quad (4.1)$$

the inequality(4.1) obviously holds whenever $\overline{F}_{(n)}(t_0) = 0$. For $\overline{F}_{(n)}(t_0) \neq 0$, the characterization result given by Definition(2.2-(ii)) may be reformulated as : $F \in NBUC - t_0$ iff

$$\frac{F(t_0)}{1 - F(t_0)} \int_{x+t_0}^\infty \overline{F}(u) du. \leq \int_x^{x+t_0} [1 - F(u)] du \quad (4.2)$$

the right-hand side of (4.2) satisfies

$$\int_x^{x+t_0} \overline{F}(u) du \leq \int_x^{x+t_0} \overline{F}_{(n)}(u) du \quad (4.3)$$

Also the left-hand side of (4.2) satisfies

$$\int_{x+t_0}^\infty \frac{F(t_0)}{1 - F(t_0)} \overline{F}(u) du \geq \int_{x+t_0}^\infty \frac{F^n(t_0)}{1 - F^n(t_0)} [1 - F^n(u)] du. \quad (4.4)$$

The last inequality(4.4) holds because

$$\begin{aligned}
& \int_{x+t_0}^{\infty} \left\{ \frac{F(t_0)}{1-F(t_0)} [1-F(u)] - \frac{F^n(t_0)}{1-F^n(t_0)} [1-F^n(u)] \right\} du \\
= & \int_{x+t_0}^{\infty} \frac{F(t_0)}{1-F(t_0)} [1-F(u)] \left\{ 1 - F^{n-1}(t_0) \frac{[1-F(t_0)][1-F^n(u)]}{[1-F^n(t_0)][1-F(u)]} \right\} du \\
= & \int_{x+t_0}^{\infty} \frac{F(t_0)}{1-F(t_0)} [1-F(u)] \left\{ 1 - F^{n-1}(t_0) \frac{[1+F(u)+\dots+F^{n-1}(u)]}{[1+F(t_0)+\dots+F^{n-1}(t_0)]} \right\} du \\
\geq & \int_{x+t_0}^{\infty} \frac{F(t_0)}{1-F(t_0)} [1-F(u)] \left\{ 1 - \frac{F^{n-1}(t_0) [1+F_t_0^{-1}+\dots+F_t_0^{-(n-1)}]}{1+F(t_0)+\dots+F^{n-1}(t_0)} \right\} du \\
\geq & 0
\end{aligned}$$

Since $F(u) \leq F^{-1}(u) \leq F_{t_0}^{-1}$ for $u \geq t_0$, we get

$$\int_{x+t_0}^{\infty} \frac{F(t_0)}{1-F(t_0)} [1-F(u)] du \geq \int_{x+t_0}^{\infty} \frac{F_{(n)}(t_0)}{\overline{F_{(n)}}(t_0)} \overline{F_{(n)}}(u) du. \quad (4.5)$$

From (4.3) and (4.5), we have

$$\int_{x+t_0}^{\infty} \frac{F_{(n)}(t_0)}{\overline{F_{(n)}}(t_0)} \overline{F_{(n)}}(u) du \leq \int_{x+t_0}^{\infty} \overline{F_{(n)}}(u) du,$$

or

$$\int_{x+t_0}^{\infty} \left[\frac{1}{\overline{F_{(n)}}(t_0)} - 1 \right] \overline{F_{(n)}}(u) du \leq \int_{x+t_0}^{\infty} \overline{F_{(n)}}(u) du$$

or

$$\int_{x+t_0}^{\infty} \frac{\overline{F_{(n)}}(u)}{\overline{F_{(n)}}(t_0)} du \leq \int_x^{\infty} \overline{F_{(n)}}(u) du,$$

i.e.

$$\int_{x+t_0}^{\infty} \overline{F_{(n)}}(u) du \leq \overline{F_{(n)}}(t_0) \int_x^{\infty} \overline{F_{(n)}}(u) du.$$

Thus

$$\int_{x+t_0}^{\infty} \overline{F_{(n)}}(t_0 + y) du \leq \overline{F_{(n)}}(t_0) \int_x^{\infty} \overline{F_{(n)}}(y) dy.$$

This proves the required result.

4.1 Preservation of NBUC- t_0 class under Poisson Shock models.

Suppose that a device is subjected to shocks occurring randomly in time according to a Poisson process with constant intensity λ . Suppose further that the device has probability \bar{P}_k of surviving the first K shocks, where $1 = \bar{P}_0 \geq \bar{P}_1 \geq \bar{P}_2 \geq \dots$. The survival function of the device is given by

$$\bar{H}(t) = \sum_{k=0}^{\infty} e^{-\lambda t} \frac{(\lambda t)^k}{k!} \bar{P}_k \tag{5.1}$$

For the discrete distribution P_k , it is well known that properties of P_k are reflected in corresponding properties of the continuous life distribution $H(t)$. This is shown by Esary et al (1993) for *IFR, IFRA, NBU* and *NBUE* classes by Klefsjö(1981), and Abouammoh and Ahmed (1988) for *NBUC - t_0* class. We will show that the same is true for the *NBUC - t_0* class.

Definition(5.1). A discrete distribution and its survival probabilities

$$\bar{P}_k = \sum_{j=k+1}^{\infty} P_j, K = 0, 1, 2, \dots \text{ with finite mean } \mu = \sum_{k=0}^{\infty} \bar{P}_k$$

are called discrete new better than used in convex ordering at t_0 (discrete *NBUC - t_0*) if

$$\sum_{j=k}^{\infty} \bar{P}_{i+j} \leq \bar{P}_i \sum_{j=k}^{\infty} \bar{P}_j \tag{5.2}$$

Theorem(5.2) The survival function $\bar{H}(t)$ in (5.1) is *NBUC - t_0* if $\{\bar{P}_k\}_{k=0}^{\infty}$ has the discrete *NBUC - t_0* property(5.2).

Proof:

It must be shown that

$$\int_x^{\infty} \bar{H}(u + t_0) du \leq \bar{H}(t_0) \int_x^{\infty} \bar{H}(u) du.$$

Now consider, using (5.1)

$$\begin{aligned}
\int_x^\infty \bar{H}(u+t_0)du &= \int_x^\infty \sum_{J=0}^\infty \frac{[\lambda(u+t_0)]^J}{J!} e^{-\lambda(u+t_0)} \bar{p}_J du \\
&= \frac{1}{\lambda} e^{-\lambda t_0} \sum_{J=0}^\infty \bar{p}_J \sum_{m=0}^J \binom{J}{m} \frac{(\lambda t_0)^{J-m}}{j!} \int_x^\infty (\lambda u)^j e^{-\lambda u} d(u\lambda). \\
&= \frac{1}{\lambda} e^{-\lambda(x+t_0)} \sum_{m=0}^\infty \sum_{j=m}^\infty \sum_{k=0}^m \bar{p}_j \frac{(\lambda t_0)^{j-m}}{(j-m)!} \frac{(\lambda x)^k}{k!}.
\end{aligned}$$

by integrating by parts m times. we get

$$\begin{aligned}
\int_x^\infty \bar{H}(u+t_0)du &= \frac{1}{\lambda} e^{-\lambda(x+t_0)} \sum_{m=0}^\infty \sum_{j=m}^\infty \sum_{k=0}^m \frac{\bar{P}_{j+m}}{(j)!} \frac{(\lambda t_0)^j (\lambda x)^k}{k!} \\
&= \frac{1}{\lambda} \sum_{j=0}^\infty \frac{(\lambda t_0)^j}{(j)!} e^{-\lambda(t_0)} \cdot \sum_{k=0}^\infty \sum_{m=k}^\infty \bar{P}_{j+m} \frac{(\lambda x)^k}{k!} e^{-\lambda(x)} \\
&\leq \sum_{j=0}^\infty \frac{(\lambda t_0)^j}{(j)!} e^{-\lambda(t_0)} \cdot \bar{p}_j \cdot \sum_{k=0}^\infty \sum_{m=k}^\infty \bar{P}_m \cdot \bar{P}_m \frac{1}{\lambda} \frac{(\lambda x)^k}{k!} e^{-\lambda(x)} \\
&\quad \text{by using the } NBUC - t_0 \text{ property} \\
&= \sum_{j=0}^\infty \frac{(\lambda t_0)^j}{(j)!} e^{-\lambda(t_0)} \cdot \bar{p}_j \cdot \sum_{m=0}^\infty \bar{P}_m \cdot \sum_{k=0}^m \frac{(\lambda x)^k}{k!} \frac{1}{\lambda} e^{-\lambda(x)} \\
&= \bar{H}(t_0) \sum_{m=0}^\infty \bar{P}_m \int_x^\infty \frac{(\lambda u)^m}{m!} e^{-\lambda(u)} du \\
&\quad \text{see Barlow and Proschan page 74,(5.4)} \\
&= \bar{H}(t_0) \int_x^\infty \sum_{m=0}^\infty \frac{(\lambda u)^m}{m!} e^{-\lambda(u)} \bar{P}_m du \\
&= \bar{H}(t_0) \int_x^\infty \bar{H}(u) du.
\end{aligned}$$

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