

MA20033 - Question Sheet Eight

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Attempt questions 1-3. Hand in by 5.00pm Friday 10 December 2004 either to me, or in the envelope on my door, 1W4.8.

1. Suppose that we are going to gather a sample of twenty independent observations from a $N(\mu, \sigma^2 = 1.0)$ distribution in order to make inferences about μ . Suppose we are interested in the null hypothesis $H_0 : \mu = 10$.
 - (a) State the two-tailed alternative hypothesis and find the appropriate critical values for a test of significance 0.05.
 - i. What is the p-value of this test when we observe a sample mean $\bar{x} = 10.41$?
 - ii. Evaluate the power at $\mu = 10.5$.
 - (b) State the upper one-tailed alternative hypothesis and find the appropriate critical values for a test of significance 0.05.
 - i. What is the p-value of this test when we observe a sample mean $\bar{x} = 10.41$?
 - ii. Evaluate the power at $\mu = 10.5$.
2. The Food and Nutrition Board of the National Academy of Sciences (USA) suggests that the RDA of iron for adult females under the age of 51 is 18mg. The following iron intakes, in mg, during a 24-hour period were observed for 45 randomly selected adult females in this age group:

15.0	18.1	14.4	14.6	10.9	18.1	18.2	18.3	15.0
16.0	12.6	16.6	20.7	19.8	11.6	12.8	15.6	11.0
15.3	9.4	19.5	18.3	14.5	16.6	11.5	16.4	12.5
14.6	11.9	12.5	18.6	13.1	12.1	10.7	17.3	12.4
17.0	6.3	16.8	12.5	16.3	14.7	12.7	16.3	11.5

Under the assumption that each observation represents the realisation of a $N(\mu, \sigma^2)$ random variable, derive a random interval which contains σ^2 with probability 0.95. Evaluate a 95% confidence interval for σ^2 . Use may use the fact that $\sum_{i=1}^{45} x_i = 660.6$, $\sum_{i=1}^{45} x_i^2 = 10115.88$, $\chi_{44,0.975}^2 = 27.574$ and $\chi_{44,0.025}^2 = 64.201$.

3. Under the same assumptions as question 2, and using the same data, test the hypotheses

$$H_0 : \sigma^2 = 10 \quad \text{versus} \quad H_1 : \sigma^2 \neq 10$$

Express the p-value of this test as a probability statement involving the test statistic (the tables which you have will not give the necessary quantiles to evaluate this p-value, but you could investigate using Minitab or R to find it).